Mirror Example

- Consider a concave mirror radius -10 cm then

$$f = -\frac{r}{2} = -\frac{-10}{2} = 5\,cm$$

- Now consider a 1 cm candle s = 15 cm from the vertex
- Where is the image

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{r} = \frac{1}{f}$$

$$\frac{1}{s'} = -\frac{2}{r} - \frac{1}{s} = \frac{1}{5} - \frac{1}{15} = 0.13333 \qquad s' = \frac{1}{0.1333} = 7.5\,cm$$

- Magnification $m = \frac{M'}{M} = -\frac{s'}{s} = -\frac{7.5}{15} = -0.5$
- Thus image is inverted and half size of object
- What if candle is at 10 cm (radius of curvature)

$$\frac{1}{s'} = -\frac{2}{r} - \frac{1}{s} = \frac{1}{5} - \frac{1}{10} = 0.1 \quad s' = \frac{1}{0.1} = 10\,cm \qquad m = -\frac{s'}{s} = -\frac{10}{10} = -1$$

Image is at object position (10 cm) inverted and same size (1 cm)

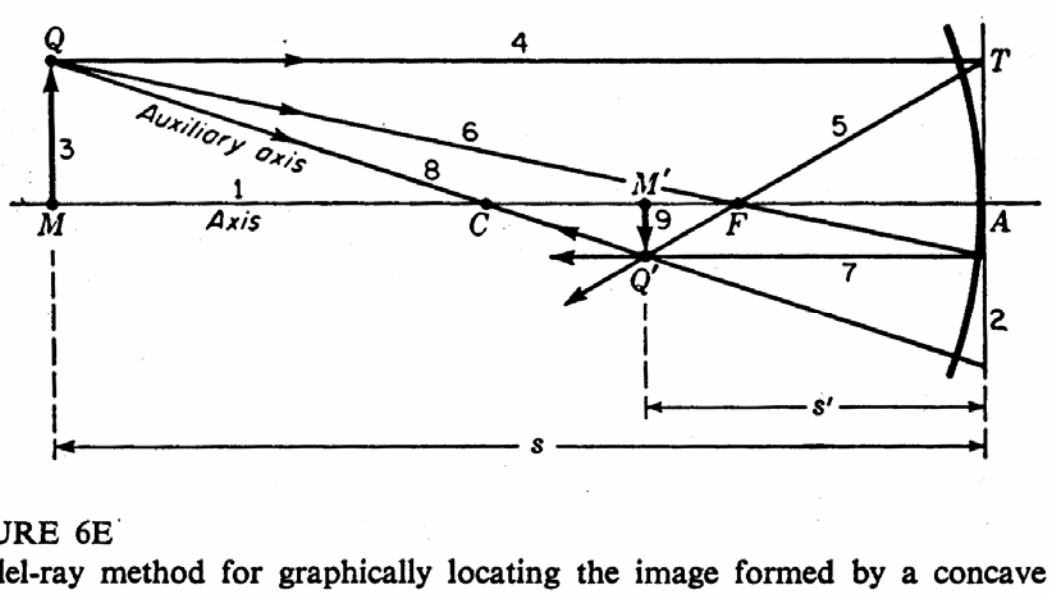


FIGURE 6E
Parallel-ray method for graphically locating the image formed by a concave mirror.

Objects less than Focal Length

- Now consider object at 2.5 cm (smaller than f = 5 cm)

$$\frac{1}{s'} = -\frac{1}{f} - \frac{1}{s} = \frac{1}{5} - \frac{1}{2.5} = -0.2 \quad s' = \frac{1}{-0.2} = -5\ cm \quad m = -\frac{s'}{s} = -\frac{-5}{2.5} = 2$$

- Image appears to be behind the mirror by 5 cm
- Image is virtual – light is expanding from mirror
- Image is erect and twice object size
- Do not see image if place something at image position

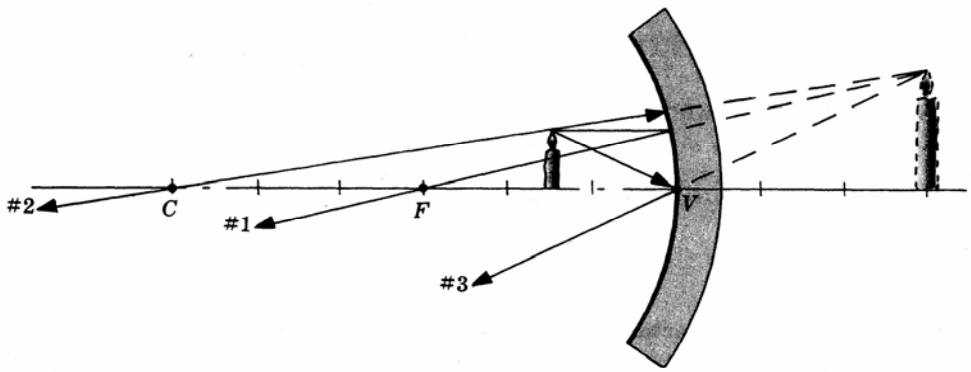


Fig. 4-49

Graphic Method of Solving Optics

- Graphic method is useful in thinking about what happens
- Use some scale (graph paper good)
- Place mirror on axis line and mark radius C & focal F points
- Draw line from object top Q to mirror parallel to axis (ray 4)
- Hits vertex line at T
- Then direct ray from T through focus point F (ray 5) and beyond
- Now direct ray from object top Q through radius C (ray 8)
- This intersects ray 5 at image Q' (point 9)
- This correctly shows both position and magnification of object
- This really shows how the light rays are travelling
- Eg Ray through the focal point F (ray 6) becomes parallel (ray 7)
- Intersects ray 5 again at image Q'

