Measurement Errors and the Lab

• In every measurement must determine the accuracy or error range
• Typically give as $Y \pm \Delta y$
• Where $Y$ is the measurement
  $\Delta y$ is the error on the measurement
• Typically follows a Gaussian distribution so
  $\pm \Delta y$ 68% of time
  $\pm 2\Delta y$ 95% of time
  $\pm 3\Delta y$ 99.7% of time
• Error is derived from the instruments or the measurement system
• In ENSC 220 lab 1:
• Error on resistors
• Measurement error from meters or instruments
• Use meter not power supply to measure $V$ and $I$
• Must always check for the error for the instrument in its manual
• Try the web for the instrument specs in general
Resistors and Errors

- Resistors are marked for their error limits
- Precision determined by the 4th band
- Typically 5%, 10%, 20%
- 5% most common in lab
- Precision resistors 1% or 2%
- Resistors of all precision are all manufactured at same time
- Then just selected for precision
- Thus not really Gaussian distribution

<table>
<thead>
<tr>
<th>Color</th>
<th>1st Band (1st figure)</th>
<th>2nd Band (2nd figure)</th>
<th>3rd Band (multiplier)</th>
<th>4th Band (tolerance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>$10^0$</td>
<td></td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>1</td>
<td>$10^1$</td>
<td>±2%</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>2</td>
<td>$10^2$</td>
<td>±2%</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>3</td>
<td>$10^3$</td>
<td>±2%</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>4</td>
<td>$10^4$</td>
<td>±2%</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>5</td>
<td>$10^5$</td>
<td>±2%</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>6</td>
<td>$10^6$</td>
<td>±2%</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>7</td>
<td>$10^7$</td>
<td>±2%</td>
</tr>
<tr>
<td>Gray</td>
<td>8</td>
<td>8</td>
<td>$10^8$</td>
<td>±2%</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>9</td>
<td>$10^9$</td>
<td>±2%</td>
</tr>
<tr>
<td>Gold</td>
<td>10</td>
<td></td>
<td>$10^{-1}$</td>
<td>±5%</td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td></td>
<td>$10^{-2}$</td>
<td>±10%</td>
</tr>
</tbody>
</table>
Worst Case Errors

• In engineering use Worst Case Error analysis
• Thus errors always add to do most damage
• Consider resistors of R₁=2.2K, and R₂=1.0K with 5% precision
• Then expected error on each is

\[ \Delta R_1 = 2200 \times 0.05 = 110 \, \Omega \]
\[ R_1 = 2200 \pm 110 \, \Omega \]
\[ R_2 = 1000 \pm 50 \, \Omega \]

• What if the resistors are in series?

\[ R_{total} = R_1 + R_2 = 2200 + 1000 = 3200 \, \Omega \]
\[ \Delta R_{total} = \Delta R_1 + \Delta R_2 = 110 + 50 = 160 \, \Omega \]

• Thus expect total resistance to be from 3040 to 3360\Omega
• Could improve if you measure the values
• For division and multiplication add percentage errors

\[ \frac{R_1}{R_2} = \frac{2200}{1000} = 2.2 \]

\[ \Delta \frac{R_1}{R_2} = \Delta \% R_1 + \Delta \% R_2 = 5\% + 5\% = 10\% \]
Digital MultiMeters and Accuracy

• Digital MultiMeter DMM accuracy is very dependent on the meter
• In lab we use a two meters: Fluke 45 and Fluke 8010A

Fluke 45 Autoranging meter
• Must look at the spec sheet for the accuracy in each range
• We have put the spec sheet in pdf form on the lab page
• Accuracy is in Appendix A

Example of Accuracy from Appendix A

<table>
<thead>
<tr>
<th>Range</th>
<th>Resolution</th>
<th>Accuracy (6 Months)</th>
<th>Accuracy (1 Year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slow</td>
<td>Medium</td>
<td>Fast</td>
</tr>
<tr>
<td>300 mV</td>
<td>—</td>
<td>10 µV</td>
<td>100 µV</td>
</tr>
<tr>
<td>3 V</td>
<td>—</td>
<td>100 µV</td>
<td>1 mV</td>
</tr>
<tr>
<td>30 V</td>
<td>—</td>
<td>1 mV</td>
<td>10 mV</td>
</tr>
<tr>
<td>300 V</td>
<td>—</td>
<td>10 mV</td>
<td>100 mV</td>
</tr>
<tr>
<td>1000 V</td>
<td>—</td>
<td>100 mV</td>
<td>1 V</td>
</tr>
<tr>
<td>100 mV</td>
<td>1 µV</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1000 mV</td>
<td>10 µV</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>10 V</td>
<td>100 µV</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>100 V</td>
<td>1 mV</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1000 V</td>
<td>10 mV</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
How to Get Reading Accuracy in a DMM

