Nodal Circuit Analysis Using KCL

- Most useful for when we have mostly current sources
- Node analysis uses KCL to establish the currents

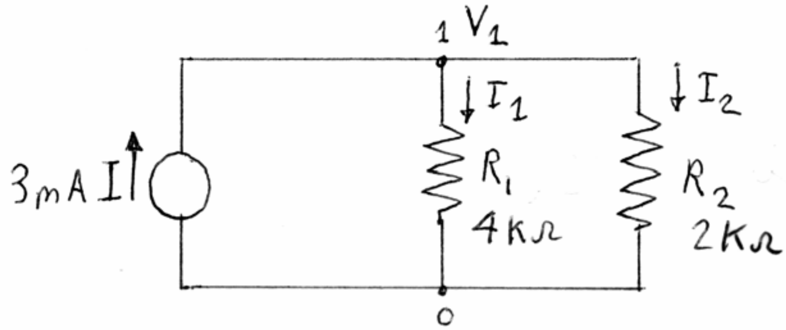
Procedure

(1) Choose one node as the common (or datum) node
- Number (label) the nodes
- Designate a voltage for each node number
- Each node voltage is with respect to the common or datum node
- Number of nodes used = number of nodes – 1 = n-1
- Note: number of nodes = branches – 1 = b-1
- Thus less equations with node analysis than mesh analysis

(2) For each node write the KCL for current flows in each node
- Use differences in the node voltages to calculate currents
- Assume the current directions and write the KCL
- Generally assume the node is a positive V relative to all others
- Current directions different for same branch in each node
- Often better to use conductance equations

$$I = I_{R1} + I_{R2} = \frac{V_1}{R_1} + \frac{V_1}{R_2} = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

(3) Solve the equations for the node voltages
- Get currents in each branch from the voltage differences



Example Nodal Circuit Analysis

- Consider the 2 node, 3 loop circuit below

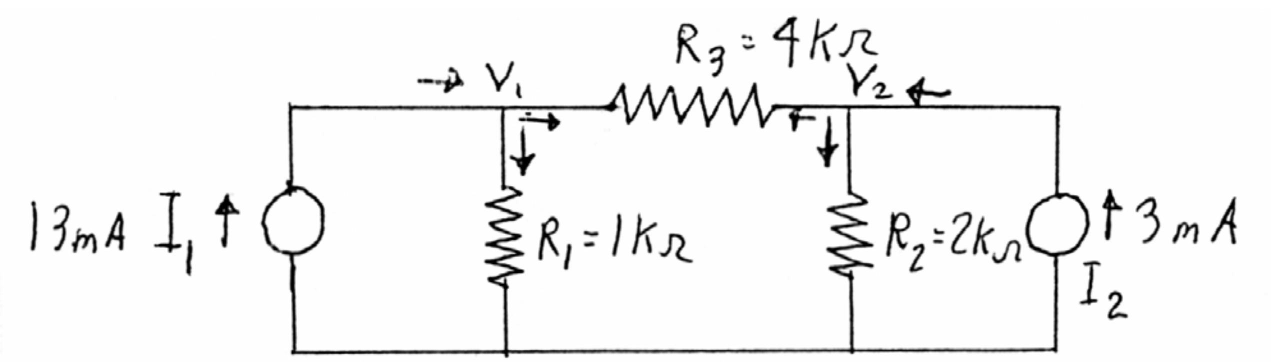
(1) Setting the base node, and node voltages
- Set the common node to ground
- Label voltages on the others

(2) For each node write KCL for current flows
- For node 1,
- Defining the current directions for I_1 as into the node
- Use differences in the node voltages for currents
- Collect each voltage into one term

$$\frac{V_1}{R_1} + \frac{(V_1 - V_2)}{R_3} = V_1\left[\frac{1}{R_1} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} = I_1$$

- Or in the conductance form

$$V_1G_1 + (V_1 - V_2)G_3 = V_1(G_1 + G_3) - V_2G_3 = I_1$$



Example Nodal Circuit Analysis continued

- For node 2,
- Defining the current directions for I_2 as into the node
- This means current in R_3 different from node 1

$$\frac{V_2}{R_2}+\frac{(V_2-V_1)}{R_3}=-\frac{V_1}{R_3}+V_2\left[\frac{1}{R_2}+\frac{1}{R_3}\right]=I_2$$

