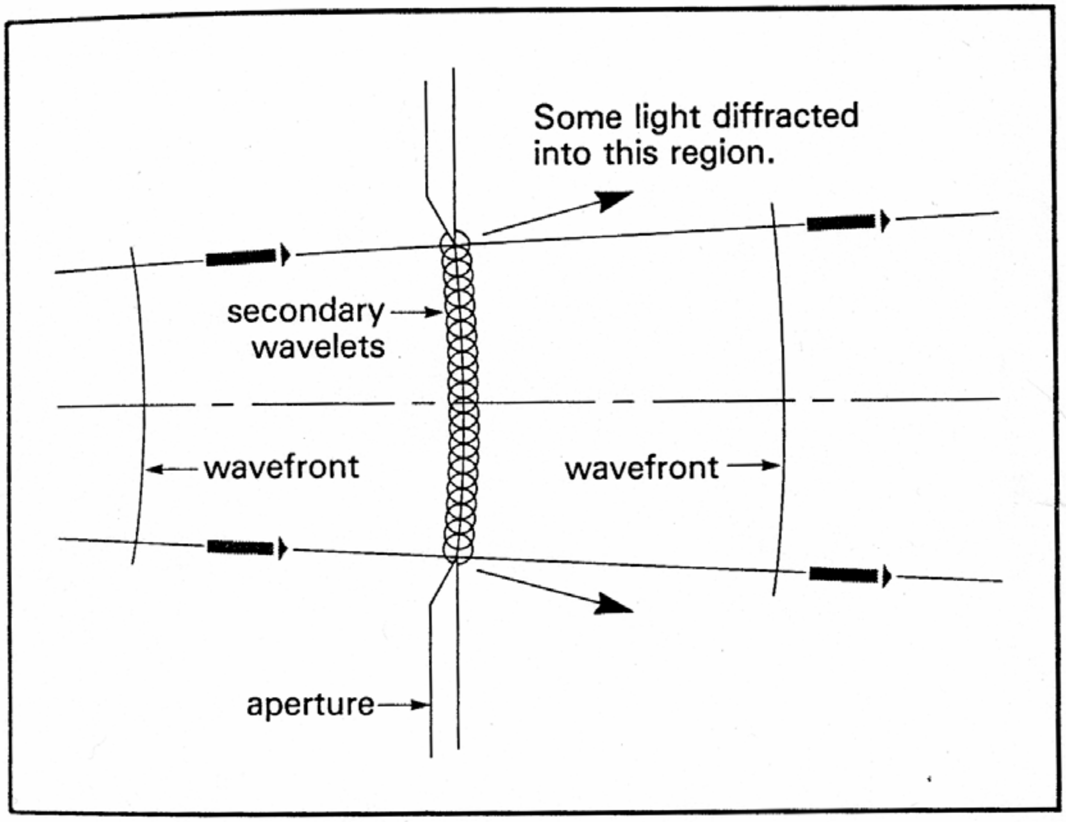
Diffraction

- Diffraction occurs when light waves pass through an aperture
- Huygen's Principal: each point on wavefront
  acts as source of another wave
- If light coming from infinity
  point source at infinity or parallel beam (laser)
- Light at slit edge diffracts
- Interference effects between the waves at each point changed



HUYGEN'S PRINCIPLE states that each point on a propagating wavefront is an emitter of secondary wavelets.

Fresnel and Fraunhofer Interference

- Both assume light source at infinity
- Near parallel light – e.g. laser beam

Fresnel interference

- Pattern created near the diffraction point
- Much more complex equations
- Pattern very dependent on the distance & slit

Fraunhofer Interference

- Diffracted light sensed at infinity
- If focus slit with lens get the same as at infinity
- Most common effect

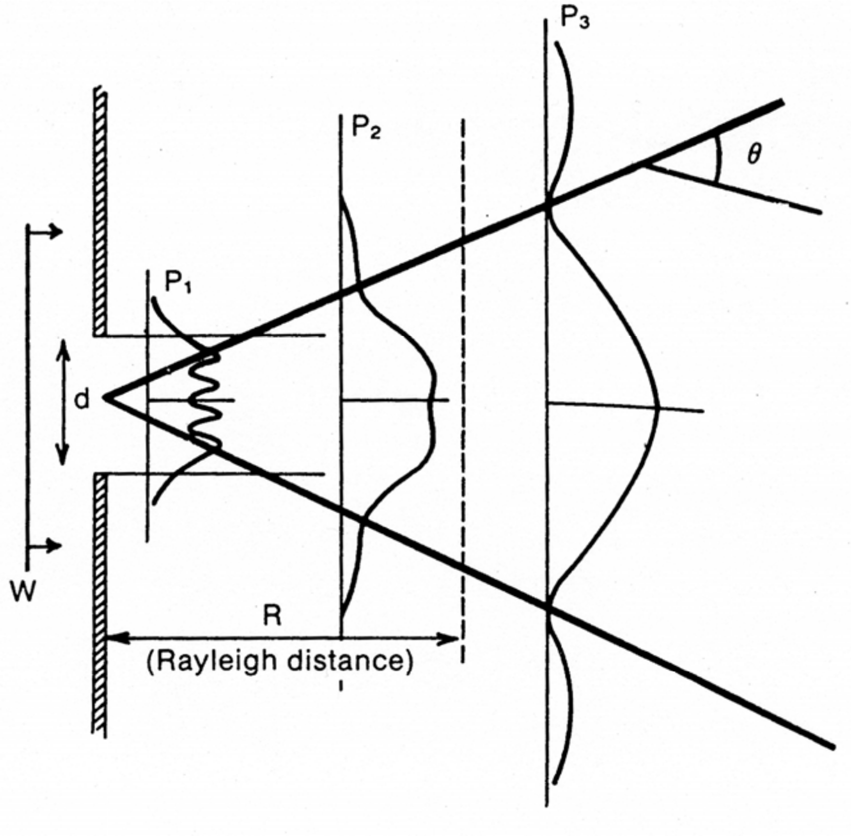


Fig. 2.8.14. Transition from Fresnel to Fraunhofer diffraction. A portion of a wave, *W*, passes through a slit of width, *d*. Intensity distributions across the wave are shown for planes P_1 (close to the slit), P_2 (just inside the Fresnel distance), and P_3 (beyond the Fresnel distance).