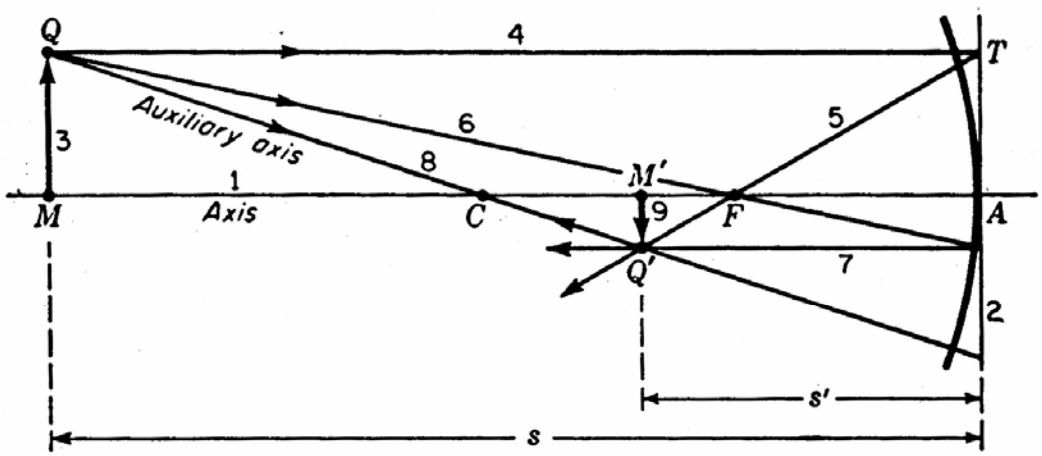
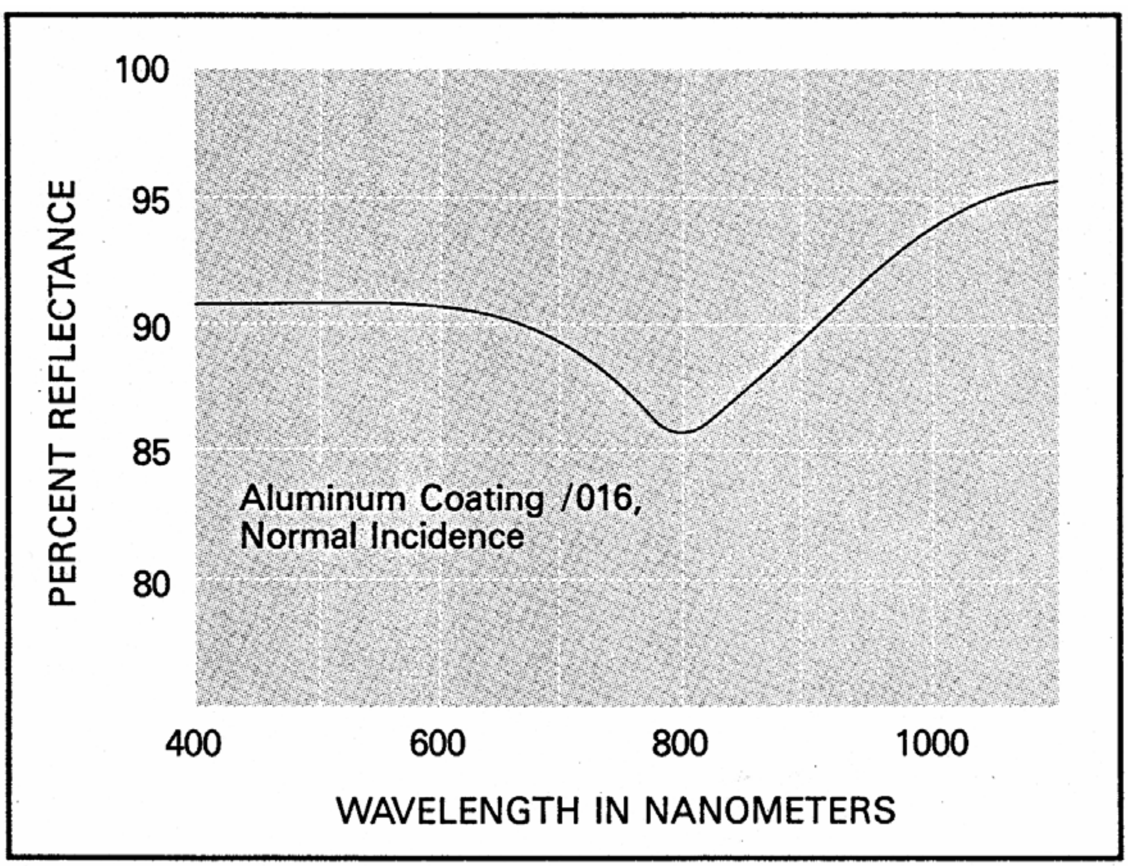


FIGURE 6E
Parallel-ray method for graphically locating the image formed by a concave mirror.