- Or if changing to conductance form

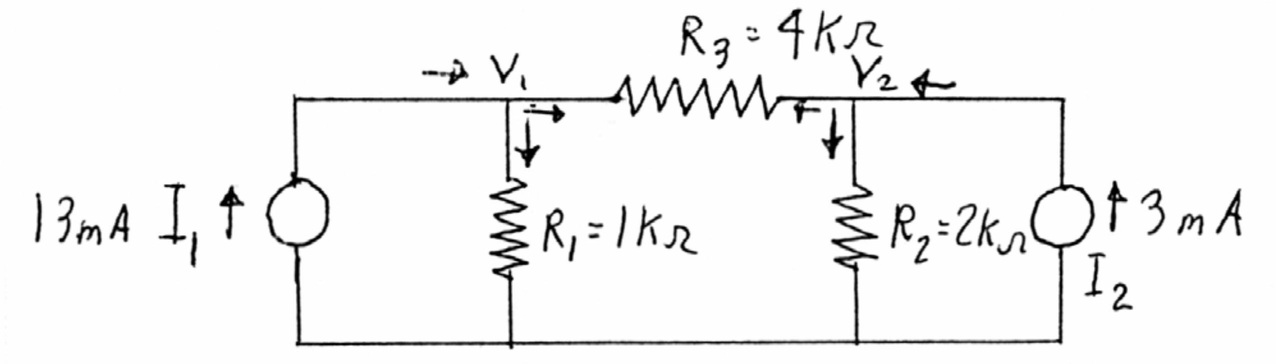
$$V_2G_2+(V_2-V_1)G_3=-V_1G_3+V_2(G_2+G_3)=I_2$$

- Now have two equations and unknowns (V's)

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_3}\right]-\frac{V_2}{R_3}=I_1$$

$$-\frac{V_1}{R_3}+V_2\left[\frac{1}{R_2}+\frac{1}{R_3}\right]=I_2$$

- Could solve this algebraically
- Instead use the numerical methods



Example Nodal Circuit Analysis

- Putting the equations in numerical form
- for node 1:

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_3}\right]-\frac{V_2}{R_3}=V_1\left[\frac{1}{1000}+\frac{1}{4000}\right]-\frac{V_2}{4000}=0.013$$

$$V_1(0.00125)-V_2(0.00025)=0.013$$

- For node 2:

$$-\frac{V_1}{R_3}+V_2\left[\frac{1}{R_2}+\frac{1}{R_3}\right]=-\frac{V_1}{4000}+V_2\left[\frac{1}{2000}+\frac{1}{4000}\right]=0.003$$

$$-V_1(0.00025)+V_2(0.00075)=0.003$$

- Using substitution method for node 1

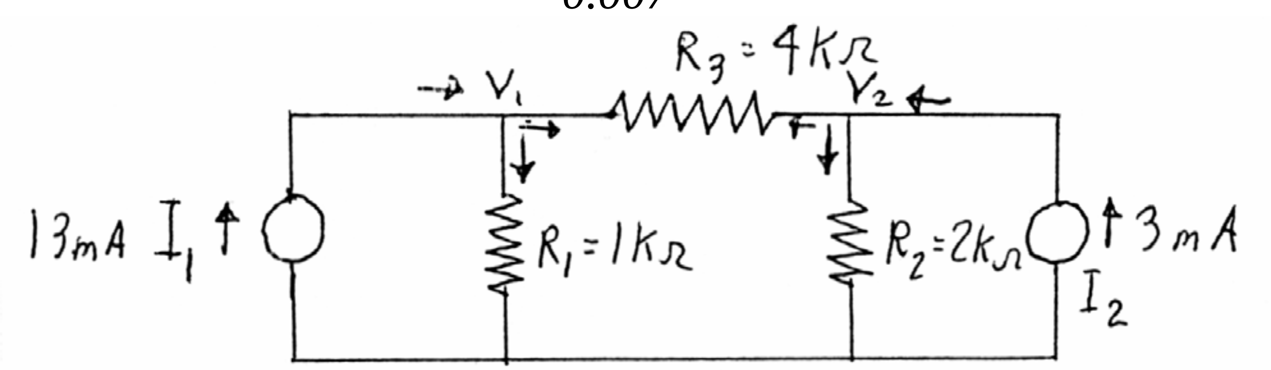
$$V_1=\frac{0.013+V_2(0.00025)}{0.00125}$$

