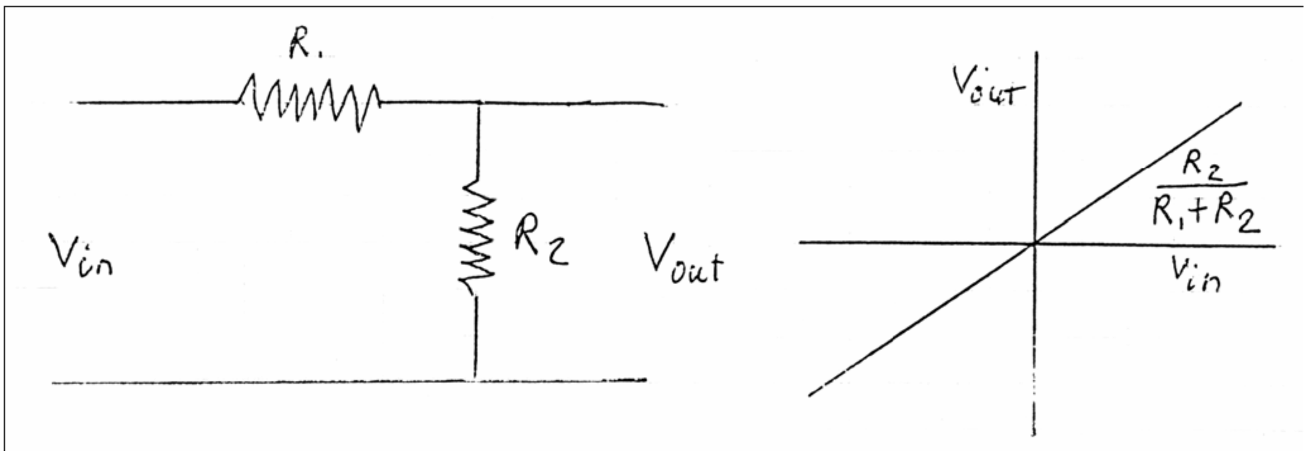
# Transfer Characteristics

- Often define circuits by their **"Transfer Characteristics"**
- Apply an input voltage to one side of a circuit
- Output voltage measured across some part of the circuit
- Transfer characteristics: Plots the output against input
- Thus that state what the output will be for any input



# Op Amp Integrator

- Recall resistor followed by a capacitor RC integrator
- If the RC time constant is long relative to period
- The resistor dominates the voltage drop and
- The voltage across the capacitor becomes the integral
- Consider an inverting op amp circuit
- But replace $R_f$ with a capacitor $C_f$
- Since summing point SP = a virtual ground.