Mirror Coatings

- Classic mirrors use metallic coatings
- Most optics mirrors front surface mirror
- Regular mirrors back surface (coating on glass)
- Problem for optics (reflection both from glass & metal surface)
- Aluminium most commons now: 90-92% reflective
- Often coated for protection with transparent film (aluminium oxide)
- Silver mirrors higher reflection 95%
- Must be coated or fail in 6 months
- Gold mirrors for IR systems
- For lasers Al mirrors problem is ~8% absorption
- Film gets damaged by laser energy absorbed
- Typical limit 50 W/cm^2 CW, 10 mJ/cm^2 for 10 nsec pulse
- Need to watch cleaning as they scratch easily



Mirror Substrates

Pyrex

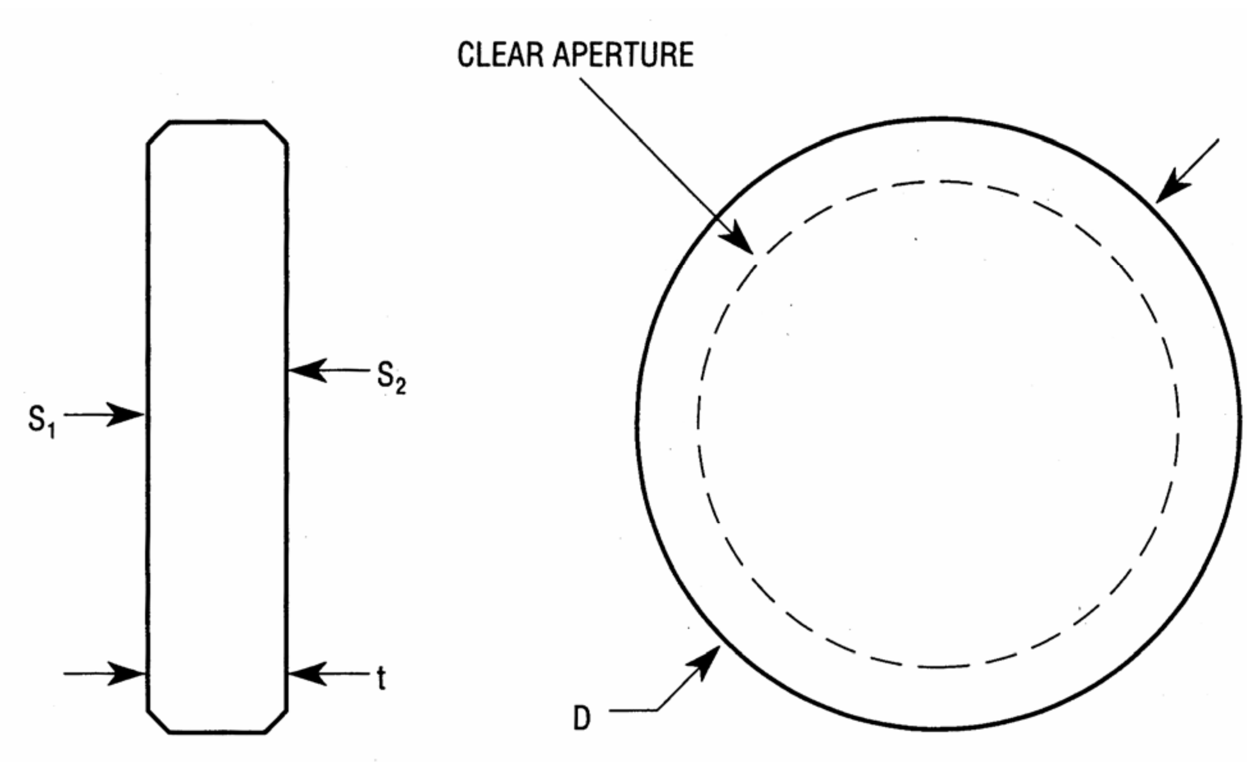
- Typical substrate pyrex: BK7
- Low deformation with heating
- Good surface polish
- Typical size: 1 inch diameter, 0.5 inch thick
- Must be platinum free
- Price of substrate ~$100

Glass-Ceramic materials

- eg Newport's Zerodur
- designed for low thermal expansion
- Used were there must be not thermal changes
- Price of Substrate ~$130

Fused Silica (Quartz)

- High thermal stability
- Extremely good polishing characteristics
- 3 times price of Pyrex



Lenses & Prism

- Consider light entering a prism
- At the plane surface perpendicular light is unrefracted
- Moving from the glass to the slope side
  light is bent away from the normal of the slope
- Using Snell's law