- Thus using node 2 for the solution

$$\left[\frac{0.013+V_2(0.00025)}{0.00125}\right]0.00025+V_2(0.00075)=0.003$$

$$V_2(-0.00005+0.00075)=0.003+0.0026$$

$$V_2=\frac{0.0056}{0.007}=8\,V$$



Example Nodal Circuit Analysis Continued

- Solving for node 1

$$V_1 = \frac{0.013 + V_2(0.00025)}{0.00125} = \frac{0.013 + 8(0.00025)}{0.00125} = 12\,V$$

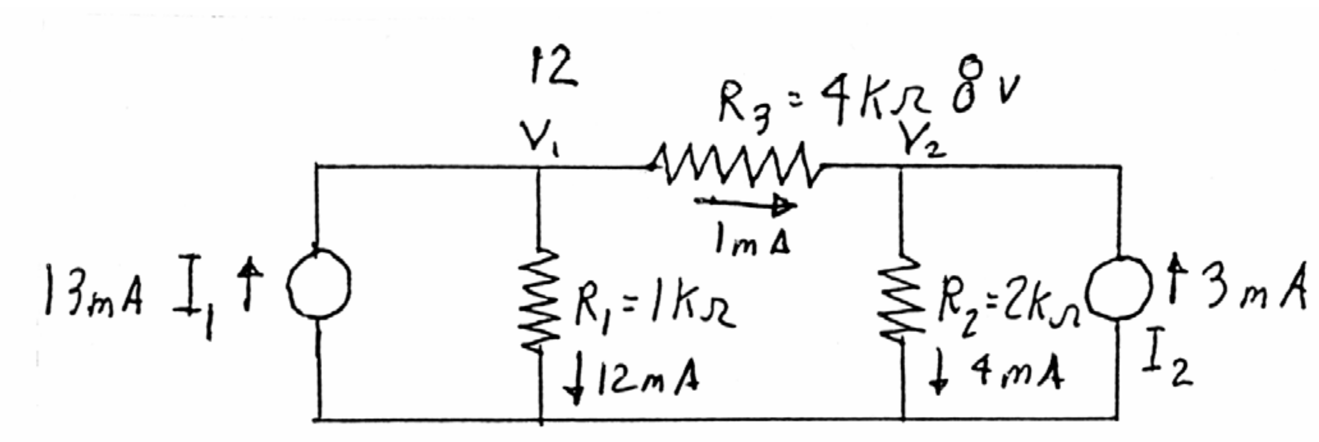
- Current in resistances is

$$I_{R3} = \frac{(V_1 - V_2)}{R_3} = \frac{12 - 8}{4000} = 1\,mA$$

- In the direction of node 2

$$I_{R1} = \frac{V_1}{R_1} = \frac{12}{1000} = 12\,mA$$

$$I_{R2} = \frac{V_2}{R_2} = \frac{8}{2000} = 4\,mA$$



Nodal Analysis General Equations

- In general the nodal equations have the form:

$$+V_1 g_{11} - V_2 g_{12} - V_3 g_{13} \cdots\cdots - V_n g_{1n} = I_1$$
$$-V_1 g_{21} + V_2 g_{22} - V_3 g_{23} \cdots\cdots - V_n g_{2n} = I_2$$

until

$$-V_1 g_{n1} - V_2 g_{n2} - V_3 g_{n3} \cdots\cdots + V_n g_{nn} = I_n$$

where
g_{ij} = branch conductance between "i"th node and "j"th node
g_{ii} = all branch conductance seen by "i"th node

- Thus in the example circuit

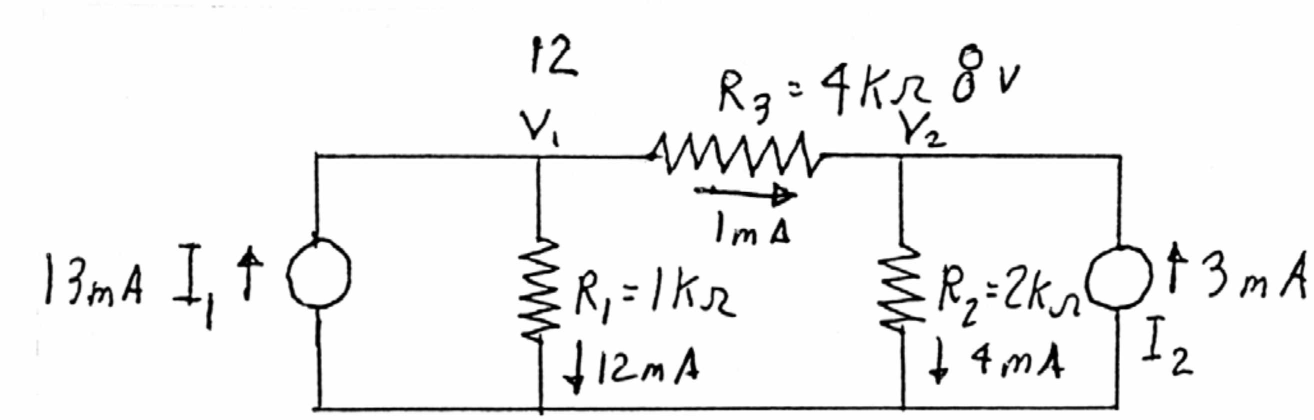
$$+V_1(G_1 + G_3) - V_2 G_2 = +V_1\left[\frac{1}{R_1} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} = I_1$$