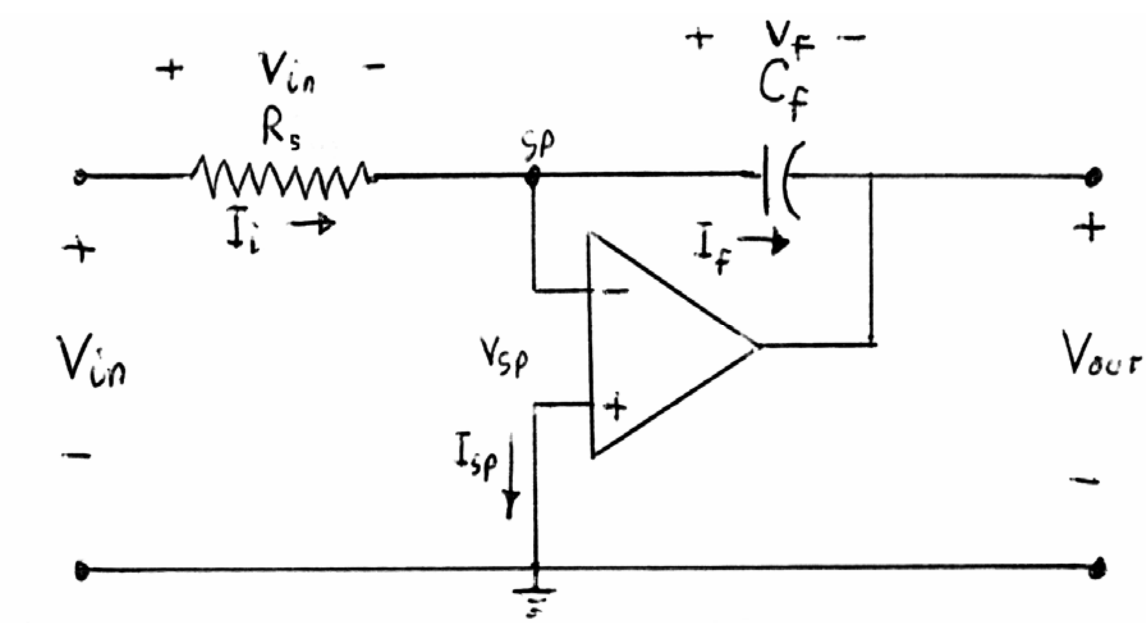
$$V_{sp} = 0 \qquad I_{sp} = 0$$

- As with the regular inverting op amp

$$I_{in} = I_s = I_i = \frac{V_{in}}{R_s}$$

- For the following capacitor then the current is

$$I_f = C_f \frac{dV_f}{dt}$$



# Op Amp Integrator Cont'd

- Since there can be no current through the op amp

$$I_s = I_f$$

$$I_f = C_f \frac{dV_f}{dt} = \frac{V_{in}}{R_s} = I_s$$

- Thus the voltage across the output capacitor is

$$V_f = \frac{1}{R_s C_f} \int V_{in} dt$$

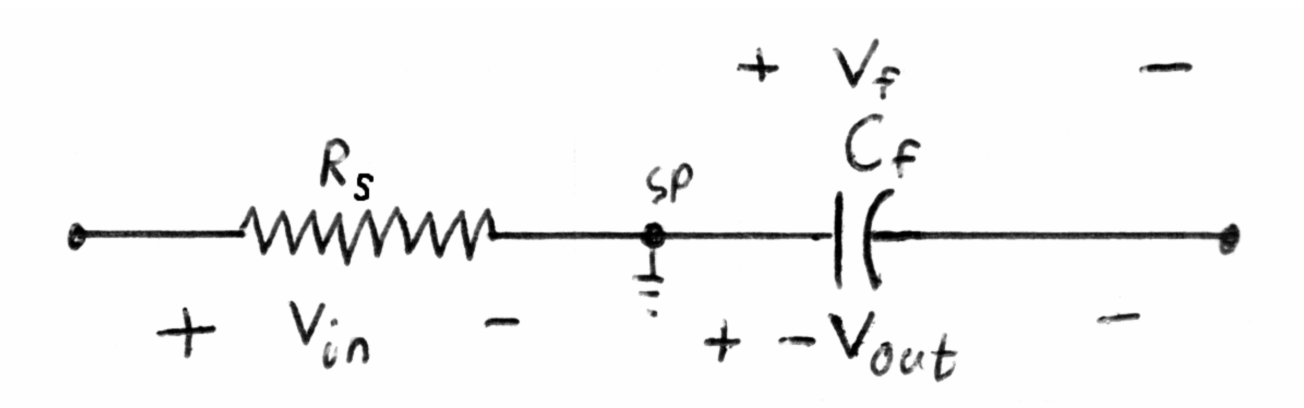
- Since

$$V_{out} = -V_f$$

- Thus the op amp output voltage is

$$V_{out} = -\frac{1}{R_s C_f} \int V_{in} dt$$

- Where $\tau = R_s C_f$ = time constant of RC circuit
- However the op amp supplies the current
- And the summing point is a ground
- Thus RC need not be longer than the input period.



# Op Amp Integrator Single Pulse Input

- Consider an op amp integrator circuit for a single square pulse
- 4 V for 10 ms duration
- What is the response?
- Assuming C is initially uncharged then