$$n \sin(\varphi) = n' \sin(\varphi')$$

$$1 \sin(\varphi') = 1.75 \sin(30^o) = 0.875$$

$$\varphi' = \arcsin(0.875) = 61^o$$

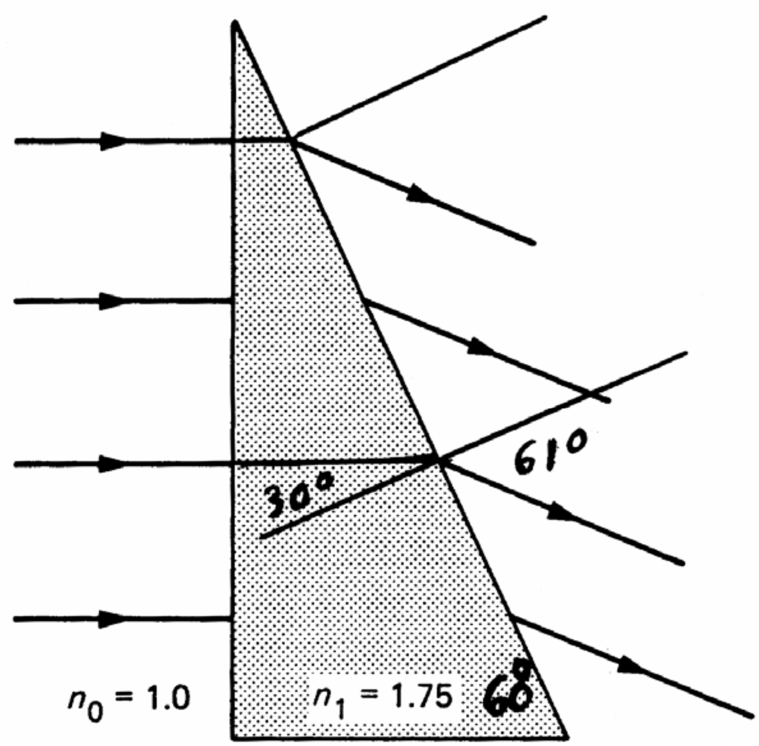


Figure 2.5 A translation into the ray language of Figure 2.3

Prisms & Index of Refraction with Wavelength

- Different wavelengths have different index of refraction
- Index change is what makes prism colour spectrum
- Generally higher index at shorter wavelengths
- Most effect if use both sides to get max deviation & long distance
- Angle change is ~ only ratio of index change – 1-2%
- Eg BSC glass red 1.5, violet 1.51, assume light leaves at 30^o

Red ϕ_R = arcsin [1.5 sin(60)] = 48.59^o
Violet ϕ_v = arcsin [1.51 sin(60)] = 49.03^o

- This 0.43^o difference spreads spectrum 7.6 mm at 1 m distance

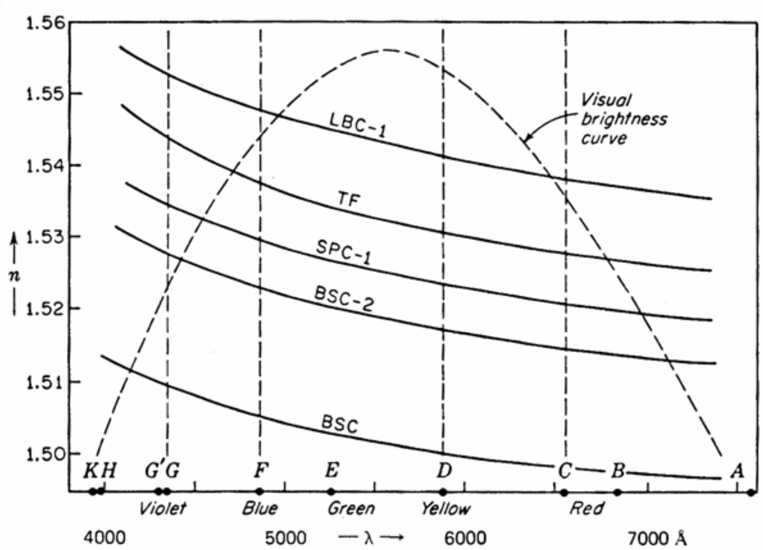
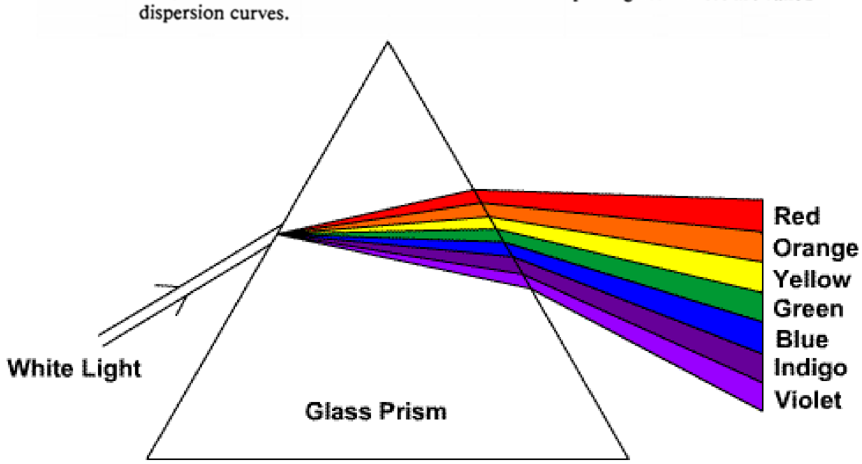


FIGURE 9Y
Graphs of the refractive indices of several kinds of optical glass. These are called dispersion curves.