$$-V_1 G_3 + V_2(G_2 + G_3) = -\frac{V_1}{R_3} + V_2\left[\frac{1}{R_2} + \frac{1}{R_3}\right] = I_2$$

- And the terms are

$$g_{11} = G_1 + G_3 = \left[\frac{1}{R_1} + \frac{1}{R_3}\right] \qquad g_{12} = G_2 = -\frac{1}{R_3}$$

$$g_{21} = G_3 = -\frac{V_1}{R_3} \qquad g_{22} = (G_2 + G_3) = \left[\frac{1}{R_2} + \frac{1}{R_3}\right]$$



Dummy Nodes and Voltage Sources

- How do we solve a Node circuits containing voltages sources?
- A Voltage source fully defines the node voltage
- This creates a "dummy node" or supernode
- Creates a node constraint equation that defines the voltage

- Example: the circuit below has current and voltage sources
- But V1 is fully defined by voltage source

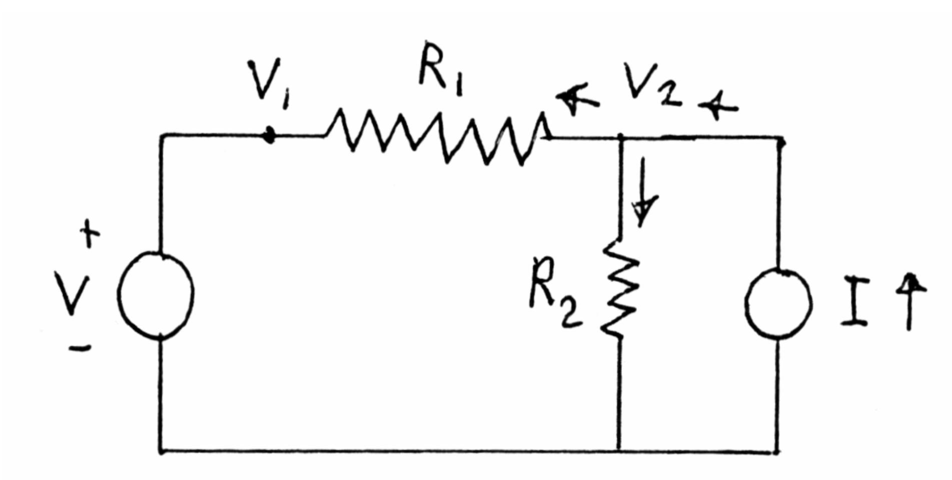
$$V_1 = V$$

- Use this constraint equation to remove one unknown
- This reduces the number of equations to solved by 1 node
- Thus eliminate the unknown current of the voltage source
- Thus node 1 can be eliminated and node 2 becomes

$$\frac{V_2}{R_2} + \frac{(V_2 - V)}{R_1} = V_2\left[\frac{1}{R_2} + \frac{1}{R_1}\right] - \frac{V}{R_1} = I$$

- Thus node 2 can be solved directly

$$V_2 = \left[I + \frac{V}{R_1}\right]\left[\frac{1}{R_2} + \frac{1}{R_1}\right]^{-1}$$



Mesh Analysis using KVL (EC 4)

- Most useful when we have mostly voltage sources
- Mesh analysis uses KVL to establish the currents

Procedure

(1) Define a current loop

- Set a direction for each simple closed path
- Number of loops needed = number of branches – 1 = b-1
- Loop currents can overlap: often many possible combinations
- Must cover all branches with the loop set
- Each loop is called a Mesh