- Meter has certain number of displayed digits
- Accuracy depends on the range of the units measured
- Also number of digits displayed for your reading
- Typically has two parts
- A percentage of the actual reading eg 1%
- Percentage is overall accuracy of the internal resistors etc
- A count for the last digit
- Measures accuracy of the Digital to Analog converter
- The count is how many tenths of final digit will cause it to flip
- Eg. if meter accuracy was 1% + 2 in spec sheet
- Meter reading was reading 100 V
- Then accuracy due to last digit
- Digit is 0 but will stay 0 for 0.2 below to 0.2 above
- Thus from % expect a range of 99 to 101 V
- From last digit it is 98.8 to 101.2
- Some DMM are 3 ½ digits
- There error is % of reading ±0.5 of last digit value

- Good reference
Fluke 8010A DMM

- Fluke 8010 is 3 1/2 digit meter autoranging
- Error specifications to it
- Error is % plus ½ of last digit
- ie. ±0.5 of last digit value
Example of Errors in Experiments

- Consider a circuit with 2 resistors in series
- $R_1=2.2\,\text{K}$, and $R_2=1.0\,\text{K}$ with 5\% precision

Apply a voltage source based on its meter of 3.2 V to it
Expect a current of

$$I = \frac{V}{R_1 + R_2} = \frac{3.2}{2200 + 1000} = 1\,\text{mA}$$

Voltages you get not those you expect
Assume the meters have 1\% error & source $\frac{1}{2}$ digit

<table>
<thead>
<tr>
<th></th>
<th>$V_{\text{expected}}$</th>
<th>$V_{\text{meter}}$</th>
<th>$V_{\text{max error}}$</th>
<th>$V_{\text{min error}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>6.40</td>
<td>6.40</td>
<td>6.405</td>
<td>6.395</td>
</tr>
<tr>
<td>$R_{\text{source}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>4.40</td>
<td>4.47</td>
<td>4.49</td>
<td>4.12</td>
</tr>
<tr>
<td>$R_2$</td>
<td>2.00</td>
<td>1.82</td>
<td>2.04</td>
<td>1.87</td>
</tr>
<tr>
<td>$V_{\text{total}}$</td>
<td>6.40</td>
<td>6.32</td>
<td>6.53</td>
<td>5.99</td>
</tr>
<tr>
<td>$V_{R_1/V_{R_2}}$</td>
<td>2.20</td>
<td>2.46</td>
<td>2.42</td>
<td>1.98</td>
</tr>
</tbody>
</table>

- Sources of error
- Resistance of source $\sim 75\,\Omega$
- Errors in resistors 5\%
- Errors in meters 1\%
Inverting Op Amplifier

- Adding resistors to op amp can control the gain
- Gain controlled by "Feedback":
- Feeding input back into output
- This circuit gives negative gain:
- Called Inverting op amp

- SP = Summing Point, where output and input signals sum
Inverting Op Amplifier

- Place a feedback resistor $R_f$ from op amp output to positive input
- The $R_s$ between the summing point SP and input $V_s = V_{in}$
- Sometimes see SP called set point
- Allows current to flow from input and output to SP
- This circuit gives negative gain:
- Increases signal but makes it negative
- Called Inverting op amp
Inverting Op Amplifier

- Putting the input into $v_n$ gives negative gain
- Output becomes the opposite polarity to input

$$V_{sp} = 0 \quad I_{sp} = 0$$

- Thus SP = a virtual ground
- In practice has small voltage offset

$$V_s = V_i = V_{in}$$

$$I_s = I_i = \frac{V_{in}}{R_s} = I_{in}$$

- Note $I_f$ is not supplied by input current $I_{sp}$
- $I_f$ is supplied by power supplies $V_{cc}$ and $V_{ee}$ of amp
- If load (e.g. a resistor) is added to $V_{out}$
- The power supplies create that current also up to amp’s I limit
Inverting Op Amplifier Gain Equations