$$\tau = R_s C_f = 5000 \times 10^{-6} = 5\, m\text{sec}$$

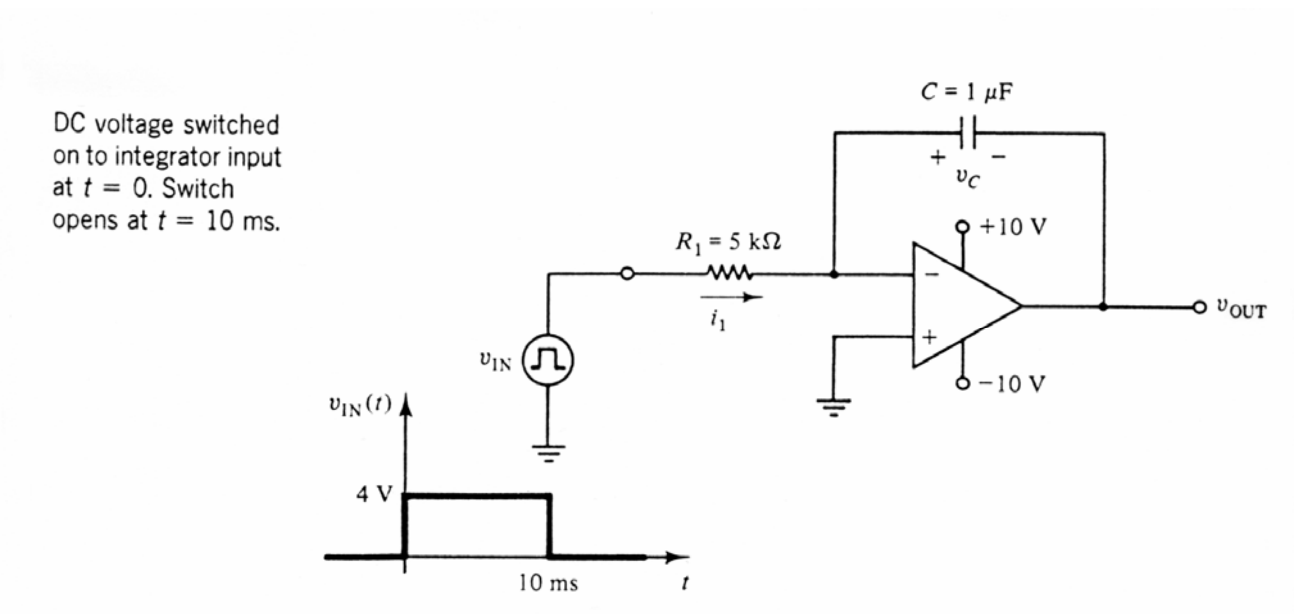
- During the pulse; t <10 msec

$$V_{out} = -\frac{1}{R_s C_f} \int V_{in} dt = -\frac{1}{0.005} \int_0^t 4\, dt = -800t\, V$$

- After the pulse t =10 msec for all times
- Because only period when input current flows is important