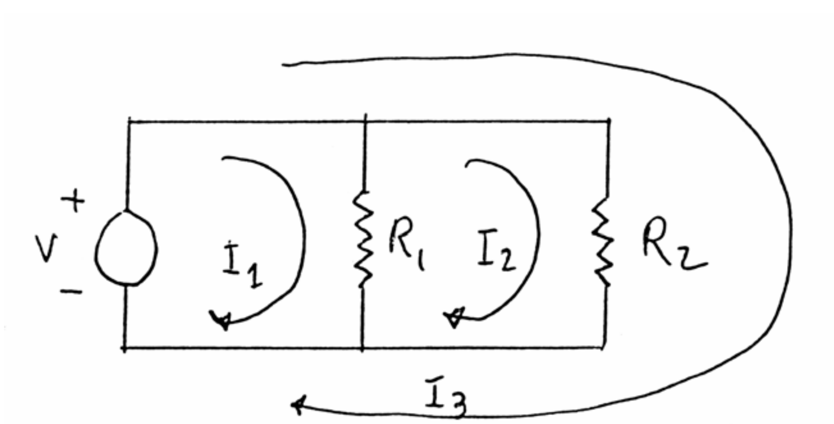
(2) For each mesh write the KVL equation for the loops

- When loop currents overlap:
- Add currents if in same direction
- Subtract currents if in opposite direction
- Voltage sources add if in the direction of loop current
- Voltage sources subtract if opposite to the loop current

$$V = V_{R2} = I_3 R_2$$

(3) Solve the simultaneous equations for the loop currents

- Get currents in each branch from the loop currents
- Voltages calculated from the currents



Example Mesh Analysis of Circuit

- Simple two source network, with 3 branches

(1) Establish two mesh currents (other loops ignored)
- Number of loops = b-1 = 3-1 =2

(2) Now write the KVL equations
- For loop 1:

$$V_1 - R_1 I_1 - R_3 (I_1 - I_2) = 0$$

- Or more commonly putting V on the right

$$I_1 (R_1 + R_3) - I_2 R_3 = V_1$$

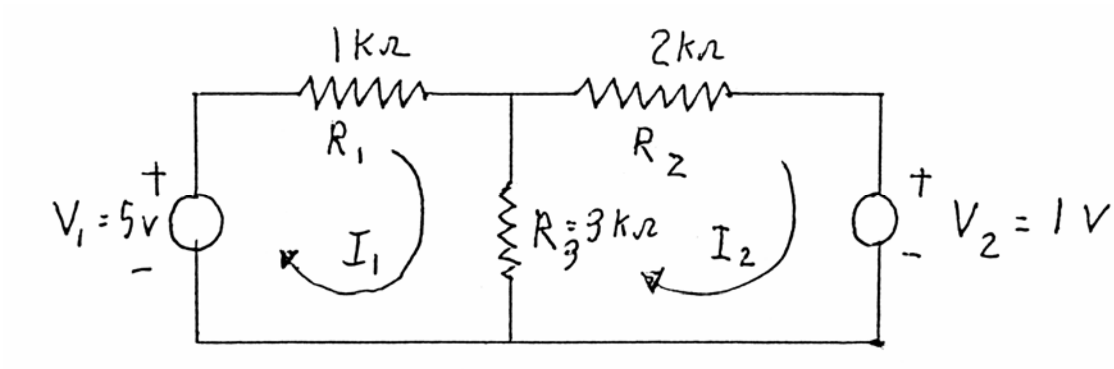
- For loop 2:

$$-V_2 - R_2 I_2 - R_3 (I_2 - I_1) = 0$$

- Again getting V on the right

$$-I_1 R_3 + I_2 (R_2 + R_3) = -V_2$$

- These are the basic equations of the network



Example Mesh Analysis of Circuit Cont'd (EC 4.5)

- Solving these two equations and unknowns
- Typically use substitution methods for simple equations
- Use matrix methods for more complex circuits
- First solving the loop 1 equations for I_1