Lens

- Lens is like a series of prisms
- Straight through at the centre
- Sharper wedge angles further out
- More focusing further out
- Snell's law applied to get the lens operation

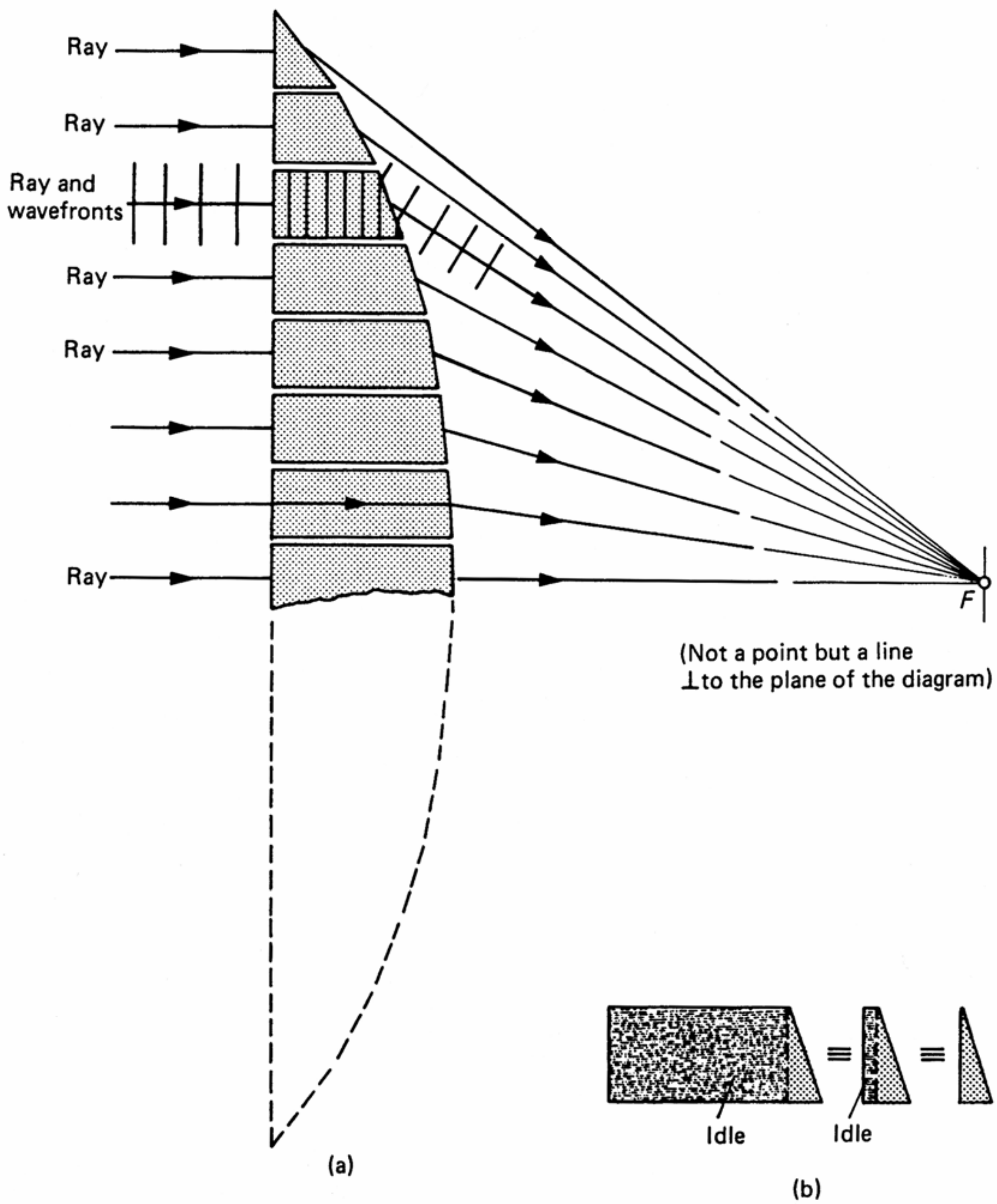


Figure 2.6 Rays corresponding to wavefronts incident upon a succession of small prisms

Why is Light Focus by a Lens

- Why does all the light focus by a lens
- Consider a curved glass surface with index n' on right side
- Radius of curvature r is centered at C
- Let parallel light ray P at height h from axis hit the curvature at T
- Normal at T is through C forming angle ϕ to parallel beam
- Beam is refracted by Snell's law to angle ϕ' to the normal

$$n \sin(\varphi) = n' \sin(\varphi')$$

Assuming small angles then sin(ϕ)~ϕ and

$$\sin(\varphi) = \frac{n'}{n} \sin(\varphi') \quad or \quad \varphi \cong \frac{n'}{n}\varphi'$$

From geometry for small angles

$$\sin(\phi) = \frac{h}{r} \quad or \quad \phi \cong \frac{h}{r}$$

Angle θ' the beam makes to the axis is by geometry

$$\theta' = \phi' - \phi = \frac{n'}{n}\phi - \phi = \frac{n'-n}{n}\phi \cong \frac{h}{r}\left[\frac{n'-n}{n}\right]$$

Thus the focus point is located at

$$f = \frac{h}{\sin(\theta')} \cong \frac{h}{\theta} \cong h\frac{r}{h}\left[\frac{n}{n'-n}\right] \cong \frac{nr}{n'-n}$$