- Virtual ground requires $V_f$ to reverse relative to $V_{\text{in}} = V_s$

$$I_s = I_f = \frac{V_s}{R_s} = \frac{V_{\text{out}}}{R_f}$$

$$V_{\text{out}} = V_f = I_f R_f = -V_s \frac{R_f}{R_s}$$

- The amplification (or gain) is:

$$A_v = \frac{V_o}{V_s} = -\frac{R_f}{R_s}$$
Example Inverting Op Amplifier

- For an inverting op amp with
  \[ R_s = 1 \text{ K}\Omega \quad \text{and} \quad R_f = 10 \text{ K}\Omega \]
  \[ V_{CC} = +15 \text{ V} \quad \text{and} \quad V_{EE} = -15 \text{ V} \]

- What is the output for a 0.5 V input?
- The voltage gain is:
  \[ A_v = \frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{10000}{1000} = -10 \]

- Thus the output is:
  \[ V_o = A_v V_s = -10 \times 0.5 = -5 \text{ V} \]
Non-inverting Op Amplifier

- Place a feedback resistor $R_f$ from op amp output to neg input
- The $R_i$ between summing point and ground
- Allows current to flow from output to input
- Voltage divider $R_f/R_s$ sets voltage at input
- This circuit gives positive gain
- Called Non-inverting op amp
- Note: text uses $V_g$ for $V_{in}$
- Also shows a $R_g$ on input which is not needed
Non-inverting Op Amplifier Gain

- Key point: Infinite op amp input resistance means no input current
- Thus voltage across the op amp input $V_{sp}$ must be zero
  \[ I_{in} = 0 \quad thus \quad V_{sp} = 0 \]
- Hence voltage at summing point $sp$ must equal input voltage
  \[ V_s = V_{in} \]
- Since no input current to the op amp neg input
  \[ I_s = \frac{V_{in}}{R_s} \quad I_s = I_f \]
- Thus voltage across the feedback resistor becomes
  \[ V_f = I_f R_f = I_s R_f = V_{in} \frac{R_f}{R_s} \]
  \[ V_o = V_s + V_f = V_{in} \frac{R_i + R_f}{R_i} \]
- The voltage amplification (or gain) is:
  \[ A_v = \frac{V_o}{V_{in}} = \frac{R_i + R_f}{R_i} \]
Example Non-inverting Op Amplifier

• For an non-inverting op amp with

\[ R_s = 1 \, K\Omega \quad \text{and} \quad R_f = 9 \, K\Omega \]

\[ V_{CC} = +15 \, V \quad \quad V_{EE} = -15 \]

• What is the output for a 0.5 V input?

• The voltage gain is:

\[ A_v = \frac{V_o}{V_{in}} = \frac{R_i + R_f}{R_i} = \frac{1000 + 9000}{1000} = 10 \]

• Thus the output is:

\[ V_o = V_{in} \frac{R_i + R_f}{R_i} = V_{in} A_v = 0.5 \times 10 = 5 \, V \]

• NOTE: must keep output < power supply voltages - input limited

• In real circuits use resistors in Kilo-ohm range

• Thus reduce effects of smaller resistance eg. contacts
Summing Inverting Op Amplifier (EC 6.4)

- Using inverting op amps to combine many signals
- Have each input resistance connected to SP
- But only one feedback resistor
- Each signal can have different amplification
- Again the SP is a virtual ground

\[
V_{sp} = 0 \quad I_{sp} = 0
\]

- Current from each input \( V_{sj} \) is

\[
I_{sj} = \frac{V_{sj}}{R_{sj}}
\]

- By KCL the total input current is

\[
I_{s-total} = \sum_{j=1}^{N} I_{sj} = \sum_{j=1}^{N} \frac{V_{sj}}{R_{sj}}
\]

- True because summing point a virtual ground
- Note each input is not affected by the other inputs
Summing Inverting Op Amp Gain

- Then by KCL the feedback current = input current

\[ I_f = -\frac{V_f}{R_f} = I_{s\text{-total}} = \sum_{j=1}^{N} I_{sj} = \sum_{j=1}^{N} \frac{V_{sj}}{R_j} \]