$$I_1(R_1 + R_3) - I_2R_3 = V_1$$

- Using substitution methods

$$I_1 = \frac{V_1 + R_3I_2}{R_1 + R_3}$$

Now substituting for I_1 in the loop 2 equation

$$-I_1R_3 + I_2(R_2 + R_3) = -V_2$$

$$-\left[\frac{V_1 + R_3I_2}{R_1 + R_3}\right]R_3 + I_2(R_2 + R_3) = -V_2$$

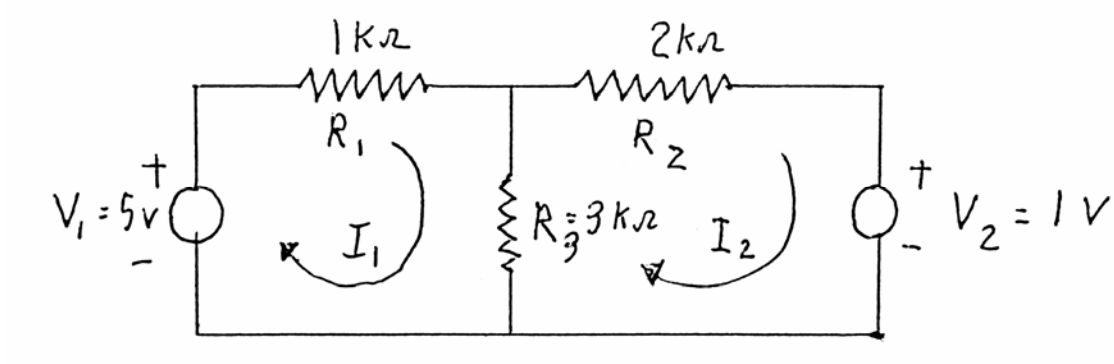
Solving for I_2 and bringing everything to a common denominator

$$I_2\left[\frac{-R_3^2 + (R_1 + R_3)(R_2 + R_3)}{(R_1 + R_3)}\right]R_3 = \frac{-V_2(R_1 + R_3) + V_1R_3}{(R_1 + R_3)}$$

$$I_2\left[R_1R_2 + R_1R_3 + R_2R_3\right] = -V_2(R_1 + R_3) + V_1R_3$$

$$I_2 = \frac{-V_2(R_1 + R_3) + V_1R_3}{\left[R_1R_2 + R_1R_3 + R_2R_3\right]}$$

- Much more difficult solving if do everything algebraically



Example Mesh Analysis of Circuit Cont'd (EC 4.5)

- Consider the specific circuit then

$$I_2 = \frac{-V_2\left(R_1 + R_3\right) + V_1 R_3}{\left[R_1 R_2 + R_1 R_3 + R_2 R_3\right]} = \frac{-1 \times \left(1000 + 3000\right) + 5 \times 3000}{1000(2000) + 1000(3000) + 2000(3000)}$$

$$I_2 = 1mA$$

- Solving for I_1 then

$$I_1 = \frac{V_1 + R_3 I_2}{R_1 + R_3} = \frac{5 + 3000 \times 0.001}{1000 + 3000} = 2\ mA$$

- Then the current through R_3 is

$$I_{R3} = I_1 - I_2 = 0.002 - 0.001 = 1\ mA$$

- Then solving for the voltage across the resistors
- Use current through each resistor

$$V_{R1} = I_1 R_1 = 0.002 \times 1000 = 2\,V$$

$$V_{R2} = I_2 R_2 = 0.001 \times 2000 = 2\,V$$

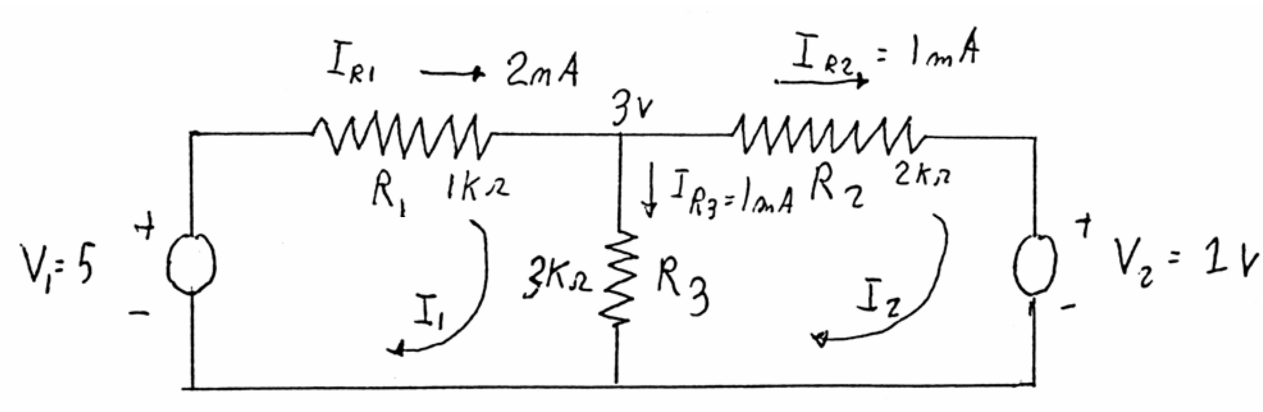
$$V_{R3} = I_{R3} R_3 = 0.001 \times 3000 = 3\,V$$