Fraunhofer Interference

- For single slit width b
- Intensity follows the pattern of a synch function

$$I(\beta) = I_0 \left[\frac{sin(\beta)}{\beta} \right]^2$$

where

$$\beta = \frac{\pi b \, sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

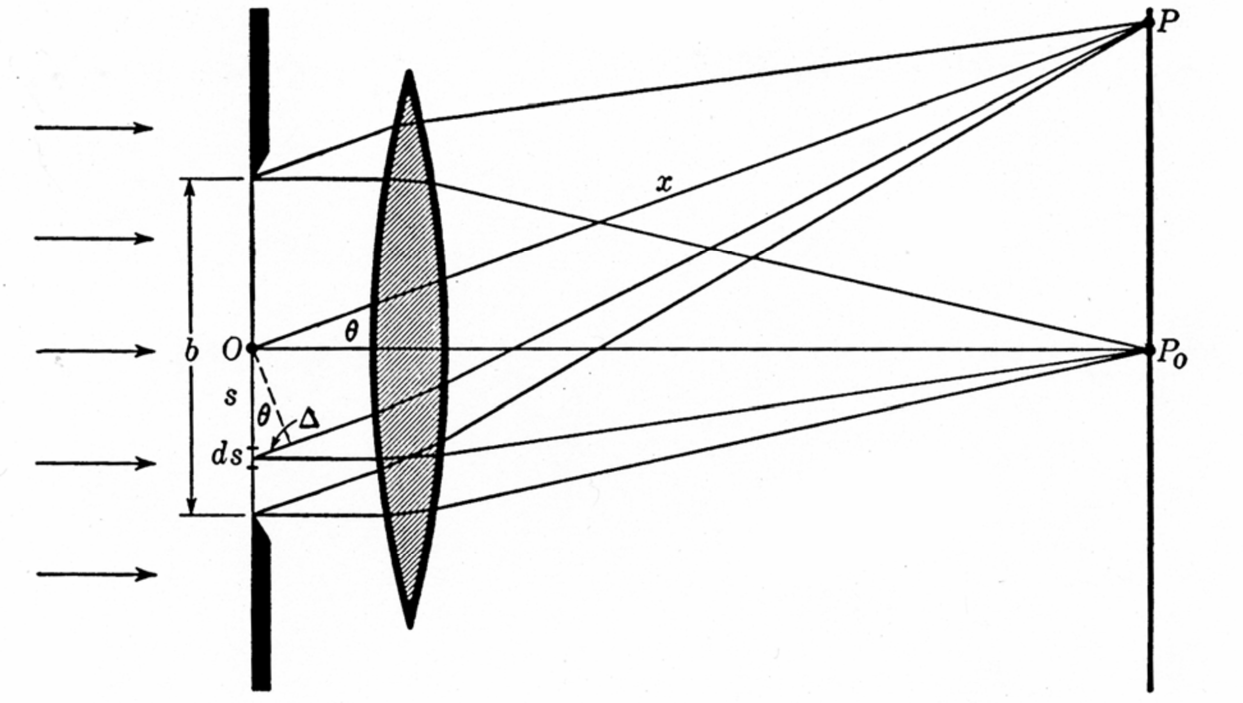


FIGURE 15C
Geometrical construction for investigating the intensity in the single-slit diffraction pattern.

Fraunhofer Interference Pattern

- Zeros are at

$$\beta = \pm N\pi$$

where N is any integer

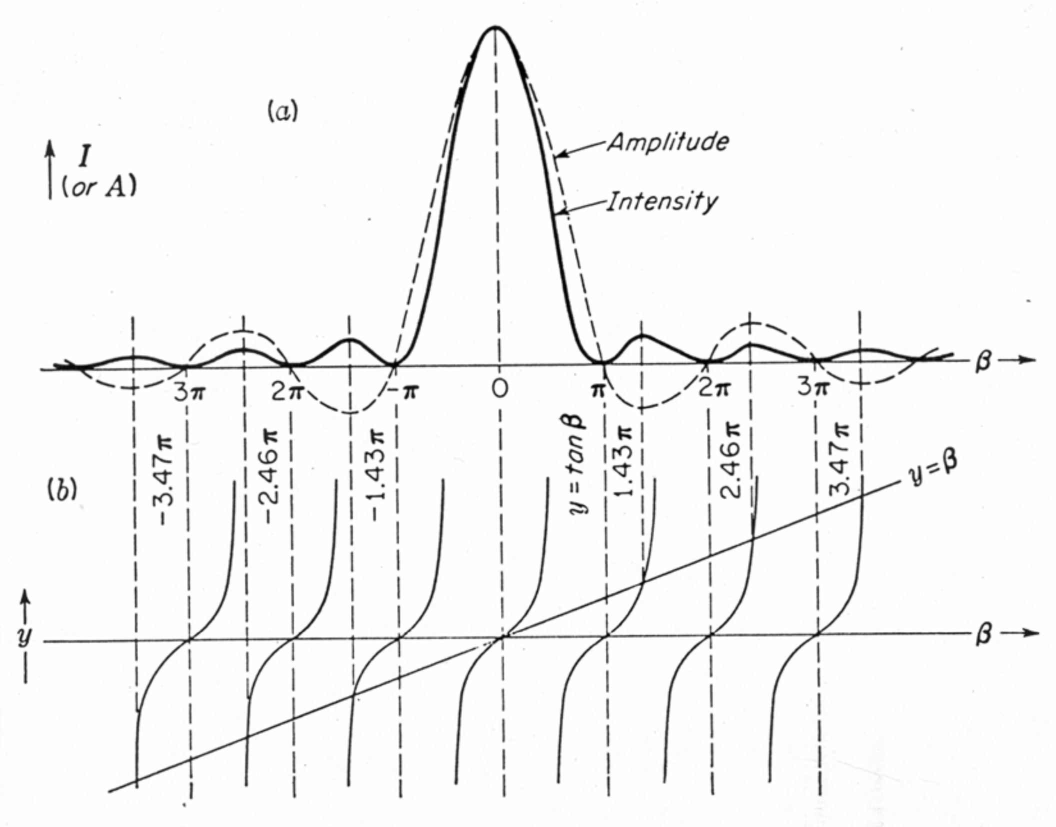


FIGURE 15D
Amplitude and intensity contours for Fraunhofer diffraction of a single slit, showing positions of maxima and minima.

- Large d little pattern, small d pattern spreads out

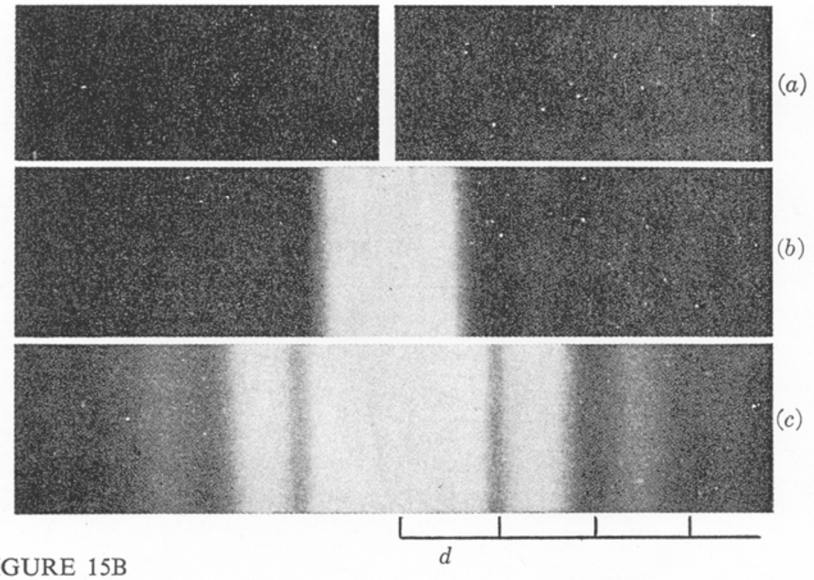


FIGURE 15B
Photographs of the single-slit diffraction pattern.

Circular Fraunhofer Interference

- Interference changes for circular opening
- Most important for laser systems Lenses act as circular apertures
- Called an Airy Disk
- For single circular aperture diameter D
- Intensity follows the pattern

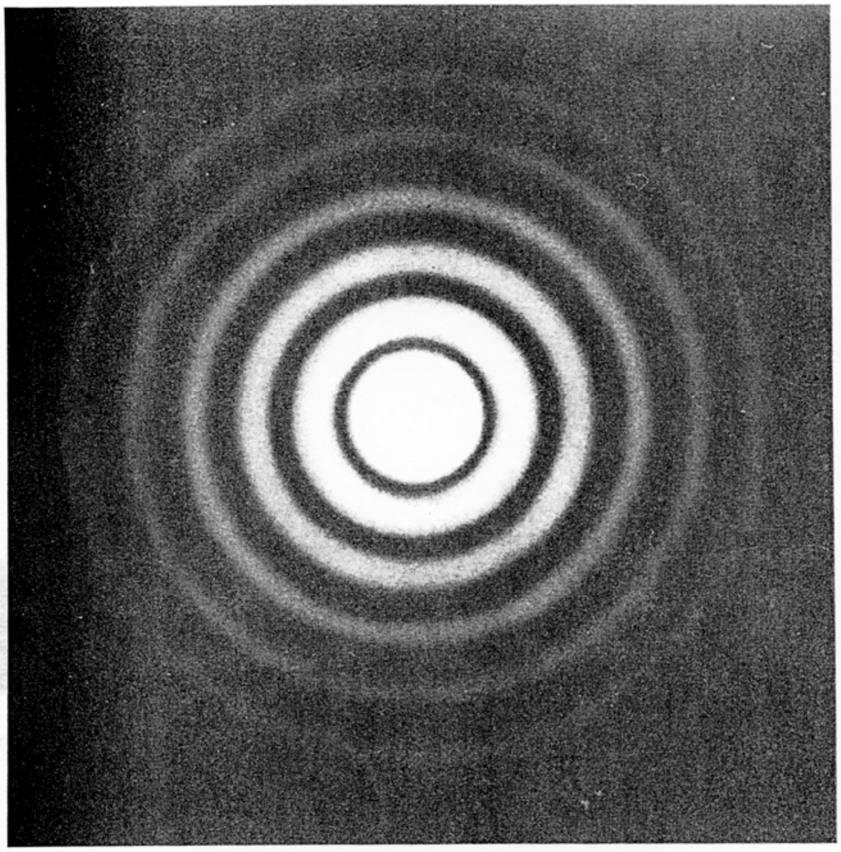
$$I(\beta) = I_0 \left[\frac{J_1(\beta)}{\beta} \right]^2$$