$$V_{out} = [-800t]_0^{0.01} = -8\, V$$

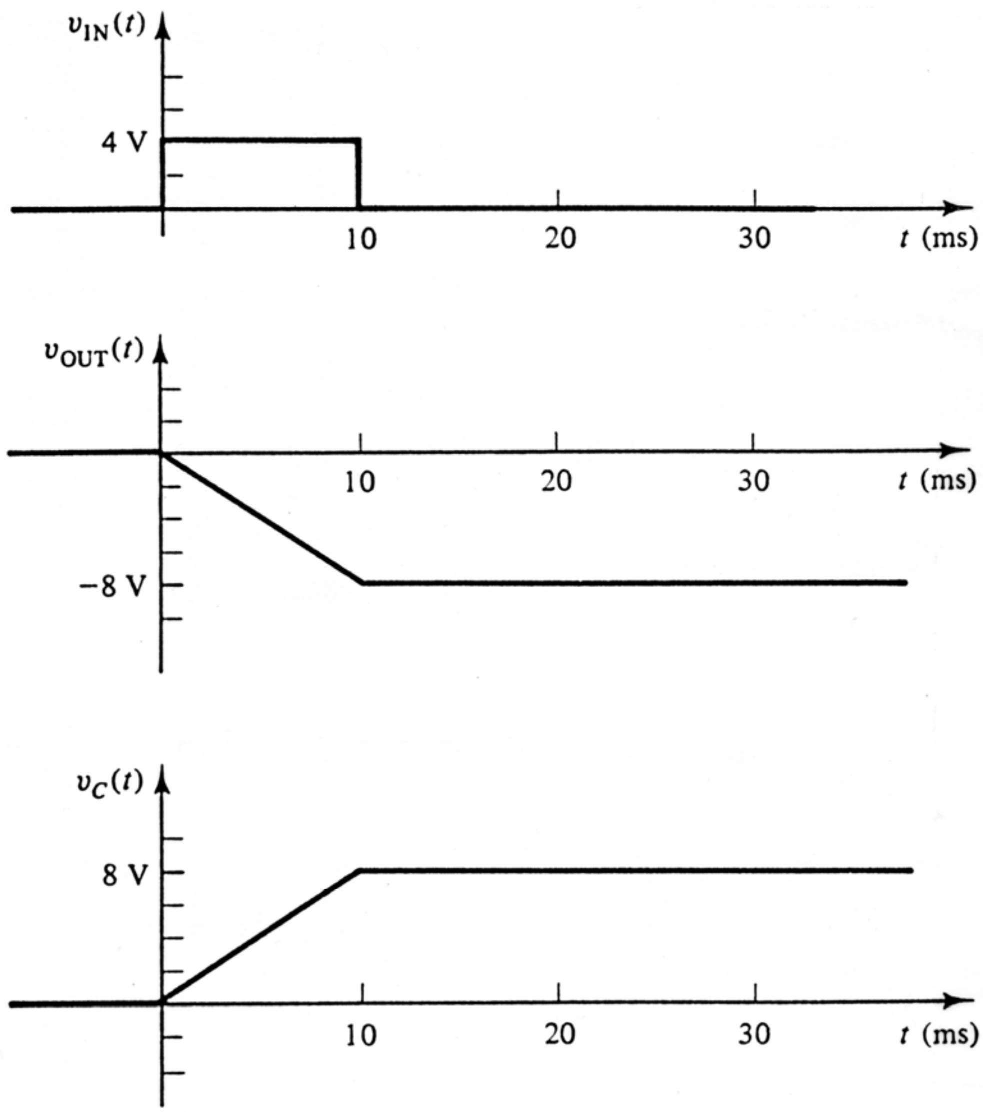
- Op amp will maintain this



# Op Amp Integrator For a Single Pulse

- Result: slope to a constant value of 8 V
- Falling edge of pulse does not matter
- Only the period of voltage input

Plot of $v_{IN}$, $v_{OUT}$, and $v_C$ versus time for the integrator of Fig. 2.18.

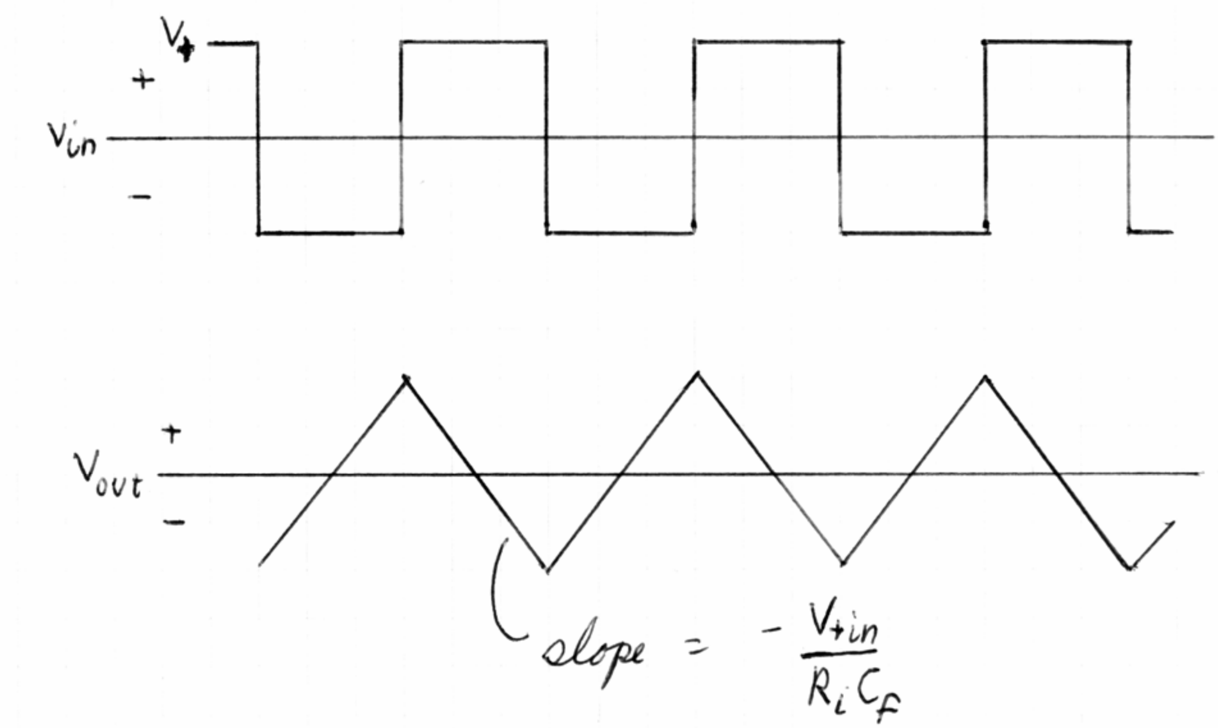


# Op Amp Integrator for a Stream of Pulses

- For a stream of pulses period T
- When period T < RC get a triangle wave output
- Negative voltage gives positive rising edge
- Slope of out wave is

$$V_{out} = -\frac{1}{R_s C_f}\int V_{in}dt = -\frac{V_{in}}{R_s C_f}$$

- Input of positive voltage starts decreasing voltage portion
- Called a sawtooth wave or triangular wave output