Thus all light is focused at same point independent of h position

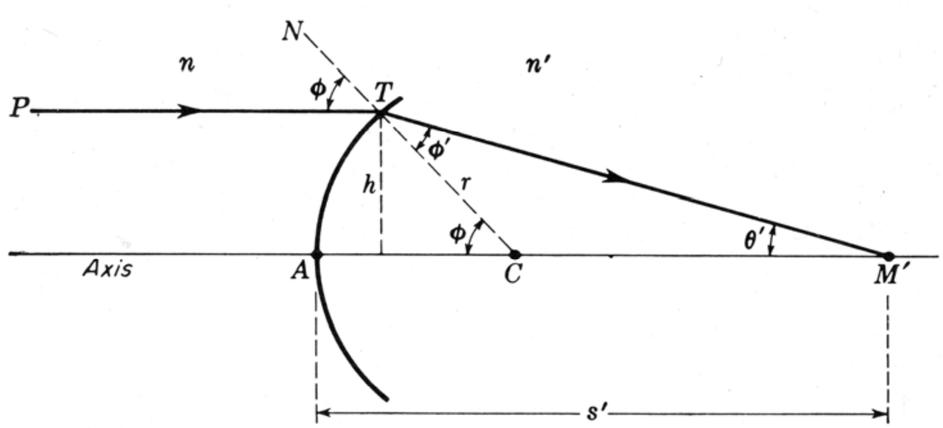


FIGURE 8E
Geometry for ray tracing with parallel incident light.

Focal Points

- Two focal points depending on surface & where light comes from
- **Primary Focal Points** are
- Convex (a) where diverge beam forms parallel light
- Concave surface (b) where light appears to converge when it is converted into a parallel beam
- **Secondary Focal Points**
- Convex (c) where parallel beam is focused
- Concave surface (d) where parallel light coming in appears to diverge from.

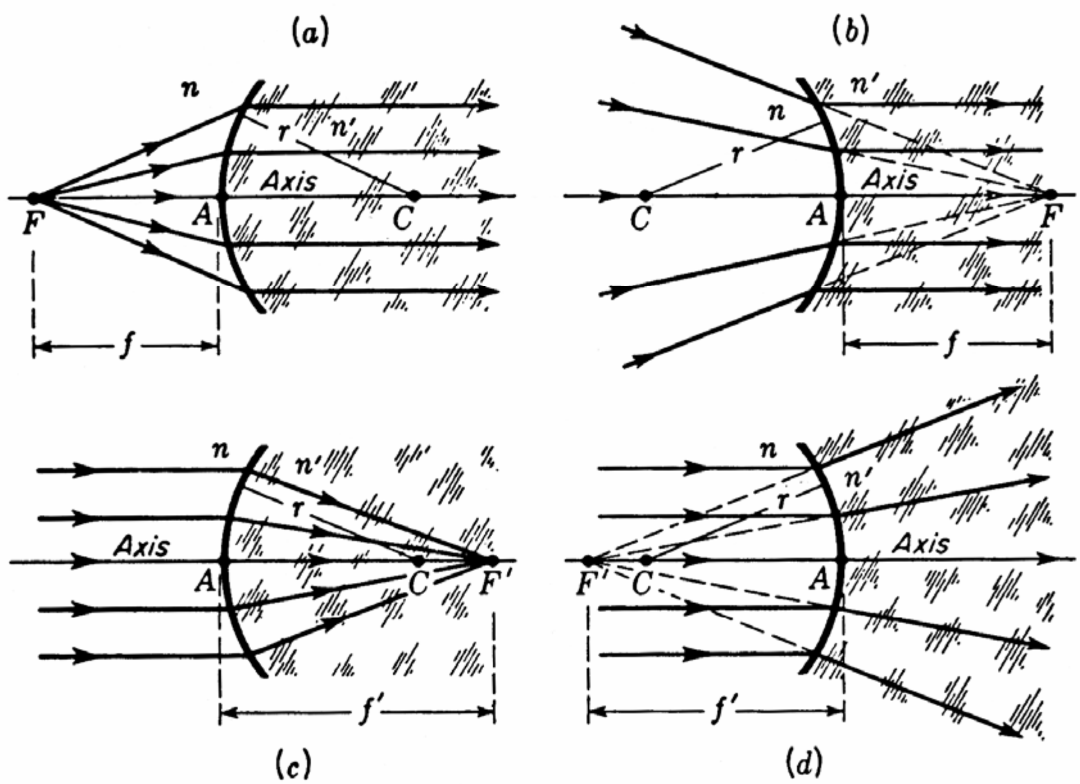


FIGURE 3B
The focal points F and F' and focal lengths f and f' associated with a single spherical refracting surface of radius r separating two media of index n and n'.

Types of Lenses

Convex

- (a) Biconvex or equiconvex
- (b) Planoconvex
- (c) positive meniscus

Concave

- (d) biconcave or equiconve
- (e) Planoconcave
- (f) negative meniscus

- Primary and secondary focal points very dependent on type
- Planoconvex/Panloconcave easiest to make
- Two surface lenses about twice the price

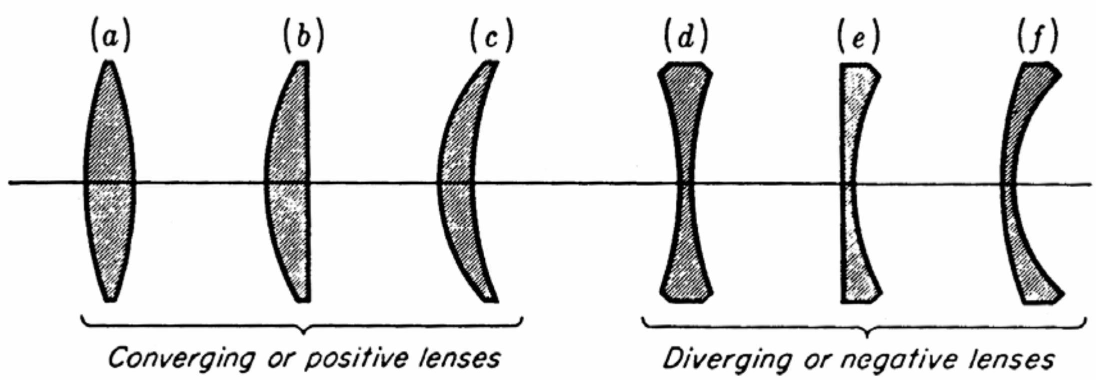


FIGURE 3A
Cross sections of common types of thin lenses.