where

$$\beta = \frac{\pi D \sin(\theta)}{\lambda}$$

J_1 = Bessel function of first kind, order 1

θ = angular deviation of pattern from minimum

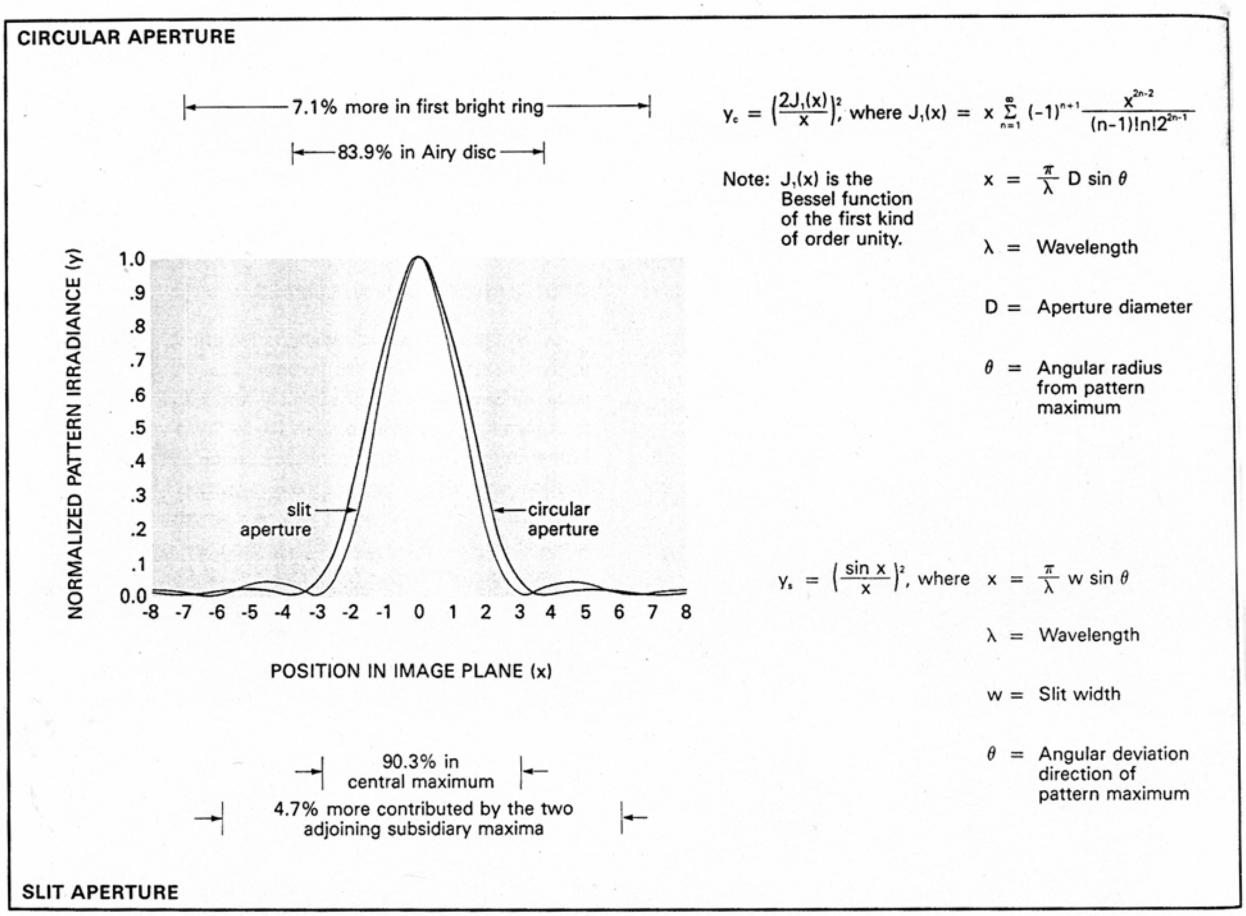


(c)

Fig. 2.8.2. Diffraction limitation of lenses: (a) square lens; (b) picture of diffraction due to a circular obstruction; and (c) a circular lens. (From M. Cagnet et al., *Atlas of Optical Phenomenon*, Springer-Verlag, New York, 1962.)

Comparison of Circular and Slit Fraunhofer Interference

- Slit produces smaller width pattern



FRAUNHOFER DIFFRACTION PATTERN of a singlet slit superimposed on the Fraunhofer Diffraction Pattern of a circular aperture.

Diffraction Limited spot

- If laser beam fills the lens then diffraction limited
- Opening of width D
- Minimum spot is to point of first zero in diffraction

$$I = \frac{b\sin(\theta)}{\lambda} = \frac{bd_{min}}{\lambda 2f}$$

$$d_{min} = \frac{2f\lambda}{D}$$

- Since circular effectively Airy diffraction
  add a factor of 1.22

$$d_{min} = \frac{1.22 f\lambda}{D}$$

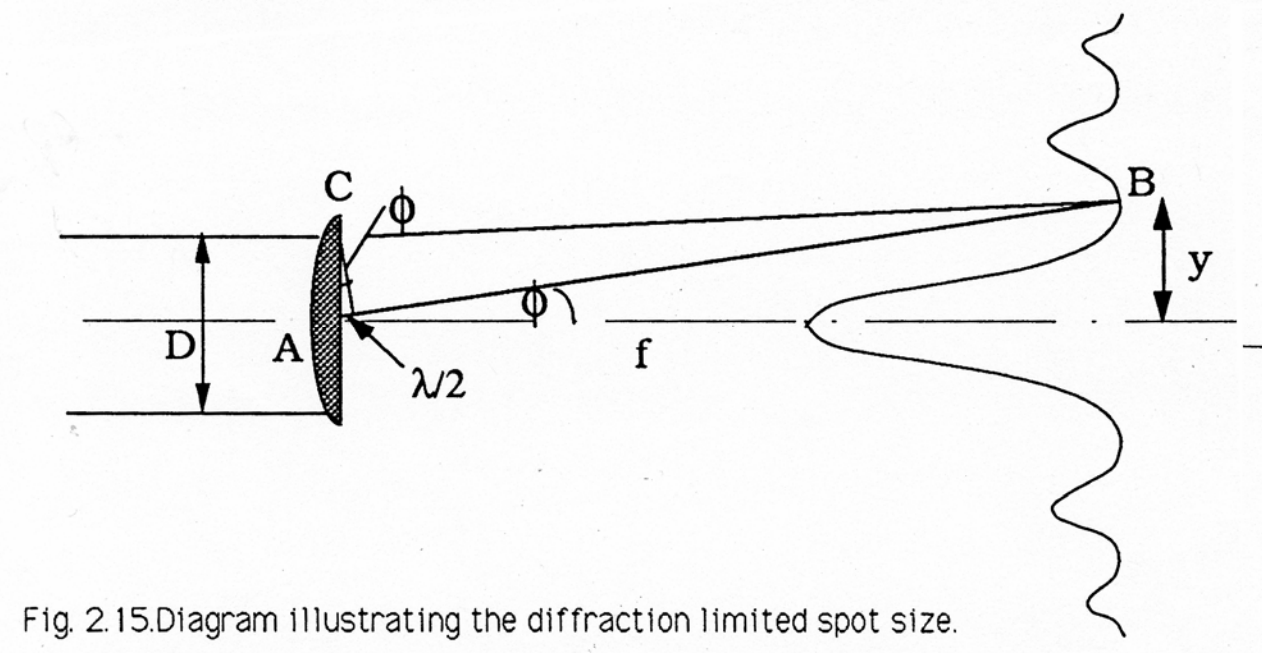


Fig. 2.15.Diagram illustrating the diffraction limited spot size.