# Op Amp Capacitive Differentiator

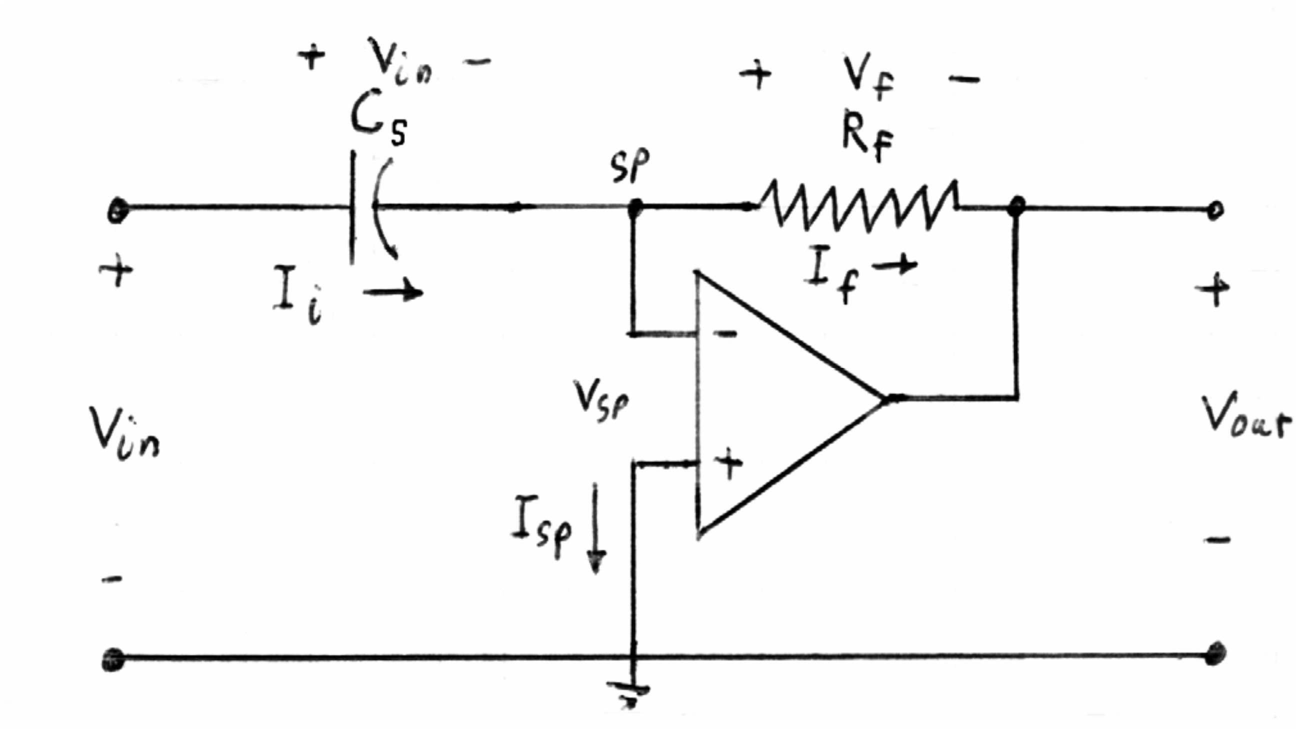
- Can change the op amp circuit to a Differentiator
- Exchange the resistor and capacitor
- Have the capacitor on the input, resistor as feedback
- Want RC time constant short relative to period of any signal
- For the feedback side