- Then in terms of output voltage

\[ V_o = V_f = \sum_{j=1}^{N} -V_{sj} \frac{R_f}{R_j} = \sum_{j=1}^{N} -V_{sj} A_{vj} \]

- Note: \( V_o \) must not exceed \( V_{EE} \) or \( V_{CC} \)
- The amplification (or gain) per channel is:

\[ A_{vj} = -\frac{R_f}{R_{sj}} \]

- Simple control systems use this summing input
Example of Summing Op Amp

- Create an op amp circuit that will generate the sum equation
  \[ V_o = -V_1 - 2V_2 \]

- To begin set \( R_f \) to the largest multiplication factor
- I.e.: times some common \( R \) for the design
- Here: largest gain \( A_{\text{max}} \) is for \( V_2 \) which is 2
- Now choose the minimum resistance: i.e. common \( R = 1 \, \text{K\Omega} \)
  \[ R_f = A_{\text{max}} R_{\text{min}} = 2 \times 1000 = 2 \, \text{K\Omega} \]
Example of Summing Op Amp Cont’d

- For the input resistances
- Set the $R_{sj}$ equal to common $R_f$ times the multiplier for that input

\[ R_j = \frac{R_f}{A_j} \]

- Thus for the example

\[ R_{i1} = \frac{R_f}{A_1} = \frac{2000}{1} = 2 \, K\Omega \]

\[ R_{i2} = \frac{R_f}{A_2} = \frac{2000}{2} = 1 \, K\Omega \]
Differential (Difference) Op Amplifier

- Want to determine the difference between two voltages
- Often used as comparators (ie. is the output near zero)
- Note Ground is no longer part of the op amp input
- Put input resistors $R_{i1}$ and $R_{i2}$ on both inputs
- Arrangement like combining inverting & non-inverting op amp

\[
\begin{align*}
R_{i1} &= R_{i2} \\
V_{sp} &= 0 \\
R_{f1} &= R_{f2} \\
I_s &= 0
\end{align*}
\]

- Consider input 2 side
- To measure $V_{in2}$ only: ground $V_{in1}$
- This reduces to an inverting amplifier

\[
V_{out2} = -V_{in2} \frac{R_{f2}}{R_{i2}}
\]
Differential Op Amplifier Cont'd

• Consider input 1 side only
• For $V_{in1}$ only; ground $V_{in2}$
• This reduces to non-inverting like amplifier but with changed input
• The effective non-inverting $V_{in}$ is:

$$V_{in} = V_{in1} \frac{R_{f1}}{R_{i1} + R_{f2}}$$

• Thus

$$V_{out1} = V_{in} \left[ \frac{R_{i2} + R_{f1}}{R_{i2}} \right] = V_{in1} \left[ \frac{R_{f1}}{R_{i1} + R_{f2}} \right] \left[ \frac{R_{i2} + R_{f1}}{R_{i2}} \right]$$

$$V_{out1} = V_{in1} \frac{R_{f1}}{R_{i1}}$$

• This reduces to a non-inverting amplifier
• But with input voltage divider on input
Differential Op Amplifier Cont'd

• The op amps are linear as long as output/input outside of saturation
• Now using superposition the total output is:

\[ V_{out} = V_{out1} + V_{out2} \]

\[ = V_{in1} \frac{R_{f1}}{R_{i1}} - V_{in2} \frac{R_{f2}}{R_{i2}} \]

• Since

\[ R_{i1} = R_{i2} \quad R_{f1} = R_{f2} \]

• Thus

\[ V_{out} = (V_{in1} - V_{in2}) \left[ \frac{R_{f1}}{R_{i1}} \right] \]

• Thus the amplification (gain) is:

\[ A_v = \frac{V_{out}}{V_{in1} - V_{in2}} = \frac{R_{f1}}{R_{i1}} \]
Non ideal Op Amplifier

- The real op amps, like the 741, have
- Finite input resistance $R_i \sim 2 \text{ M}\Omega$
- Finite amplification $A \sim 200,000$
- Real out resistance $R_o \sim 75 \text{ }\Omega$
- Commonly will have an offset current as well

**FIGURE 6.15** An equivalent circuit for an operational amplifier.