Young’s Double Slit Experiment

- Now consider 2 slits width b
- Separated by space a (centre to centre of slits)
- Now the pattern created by one slit creates interference with other

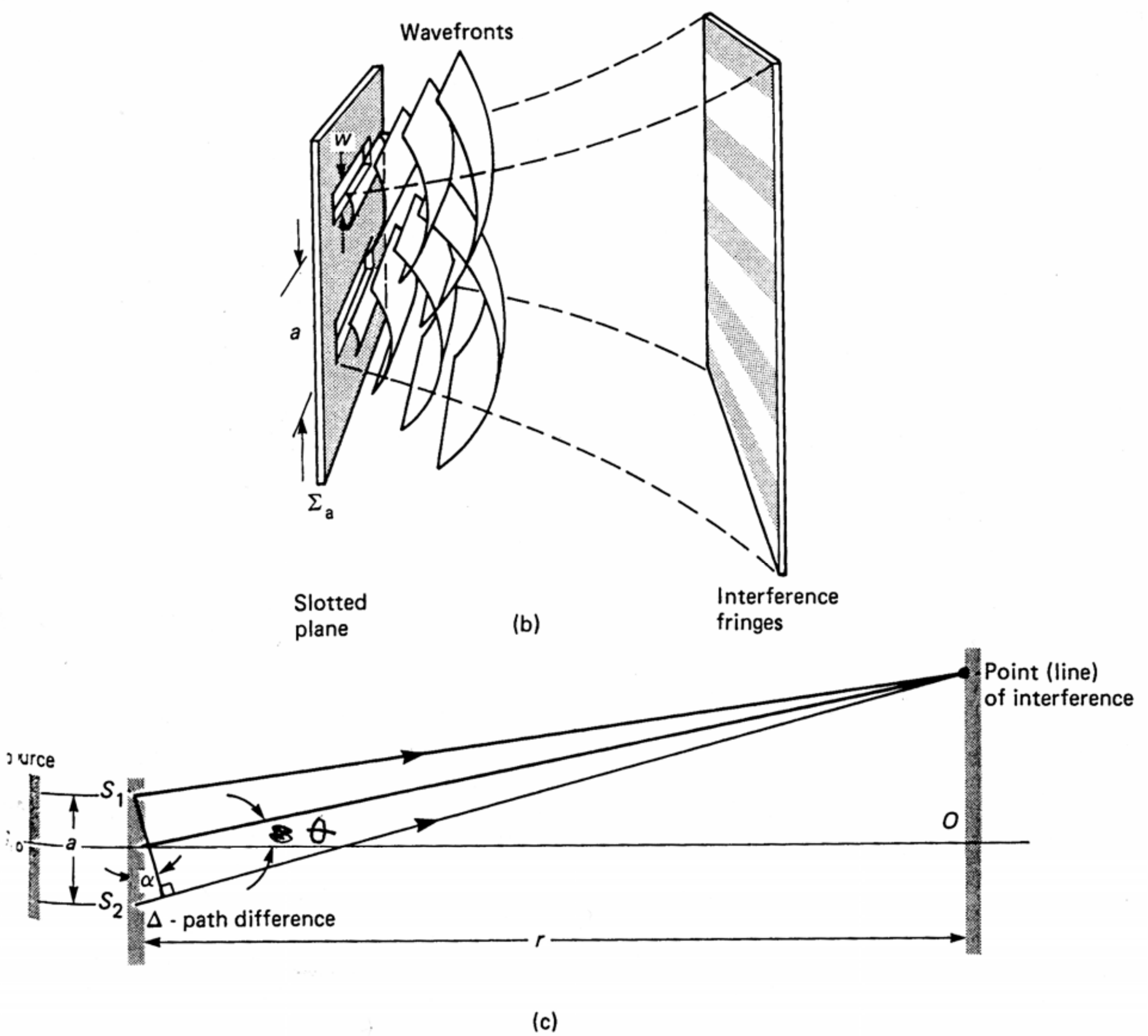


Figure 10.20 (a, b) Interference fringes formed when a monochromatic coherent plane wave strikes a thin opaque plane with two neighbouring narrow slits in it (the Young Experiment). (c) The common source S_o ensures coherence

Resolution of Spots

- Really want the separation of two spots
- When spots fully separated then can resolve

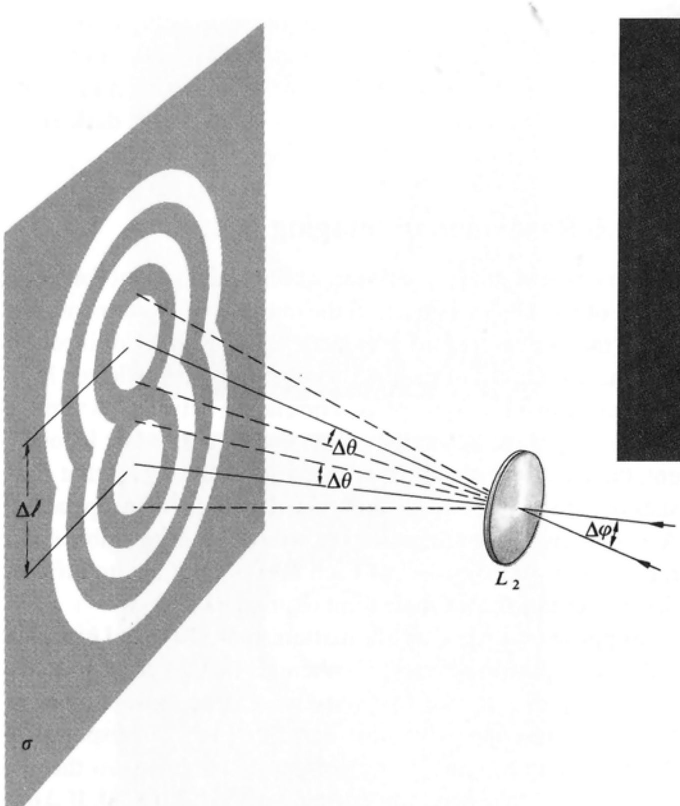
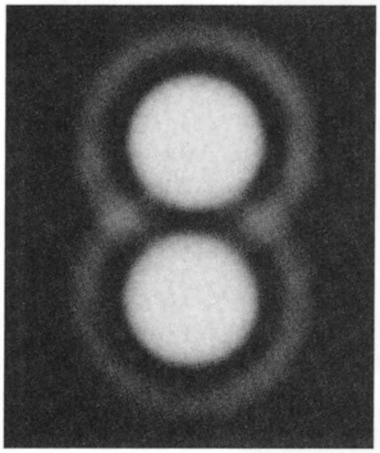


Figure 10.24 Overlapping images.



Overlapping Images

- When spots overlap cannot separate
- Different systems determine how much overlap allowed