$$I_f = \frac{V_f}{R_f}$$

- Recall that for a capacitor

$$I_{in}(t) = C_s \frac{dV_{in}}{dt}$$

- Since the summing point SP is a ground this equation is exact



# Op Amp Capacitive Differentiator Output

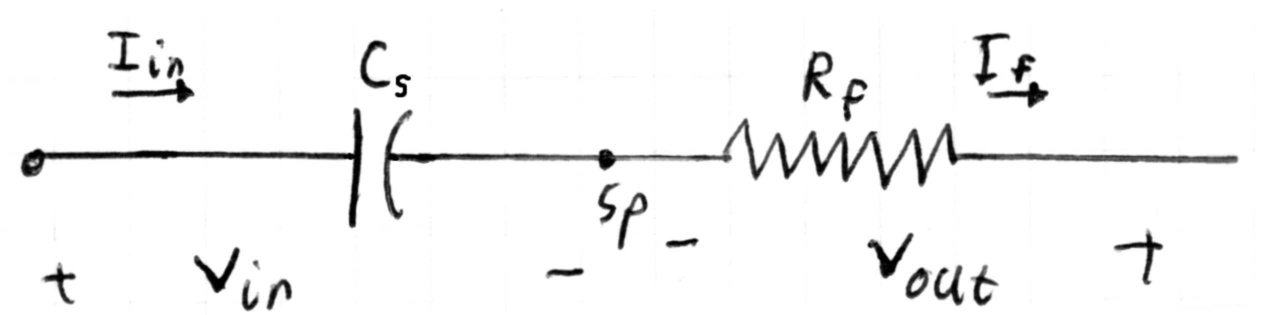
- Again for inverting op amp circuits

$$I_s = I_f \qquad V_f = -V_{out}$$

- Thus the output becomes

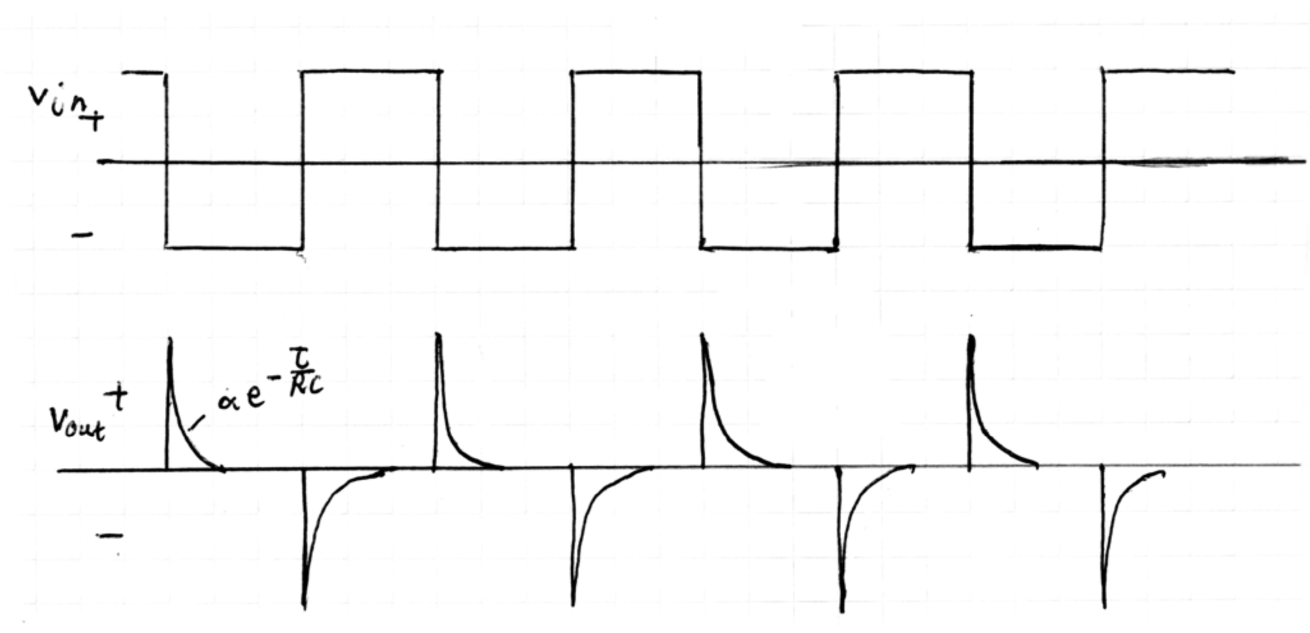
$$V_{out} = -R_f I_{in} = -R_f C_s \frac{dV_{in}}{dt}$$

- Where $\tau = R_f C_s$ is the time constant of the RC circuit.
- Note the response time of op amp limits the operation
- Even if RC is very small



