ENSC 220 – Assignment #6 solution 2005

1.

Let $Z_1 = 4 - j8\,\Omega, \qquad Z_2 = 2 + j10\,\Omega$

Then $Z_1 /\!/ Z_2 = \dfrac{(4-j8)(2+j10)}{6+j2} = \dfrac{88+j24}{(6+j2)} = \dfrac{(88+j24)(6-j2)}{40} = 14.4 - j0.8\,\Omega$

$Z_3 = 20\,\Omega, \qquad Z_4 = -j10\,\Omega$

$$Z_3 /\!/ Z_4 = \frac{-j200}{20-j10} = \frac{-j20}{2-j1} = \frac{-j20(2+j1)}{5} = 4 - j8\,\Omega$$

$$Z_{\text{ab}} = 21.6 + j38.8 + 14.4 - j0.8 + 4 - j8 = 40 + j30\,\Omega = 50\angle 36.87^\circ\,\Omega$$

2.

$$Y_1 = \frac{1}{6-j2} + \frac{1}{4+j12} + \frac{1}{5} + \frac{1}{j10} = \frac{6+j2}{40} + \frac{4-j12}{160} + \frac{32}{160} - j\frac{16}{160}$$

$$= \frac{24+j8+4-j12+32-j16}{160} = \frac{60-j20}{160} = \frac{3-j1}{8}$$

$$Z_1 = \frac{1}{Y_1} = \frac{8}{3-j1} = 2.4 + j0.8$$

$$Z_{\text{ab}} = 13.6 - j12.8 + 2.4 + j0.8 = 16 - j12 = 20\angle -36.87^\circ$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = 50\angle 36.87^\circ\,\text{m℧} \quad = 40 + j30\,\text{m℧}$$

3.

$$Z_1 = 30 + j0.25 \times 10^{-3}(40{,}000) = 30 + j10\,\Omega$$

$$Z_2 = 20 - j\frac{10^6}{(40{,}000)(2.5)} = 20 - j10\,\Omega$$

$$\mathbf{I}_g = 20\angle -73.74^\circ\,\text{A}$$

Let $\mathbf{I}_C$ represent the current down through the capacitive branch. Then

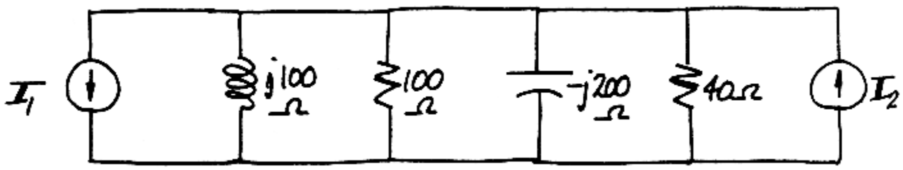
$$\mathbf{I}_C = \frac{Z_1}{Z_1 + Z_2}\mathbf{I}_g = \frac{30+j10}{50}(20\angle -73.74^\circ) = 12.65\angle -55.31^\circ\,\text{A}$$

$$\mathbf{V}_o = 20\mathbf{I}_C = 252.98\angle -55.31^\circ\,\text{V}$$

$$v_o = 252.98\cos(40{,}000t - 55.31^\circ)\,\text{V}$$

4.

$$j\omega L = j5000(20 \times 10^{-3}) = j100\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j5000} = -j200\,\Omega$$

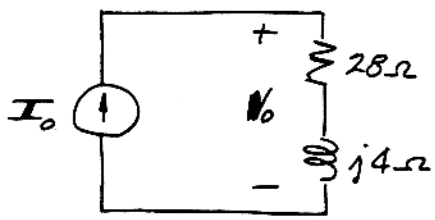


$$\mathbf{I}_1 = \frac{400\underline{/36.87^\circ}}{j100} = 4\underline{/-53.13^\circ} = 2.4 - j3.2\,\text{A}$$

$$\mathbf{I}_2 = \frac{128\underline{/-90^\circ}}{40} = 3.2\underline{/-90^\circ} = -j3.2\,\text{A}$$

$$Y = \frac{1}{j100} + \frac{1}{100} + \frac{1}{-j200} + \frac{1}{40} = \frac{-j2 + 2 + j + 5}{200} = \frac{7 - j1}{200}$$

$$Z = \frac{200}{7 - j1} = \frac{200}{50}(7 + j1) = 28 + j4\,\Omega$$



$$\mathbf{I}_o = \mathbf{I}_2 - \mathbf{I}_1 = -j3.2 - 2.4 + j3.2 = -2.4 = 2.4\underline{/180^\circ}$$

$$\mathbf{V}_o = (28 + j4)\mathbf{I}_o = (28.28\underline{/8.13^\circ})(2.4\underline{/180^\circ}) = 67.88\underline{/188.13^\circ}$$

$$v_o = 67.88\cos(5000t + 188.13^\circ)\,\text{V} \quad \text{or} \quad v_o = 67.88\sin(5000t - 81.87^\circ)\,\text{V}$$

5.