$$d_{min} = \frac{1.22\, f\lambda}{D}$$

- This is most common but also see twice this

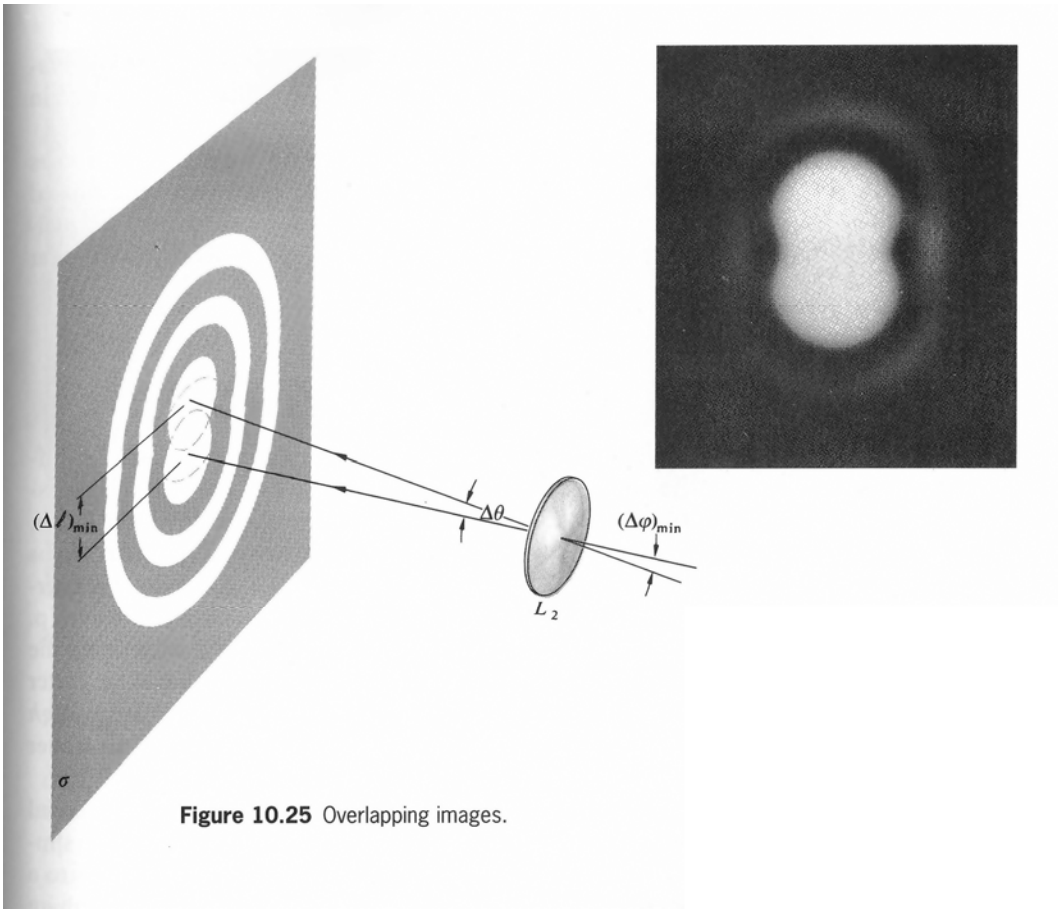


Figure 10.25 Overlapping images.

Double Slit Interference

- Get the single slit pattern forming envelope
- Interference of two slits modulating that.

$$I(\beta) = 4I_0\left[\frac{sin(\beta)}{\beta}\right]^2 cos(\alpha) \qquad \beta = \frac{\pi\, b\, sin(\theta)}{\lambda} \quad \alpha = \frac{\pi\, a\; sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

- For zeros: $\beta = \pm N\pi$
- Principal Maximums occur at

$$sin(\theta) = \frac{m\lambda}{a}$$

where m = any integer, order of the diffraction

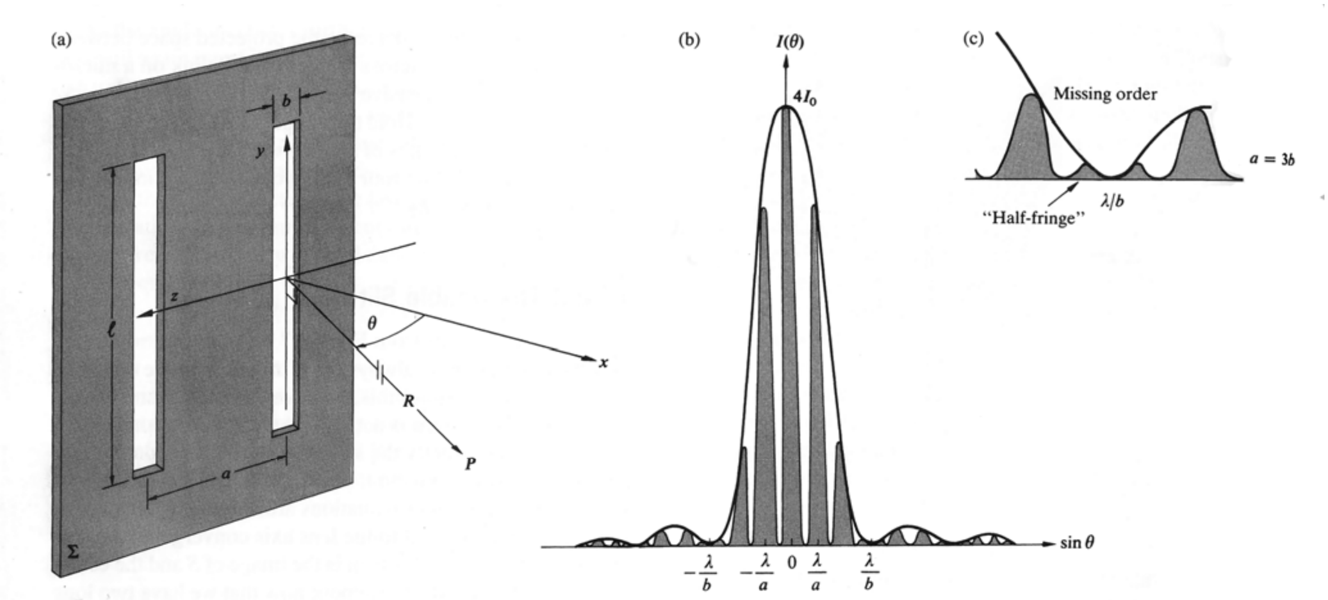
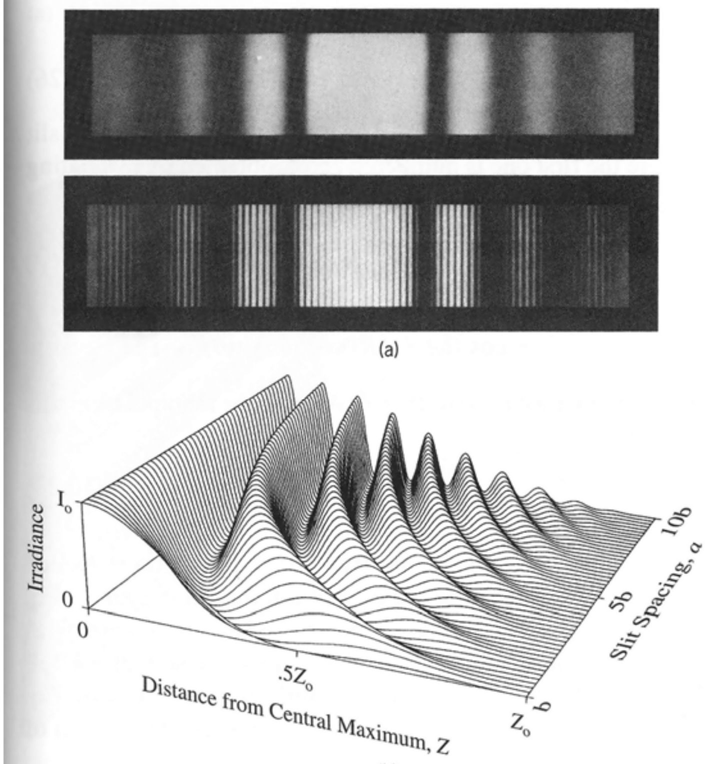


Figure 10.13 (*a*) Double-slit geometry. Point *P* on σ is essentially infinitely far away. (*b*) A double-slit pattern ($a = 3b$).

Young's Double Slit & Single Slit

- If take single slit
- Then add second slit see the one pattern on top of other



(a)

(b)

Figure 10.14 Single- and double-slit Fraunhofer patterns. (*a*) Photographs taken with monochromatic light. The faint cross-hatching arises entirely in the printing process. (Photos courtesy M. Cagnet, M. Francon, and J. C. Thrierr: *Atlas optischer Erscheinungen*, Berlin–Heidelberg–New York: Springer, 1962.) (*b*) When the slit spacing equals *b*, the two slits coalesce into one (of width 2*b*) and the single-slit pattern appears—that's the first curve closest to you. The farthest curve corresponds to the two slits separated by $a = 10b$. Notice that the two-slit patterns all have their first diffraction minimum at a distance from the central maximum of Z_0. Note how the curves gradually match Fig. 10.13*b* as the slit width *b* gets smaller in comparison to the separation *a*. (Reprinted from "Graphical Representations of Fraunhofer Interference and Diffraction" *Am. J. Phys.*, 62, 6, (1994), with permission of A. B. Bartlett, University of Colorado and B. Mechtly, Northeast Missouri State University and the American Association of Physics Teachers.)