- Now current through each V source

$$I_{V1} = I_1 = 2\ mA$$

$$I_{V2} = -I_2 = -1\ mA$$

- Note: V_2 has current into + side: thus it is being charged
- Having all V's & I's completely solves the circuit



Mesh Analysis of Circuit: Matrix Solutions

- For solving this using matrices use numerical equations
- For loop 1:

$$I_1(R_1 + R_3) - I_2 R_3 = I_1(1000 + 3000) - I_2 3000 = 5$$

- For loop 2:

$$-I_1 R_3 + I_2(R_2 + R_3) = -I_1 3000 + I_2(2000 + 3000) = -1$$

- This makes manipulation easier
- Note: some calculators have multiple equation/unknown solvers

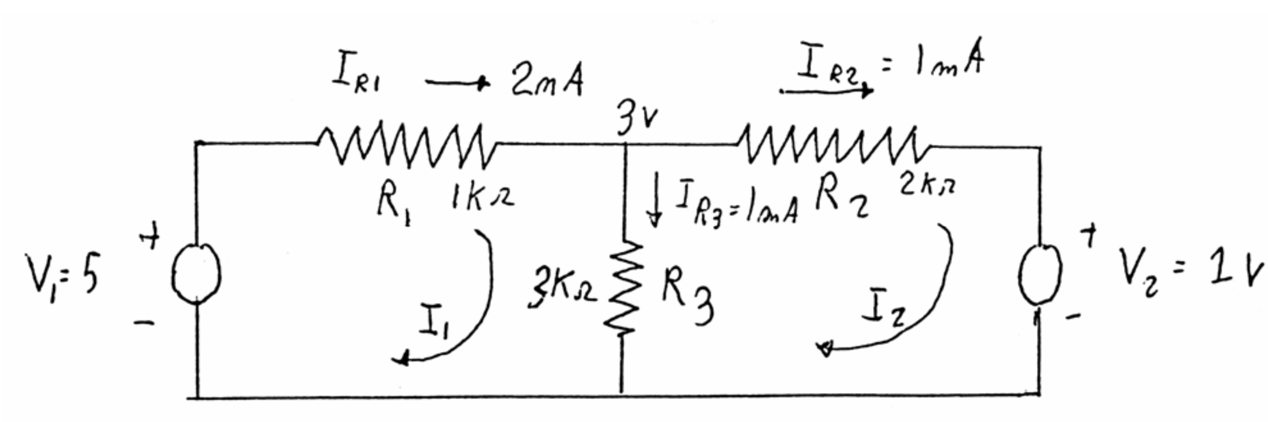
- Alternatively solve using matrixes (see EC appendix A)
- Resistors become a 2x2 R matrix
- Current a 2x1 column matrix I
- Voltage a 2x1 column matrix V

$$[R][I] = [V]$$

$$\begin{bmatrix} +4000 & -3000 \\ -3000 & +5000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} +5 \\ -1 \end{bmatrix}$$

- Then solve the equations by inverting the R matrix

$$[I] = [R]^{-1}[V]$$



Matrix Method and Spread Sheets

- Easy to use matrix method in Excel or Matlab or Maple
- Use minvert and mmult array functions in Excel
- Create the R and V matrix in a spreadsheet
- First invert the matrix: select output cells with same array size
- Enter =minverse(
- Then select the R matrix cells eg =minverse(B4:C5)
- Then press **control+shift+enter** (very important)
- Does not properly enter array if you do not do that
- This creates inverse of matrix at desired location
- Then need to multiply inverse times V column: use =mmult(
- Select output column 1 cells then comma
- Select R^{-1} cells and V cells (eg =mmult(B8:C9,D8:D6))
- Then press **control+shift+enter**
- Here is example from previous page

E220 example lesson 5

i index	R matrix		V matrix
1	4000	-3000	5
2	-3000	5000	-1

	R inverse		V matrix	I solution
1	0.000455	0.000273	5	0.002
2	0.000273	0.000364	-1	0.001

Mesh Analysis General Equations

- In general the mesh equations have the form:

$$+I_1 r_{11} - I_2 r_{12} - I_3 r_{13} \cdots\cdots - I_n r_{1n} = V_1$$
$$-I_1 r_{21} + I_2 r_{22} - I_3 r_{23} \cdots\cdots - I_n r_{2n} = V_2$$