Fresnel Lens

- Lens with thickness remove
- Cheaper, but can be lower quality
- Reason: diffraction effects at step boundries

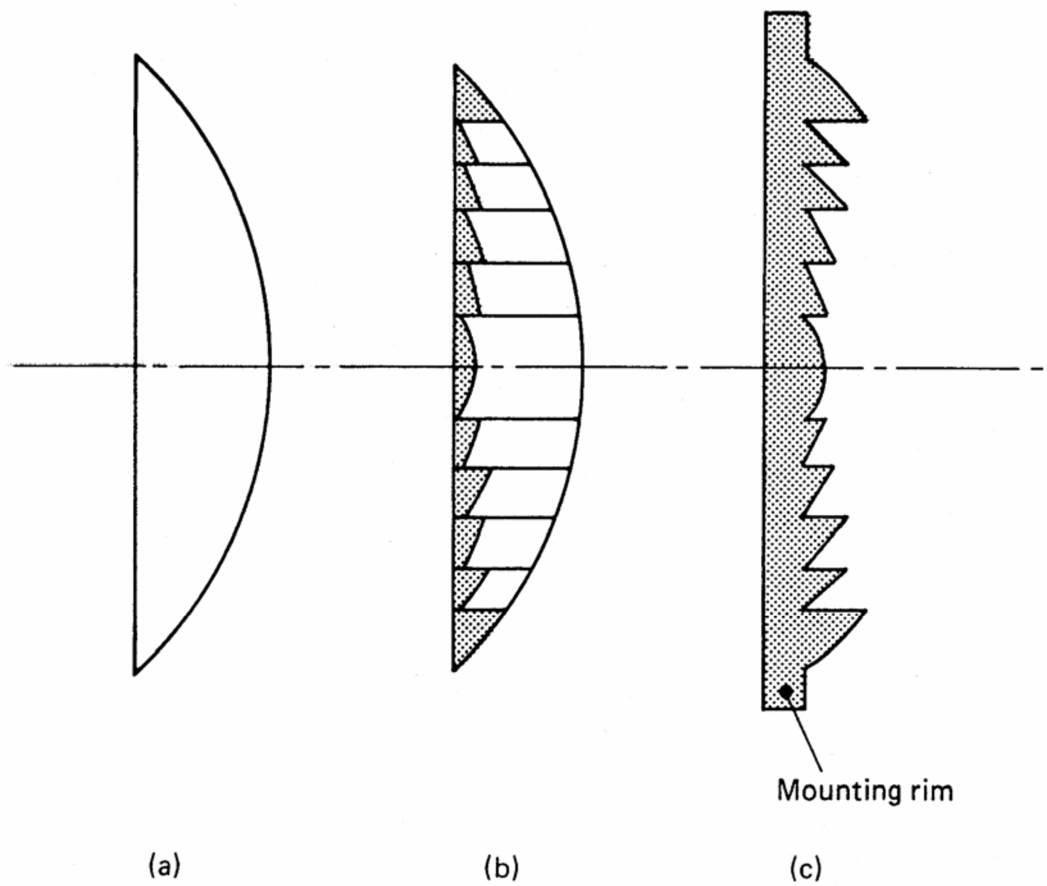


Figure 2.8 Metamorphosis of a succession of prismlets into a Fresnel lens

Lens Conventions

- From Jenkins & White: Fundamentals of Optics, pg 50
- Incident rays travel left to right
- Object distance s + if left to vertex, - if right to vertex
- Image distance s' + if right to vertex, - if left to vertex
- Focal length measured from focal point to vertex
  f positive for converging, negative for diverging
- r positive for convex surfaces
  r negative for concave
- Object and Image dimension
  + if up, - if down from axis

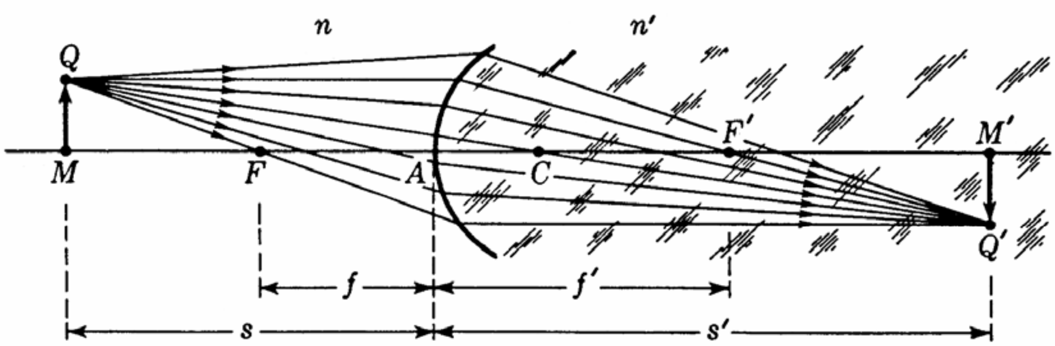


FIGURE 3D
All rays leaving the object point Q and passing through the refracting surface are brought to a focus at the image point Q'.