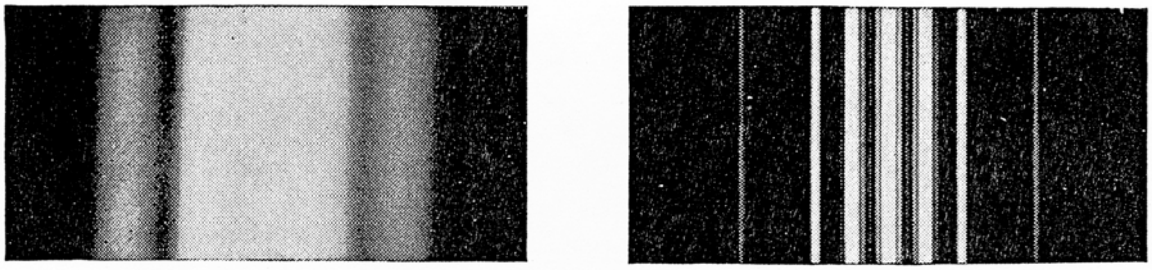
Diffraction Gratings

- Diffraction gratings used by many systems
  eg spectrometers, acousto-optic deflectors
- Recall the Interference from a single slit
  width b seen at a long distance (Fraunhofer)

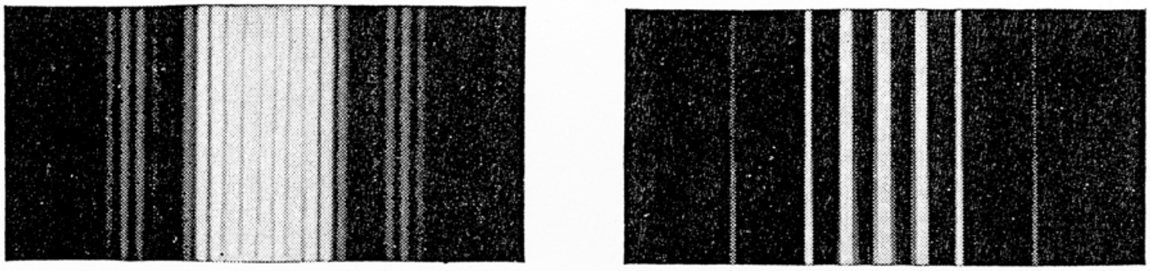
$$I(\beta) = I_0\left[\frac{\sin(\beta)}{\beta}\right]^2 \qquad \beta = \frac{\pi b \sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

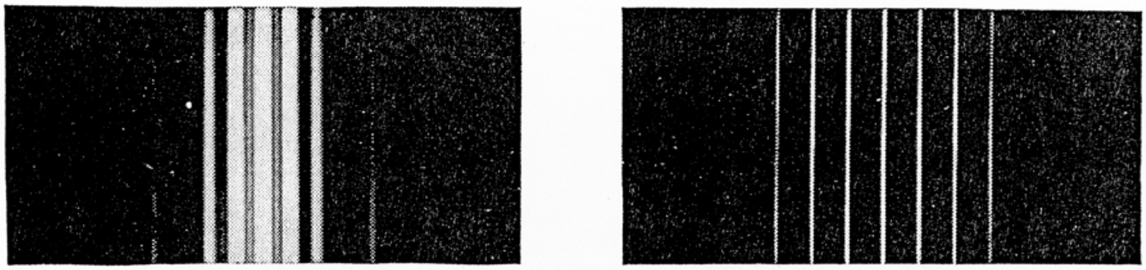
- For zeros: $\beta = \pm N\pi$
- If have several slits then waves from each interfere
- More slits narrower beams



(a) 1 slit (d) 5 slits



(b) 2 slits (e) 6 slits



(c) 3 slits (f) 20 slits

FIGURE 17A
Fraunhofer diffraction patterns for gratings containing different numbers of slits.

Diffraction Gratings

- Now consider N slits with b spaced distance d apart
- Get the diffraction pattern from each slit
- But the diffraction patterns interfere

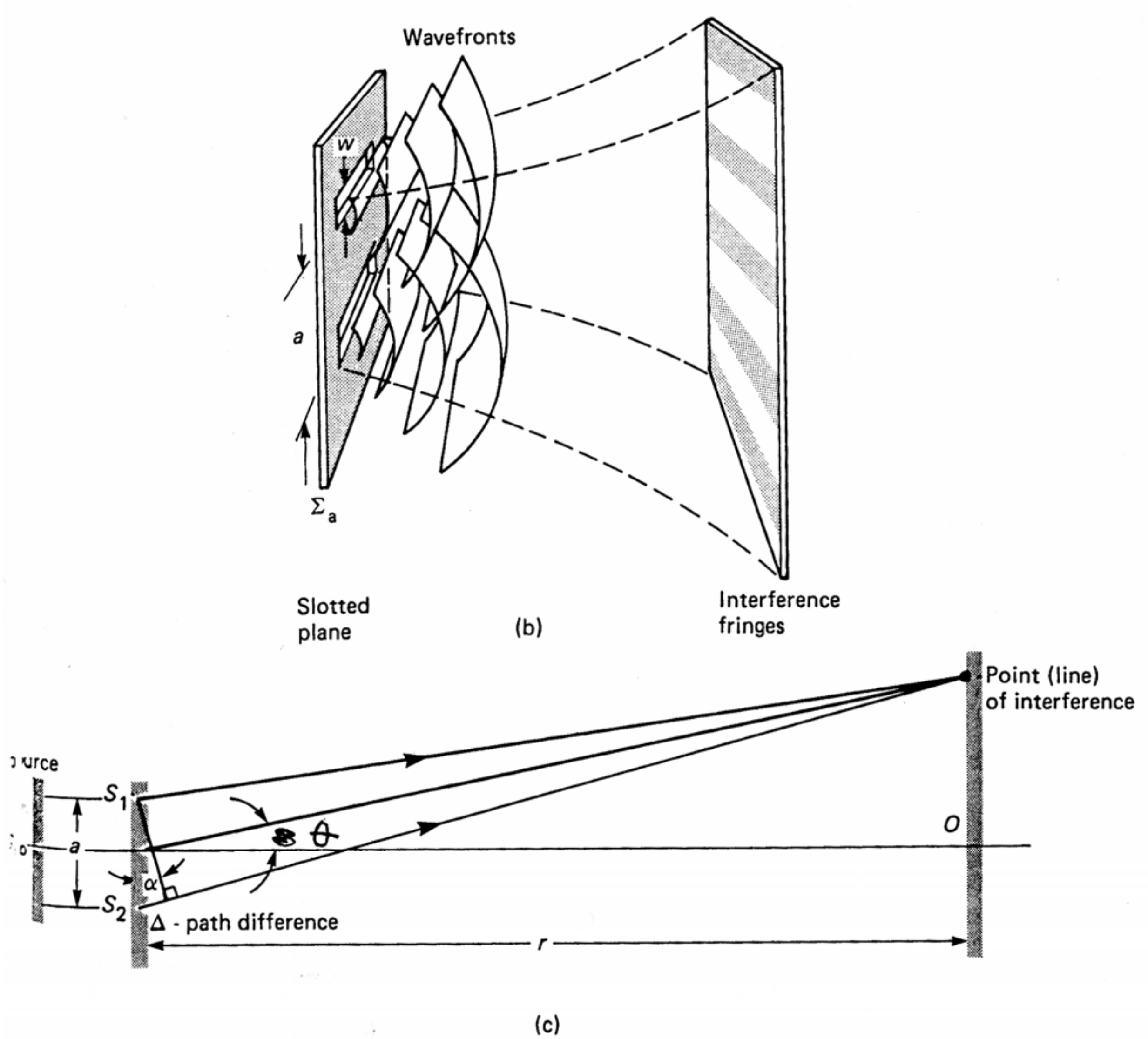


Figure 10.20 (a, b) Interference fringes formed when a monochromatic coherent plane wave strikes a thin opaque plane with two neighbouring narrow slits in it (the Young Experiment). (c) The common source S_o ensures coherence

Diffraction Gratings Formulas

- Similar to the single slit the intensity becomes for n slits

$$I(\beta)=I_0\left[\frac{sin^2(\beta)}{\beta^2}\right]\left[\frac{sin^2(n\gamma)}{sin^2(\gamma)}\right] \qquad \gamma=\frac{\pi d\ sin(\theta)}{\lambda}$$