- until

$$-I_1 r_{n1} - I_2 r_{n2} - I_3 r_{n3} \cdots\cdots + I_n r_{nn} = V_n$$

- where
  r_{ij} = total resistance in the "i"th mesh seen by current "j"
  r_{ii} = total resistance in the "i"th mesh seen by the “i”th current loop

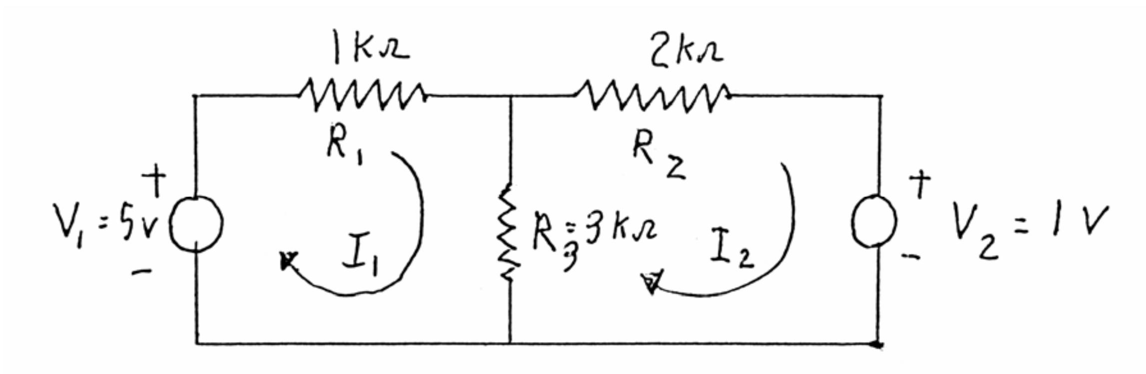
- Eg. in the example circuit for loop 1

$$I_1(R_1 + R_3) - I_2 R_3 = V_1$$

- Then the matrix terms are

$$r_{11} = (R_1 + R_3)$$
$$r_{12} = R_3$$

- This is the general form of the equations/unknowns
- Also the general matrix form



Dummy Meshes and Current Sources

- How do we do mesh circuits containing a current source?
- A Current source fully defines the mesh current
- This creates a "dummy mesh" or “supermesh”:
- Creates a mesh constraint equation that defines the currents

- Eg. I and V source the circuit below (same as in the dummy node)
- Then I_2 is fully defined by the I source

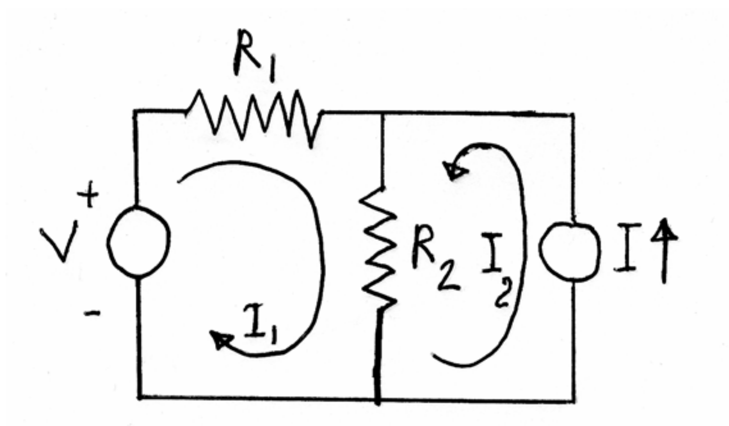
$$I_2 = -I$$

- Use this constraint equation to remove the unknown current I_2
- Reduces the number of equations to solve by 1 mesh
- Thus eliminates the unknown voltage of current source
- Thus loop 2 can be eliminated and loop 1 becomes

$$I_1(R_1 + R_2) + IR_2 = V$$

$$I_1 = \frac{V - IR_2}{(R_1 + R_2)}$$

- Thus I_1 loop 1 can be solved directly



Dual Networks

- Two networks are Duals when then have similar equations
- For the dual of a mesh network

(1) Write the mesh equations
(2) Replace the currents with voltages and vise versa
(3) Replace the resistances with conductances

- Example for the mesh circuit example below

$$+I_1(R_1+R_3)-I_2R_2=V_1$$
$$-I_1R_3+I_2(R_2+R_3)=V_2$$

- Then the dual circuit is

$$+V_1(G_1+G_3)-V_2G_2=I_1$$
$$-V_1G_3+V_2(G_2+G_3)=I_2$$

- Note: current direction of I_2 is in loop 2 direction