[a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{sL + R + R_l} = \dfrac{(R/L)}{s + (R + R_l)/L}$

[b] $H(j\omega) = \dfrac{(R/L)}{\left(\frac{R+R_l}{L}\right) + j\omega}$

$$|H(j\omega)| = \frac{(R/L)}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega^2}}$$

$|H(j\omega)|_{\text{max}}$ occurs when $\omega = 0$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_l}$

[d] $|H(j\omega_c)| = \dfrac{R}{\sqrt{2}(R+R_l)} = \dfrac{R/L}{\sqrt{\left(\frac{R+R_l}{L}\right)^2 + \omega_c^2}}$

$$\therefore\ \omega_c^2 = \left(\frac{R+R_l}{L}\right)^2; \qquad \therefore\ \omega_c = (R+R_l)/L$$

[e] $\omega_c = \dfrac{1575}{0.25} = 6300$ rad/s

$$H(j\omega) = \frac{6000}{6300 + j\omega}$$

$$H(j0) = 0.9524$$

$$H(j6300) = \frac{0.9524}{\sqrt{2}}\underline{/-45^\circ} = 0.6734\underline{/-45^\circ}$$

$$H(j1890) = \frac{6000}{6300 + j1890} = 0.9122\underline{/-16.70^\circ}$$

$$H(j18{,}900) = \frac{6000}{6300 + j18{,}900} = 0.3012\underline{/-71.57^\circ}$$

6.

[a] $H(s) = \dfrac{V_o}{V_i} = \dfrac{R}{R + R_c + (1/sC)}$

$$= \frac{R}{R+R_c} \cdot \frac{s}{[s + (1/(R+R_c)C)]}$$

[b] $H(j\omega) = \dfrac{R}{R+R_c} \cdot \dfrac{j\omega}{j\omega + (1/(R+R_c)C)}$

$$|H(j\omega)| = \frac{R}{R+R_c} \cdot \frac{\omega}{\sqrt{\omega^2 + \frac{1}{(R+R_c)^2C^2}}}$$

The magnitude will be maximum when $\omega = \infty$

[c] $|H(j\omega)|_{\max} = \dfrac{R}{R+R_c}$

[d] $|H(j\omega_c)| = \dfrac{R\omega_c}{(R+R_c)\sqrt{\omega_c^2 + [1/(R+R_c)C]^2}}$

$\therefore \quad |H(j\omega)| = \dfrac{R}{\sqrt{2}(R+R_c)}$ when

$\therefore \quad \omega_c^2 = \dfrac{1}{(R+R_c)^2C^2}$

or $\omega_c = \dfrac{1}{(R+R_c)C}$

[e] $\omega_c = \dfrac{1}{(R+R_C)C} = \dfrac{10^9}{(50\times 10^3)(2.5)} = 8000$ rad/s

$$H(j\omega_c) = \left(\frac{40}{50}\right)\frac{j8000}{8000+j8000} = 0.5657\underline{/45^\circ}$$

$$H(j0.1\omega_c) = \frac{(0.8)j800}{8000+j800} = 0.0796\underline{/84.29^\circ}$$

$$H(j10\omega_c) = \frac{(0.8)j80{,}000}{8000+j80{,}000} = 0.7960\underline{/5.71^\circ}$$

7.

[a] $\omega_o^2 = \dfrac{10^3\times 10^6}{(40)(0.25)} = 10^8, \qquad \omega_o = 10^4$ rad/s

$f_o = \dfrac{\omega_o}{2\pi} = \dfrac{5000}{\pi} = 1591.55$ Hz

[b] $V_m = \dfrac{600}{25}\times 5 = 120$ V

[c] $R_e = \dfrac{(20)(5)}{25} = 4\,\text{k}\Omega$

$Q = (10^4)(4\times 10^3)(0.25\times 10^{-6}) = 10$

$\beta = \dfrac{\omega_o}{Q} = \dfrac{10^4}{10} = 1000$ rad/s

[d] $Q = 10$

[e] $\omega_1 = \omega_o\left[-\dfrac{1}{20} + \sqrt{\dfrac{1}{400}+1}\right] = 9{,}512.49$ rad/s

$\omega_2 = \omega_o\left[\dfrac{1}{20} + \sqrt{\dfrac{1}{400}+1}\right] = 10{,}512.49$ rad/s

[f] $Q\,(\text{with}) = 10$

$Q\,(\text{without}) = (10^4)(5000)(0.25\times 10^{-6}) = 12.5$

$\therefore\ Q\,(\text{with}) = 0.80Q\,(\text{without})$

8.

[a] $\omega_o^2 = \dfrac{10^3 \times 10^9}{(500)(5)} = \dfrac{10^{12}}{2500} = \dfrac{10^{10}}{25}$

$\omega_o = 20{,}000\,\text{rad/s} = 20\,\text{krad/s}$

[b] $V_{mo} = 50\,\text{V}$

[c] $\mathbf{V}_o = \dfrac{\mathbf{V}_g}{Z}(2000) = \dfrac{(50\angle 0^\circ)(2000)}{Z}$

$|\mathbf{V}_o| = \dfrac{(2000)(50)}{|Z|} = 0.80(50); \qquad \therefore\ |Z| = 2500$

$2000\left|1 + jQ\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)\right| = 2500$

$Q = \dfrac{\omega_o L}{R} = \dfrac{20 \times 10^3(500 \times 10^{-3})}{2000} = 5$

$1 + 25\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)^2 = (1.25)^2$

$25\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{w}\right)^2 = 0.5625$

$5\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right) = \pm 0.75$

$\omega^2 - \omega_o^2 = \pm 0.15\omega\omega_o \quad \text{or} \quad \omega^2 \mp 3000\omega - 4 \times 10^8 = 0$

$\omega = \pm 1500 \pm \sqrt{2{,}250{,}000 + 4 \times 10^8} = \pm 1500 \pm 10^4\sqrt{4.0225}$

$= \pm 1500 \pm 20{,}056.17$

$\omega_1 = 18{,}556.17\,\text{rad/s}; \qquad \omega_2 = 21{,}556.17\,\text{rad/s}$

$\therefore\ 18{,}556.17 \le \omega \le 21{,}556.17\,\text{rad/s}$