- Principal Maximums occur at

$$sin(\theta)=\frac{m\lambda}{d}$$

where m = any integer, order of the diffraction

- The maxima vary with the single slit β function

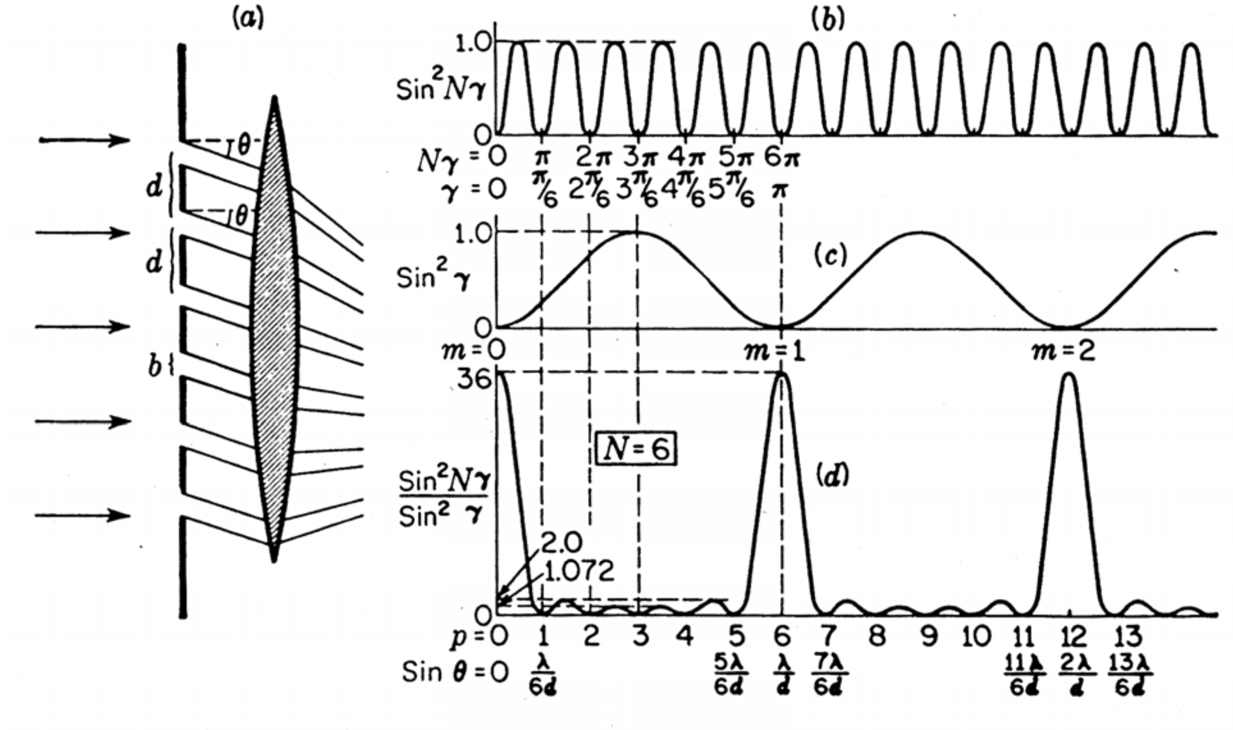


FIGURE 17C
Fraunhofer diffraction by a grating of six very narrow slits and details of the intensity pattern.

Diffraction Gratings as Deflectors

- For large N gratings the Principal Maxima are narrow angles
- Hence beams deflected to specific angles
- Can create deflector by selecting beam angle

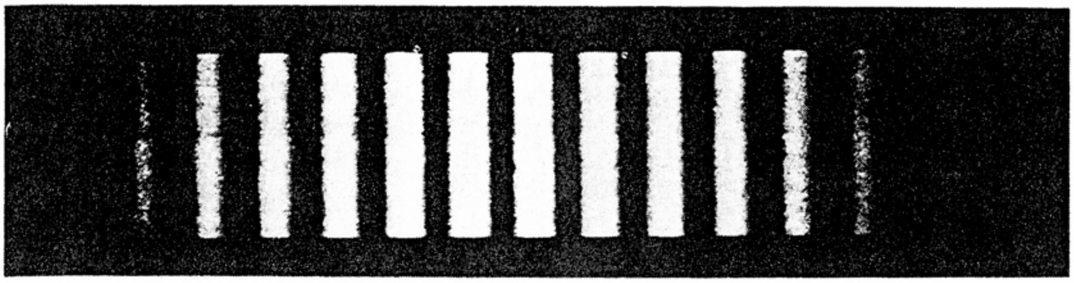


Figure 10.21 Interference fringes

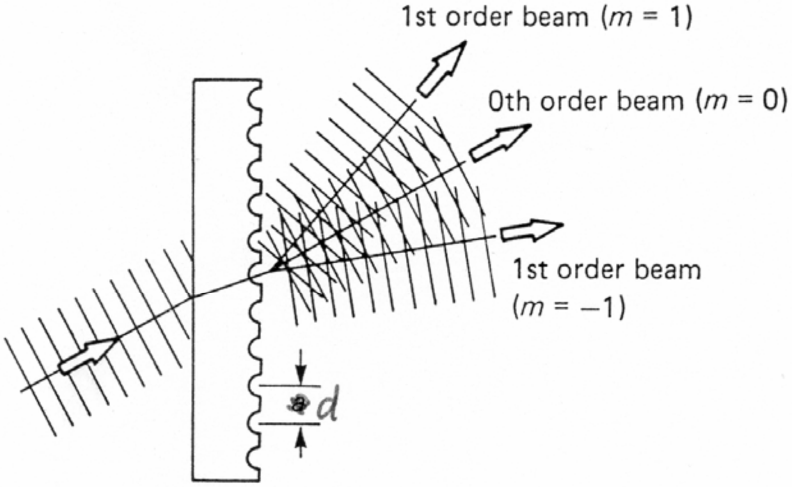
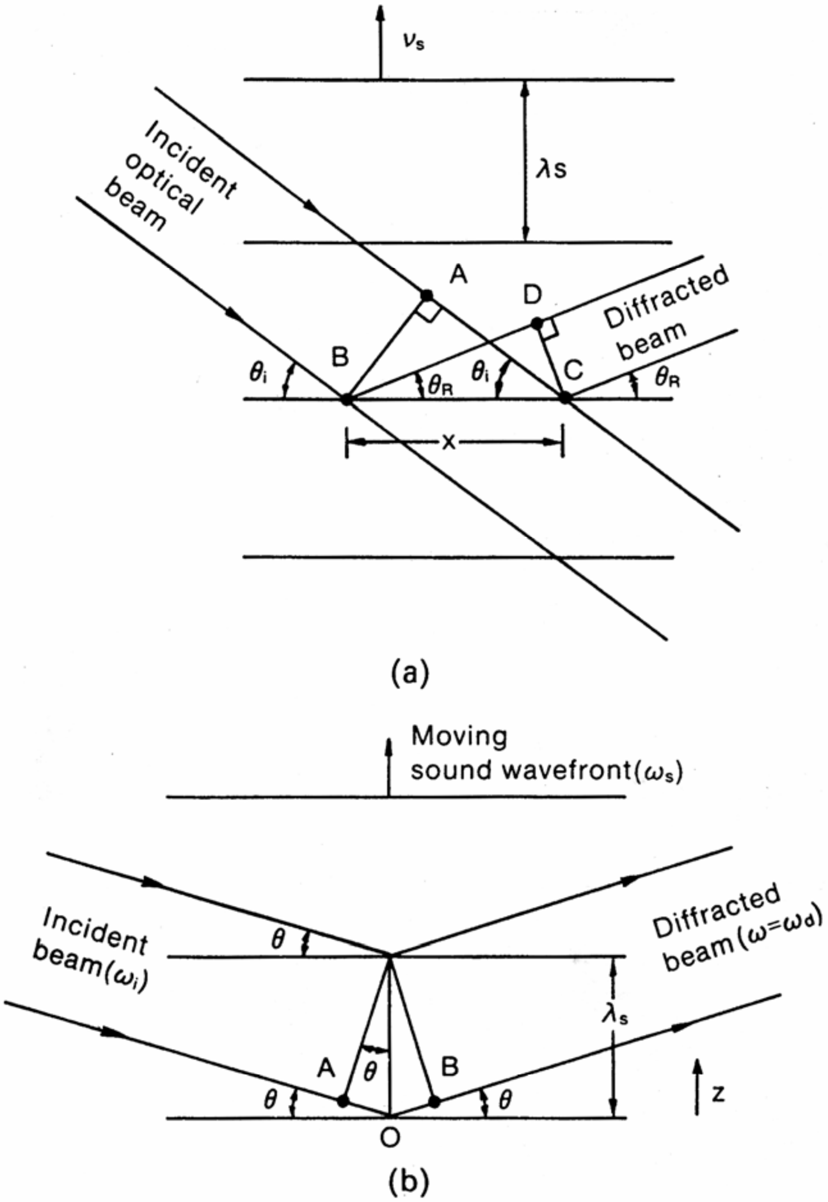