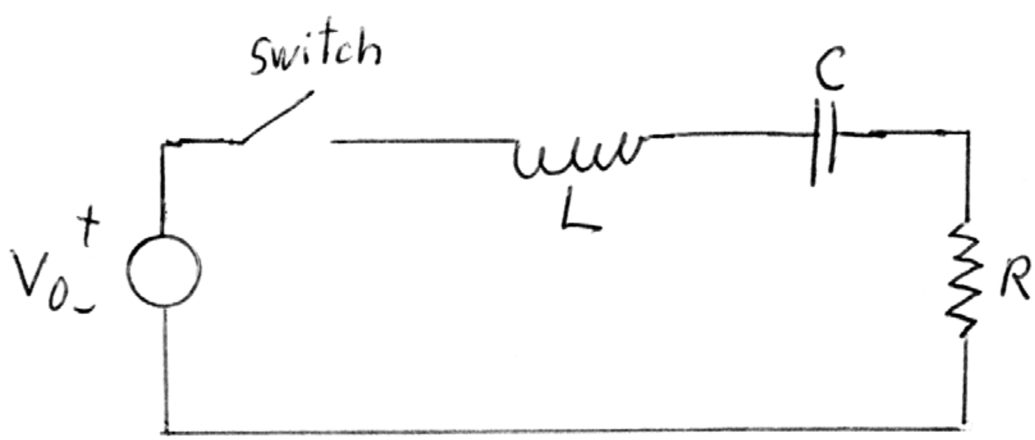
# Op Amp Capacitive Differentiator & Stream of Pulses

- Thus for a string of pulses (square wave)
- Get a sudden change called an impulse
- Direction opposite to that of falling/rising edge
- Followed by an exponential decay
- Decay is as capacitor charges/discharges
- Decay time set by the RC time constant
- Other waveforms integrated eg sin wave gives cos wave



## Second Order Systems (EC 8)

- Second Order circuits involve two energy storage systems
- Create second order Differential Equations
- Transfer of energy from one storage to another and back again
- In circuits Resistors, Inductors and Capacitors
- Called RLC circuits
- L stores energy in Magnetic field from the current
- C stores energy in Electric field from stored charge
- As L discharges energy from B field it is stored in C
- As C discharges charge it is stored in L
- Resistor is always loosing energy
- Eg. series Voltage source, Resistor, Inductor and Capacitor
- Also parallel RLC (equations different)



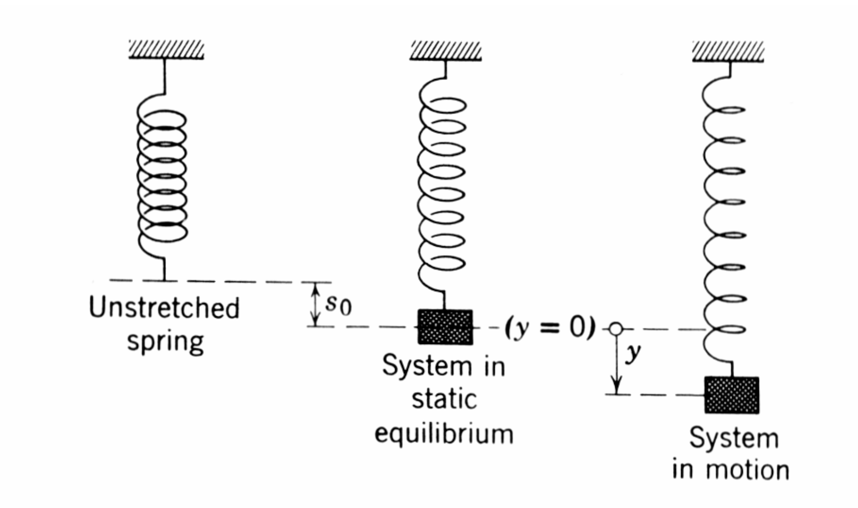
# Damped Spring DE (EC 8)

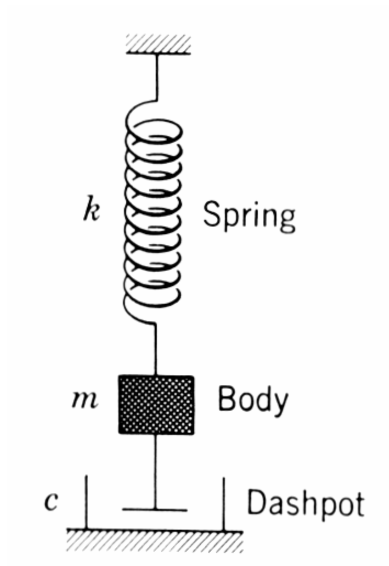
- Math often uses the damped spring with weight for 2nd order DE

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=0$$

Where m=mass of weight
c=damping constant
k=spring constant
y=vertical displacement

- Energy is stored in momentum of weight
- Energy also in position of spring
- Energy lost in damping pot



# Solution of Second Order Systems

- General solution to Second Order circuits
- Proceeds similar to First Order Circuits