Figure 10.22 A diffraction grating produces several beams, each leaving the structure at a different angle

Acousto-Optic Deflectors

- Consider a material whose index of refraction is significantly changed by acoustic waves
- Eg. Lithium Niobate, quartz
- A piezoelectric transducer attached to one end
- Apply ultrasonic waves, eg 40 MHz
  creates a diffraction grating from index changes
  wavelength λ_s



(a)

(b)

Fig. 2.12.22. Acousto-optic interaction with incident and diffracted light beams. The horizontal lines separated by the acoustic wavelength λ_s represent the moving sound beam.

Acousto-Optic Deflectors

- If beam enters crystal at angle θ
- The it will be deflected constructively when

$$2\lambda_s \sin(\theta) = m\lambda$$

where m is any integer

- Typical defection is about 0.5 degrees
- Use slits to select only the desired beam
- Called a Bragg Cell
  (Angle for only one output is Bragg angle)

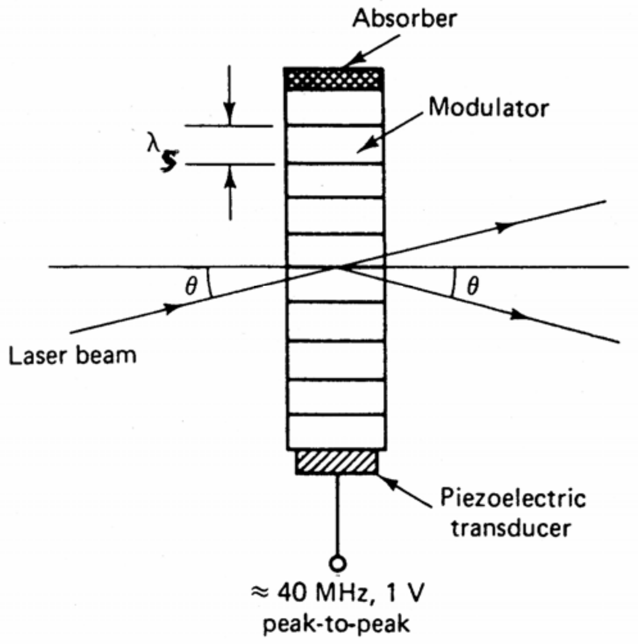


Figure 1-31 Acousto-optical modulator.

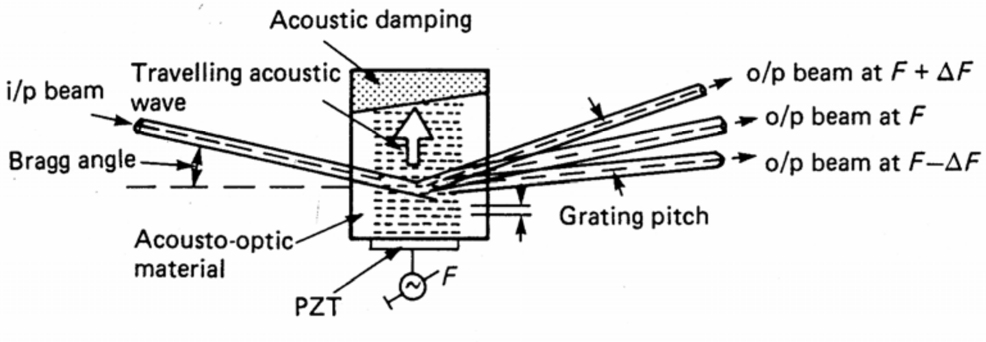


Figure 10.24 The Bragg cell. Incoming radiation must be directed at the diffraction plate at the Bragg angle. Zero-order beam not shown

Acousto-Optic Analogue Modulators

- Use the Bragg Cell for deflections
- Focus output through a slit
- By deflecting beam change intensity through slit
- Focus light from slit into parallel beam

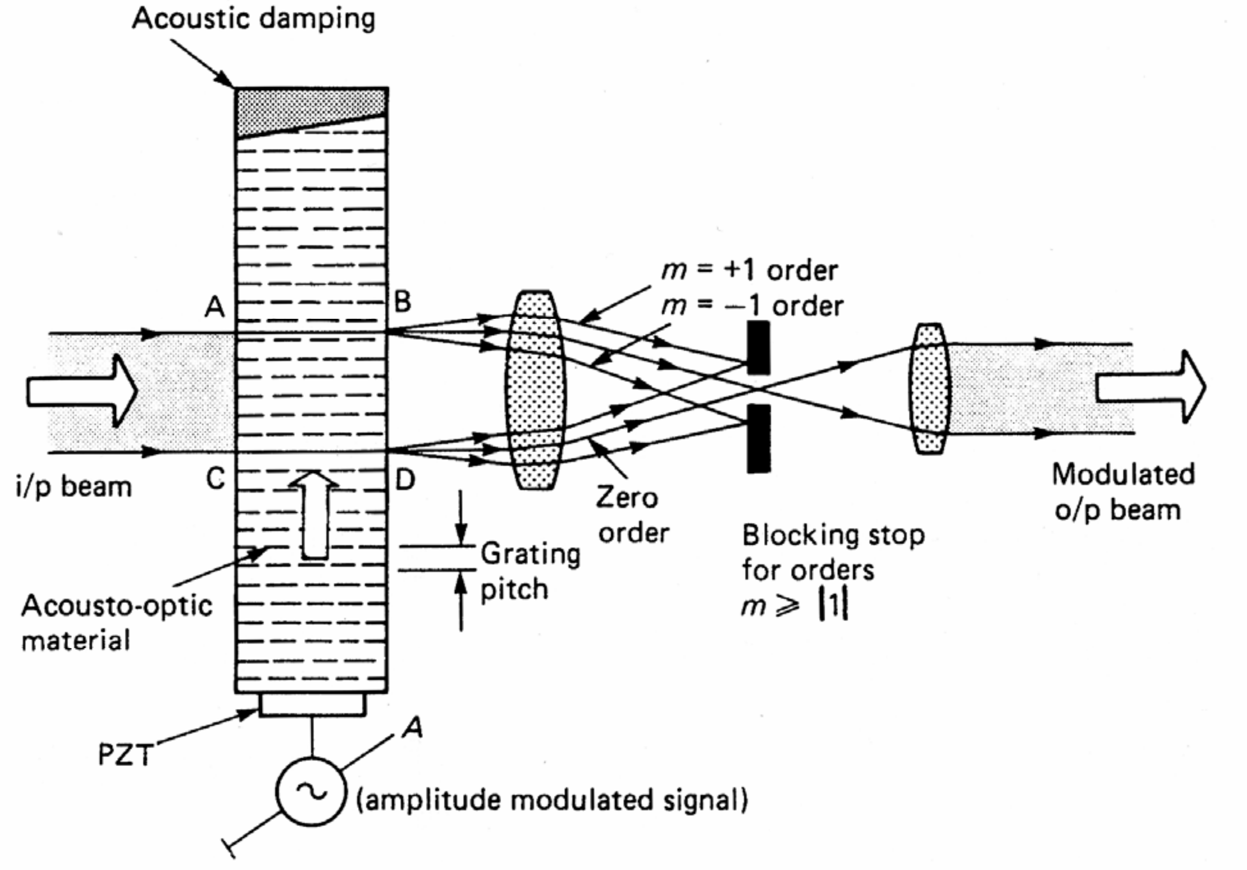


Figure 10.32 Analogue modulation with an AO crystal. Beam intensity depends on the amplitude of the modulating signal