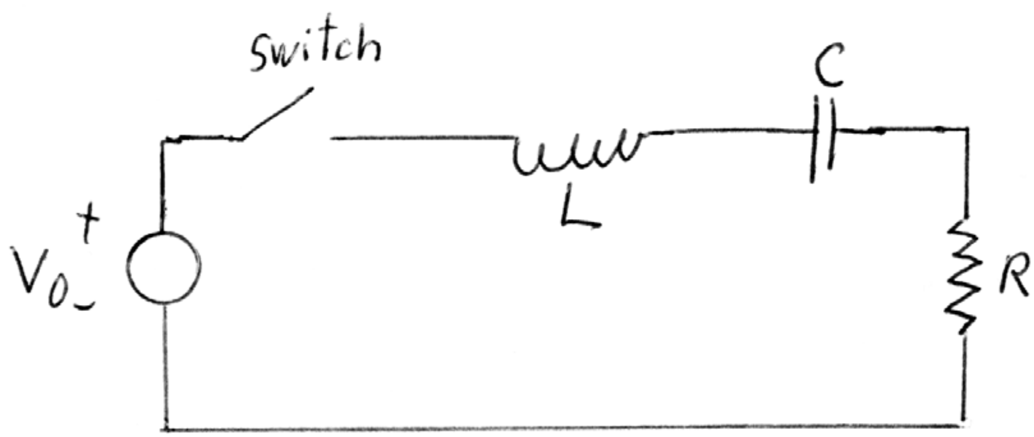
(1) Use Kirchhoff's laws for circuit equation

(2) Manipulate to get I or V in terms of derivatives in time

(3) Generate the "Differential Equation form"
- also called "Homogeneous equation form"

(4) Solve the Differential Equation:
- Solution substitution method: assume a solution
- For step change assume exponential type solution.
- Second order equations generally have two solutions
- Response is combination of both solutions

(5) Use initial or final conditions for constants of integration
- Conditions may include derivatives at those times



# Solution of Series RLC Second Order Systems

- Consider a series RLC with voltage source suddenly applied
- For series RLC used KVL
- Note for parallel will use KCL

(1) Using KVL to write the equations:

$$V_0 = L\frac{di}{dt} + iR + \frac{1}{C}\int_0^t i dt$$

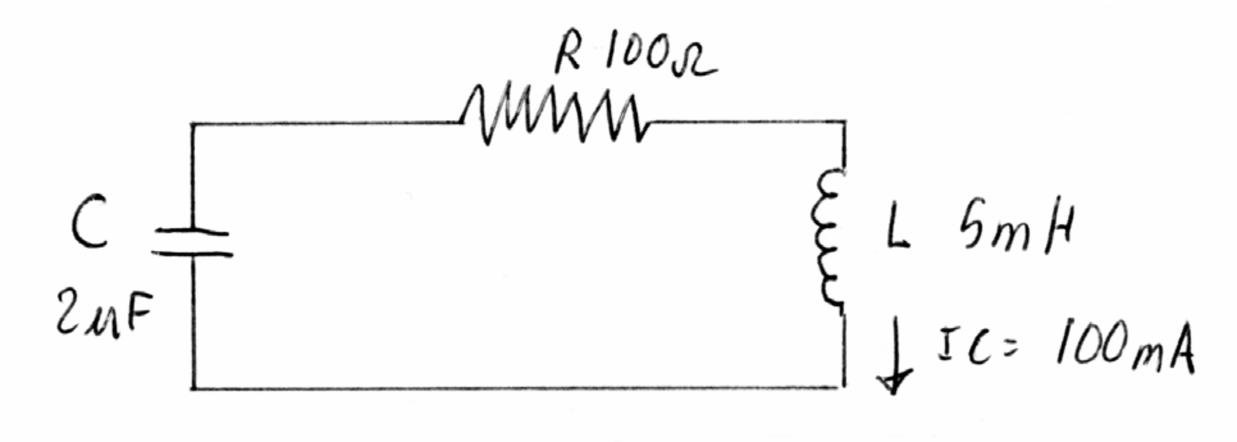
(2) Want full differential equation

- Differentiating with respect to time

$$0 = L\frac{d_2 i}{dt^2} + \frac{di}{dt}R + \frac{1}{C}i$$

(3) This is the differential equation of second order

- Second order equations involve 2nd order derivatives



# Comparison of RLC and Damped Spring DE (EC 8)

- Looking at the damped spring with weight 2nd order DE

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=0$$

Where m=mass of weight
c=damping constant
k=spring constant
y=vertical displacement

- Energy is stored in momentum of weight and spring
- For the RLC series the DE is

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=0$$

- Current i is related to the displacement y
- L is equivalent to the momentum energy stored m
- 1/C is equivalent to the spring constant k
- R is equivalent to the damping loss c