Gaussian Formula for a Spherical Surface

- The radius of curvature r controls the focus
- Gaussian Lens formula

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

where n index on medium of light origin
n' index on medium entered
r = radius of curvature of surface

- Clearly for s' infinite (parallel light output) then s = f (primary focal length)

$$\frac{n}{s} + \frac{n'}{\infty} = \frac{n}{f} = \frac{n' - n}{r}$$

$$f = \frac{nr}{n' - n}$$

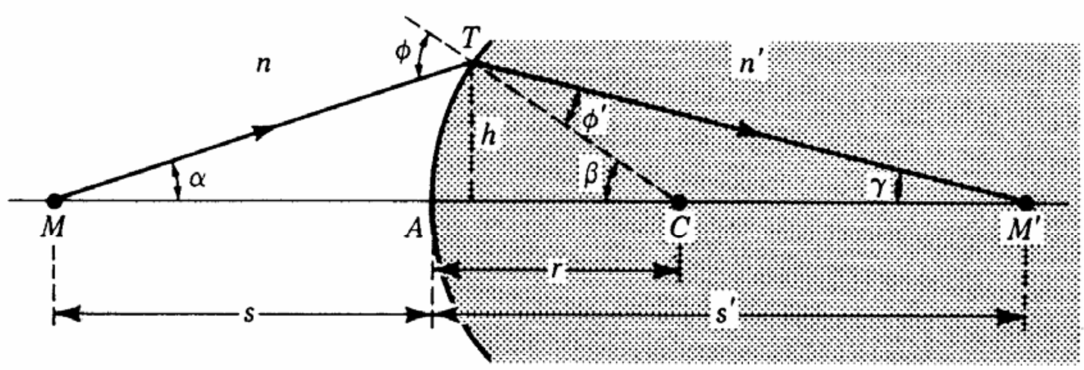


FIGURE 3K
Geometry for the derivation of the paraxial formula used in locating images.

Thin Lens

- Assume that thickness is very small compared to s, s' distances
- This is often true for large focal length lenses
- Primary focus f on left convex lens, right concave
- Secondary focus f' on right convex, left concave
- If same medium on both sides then thin lens approximation is

$$f = f'$$

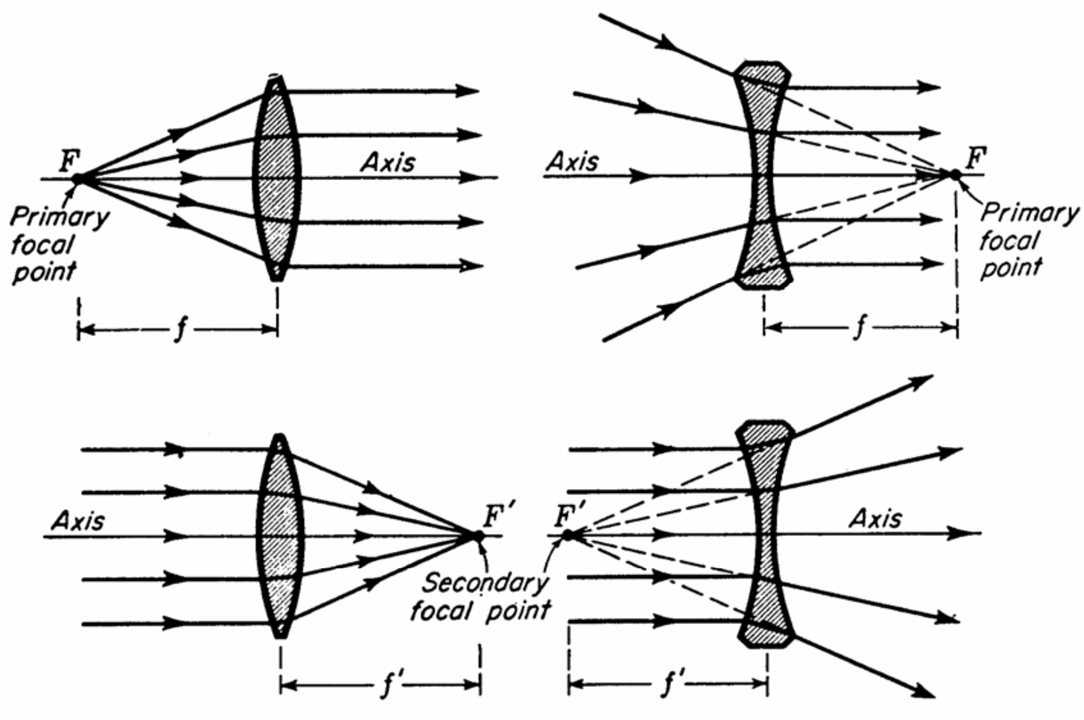


FIGURE 4A
Ray diagrams illustrating the primary and secondary focal points F and F' and the corresponding focal lengths f and f' of thin lenses.

Basic Thin Lens formula

- Basic Thin Lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- Lens Maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

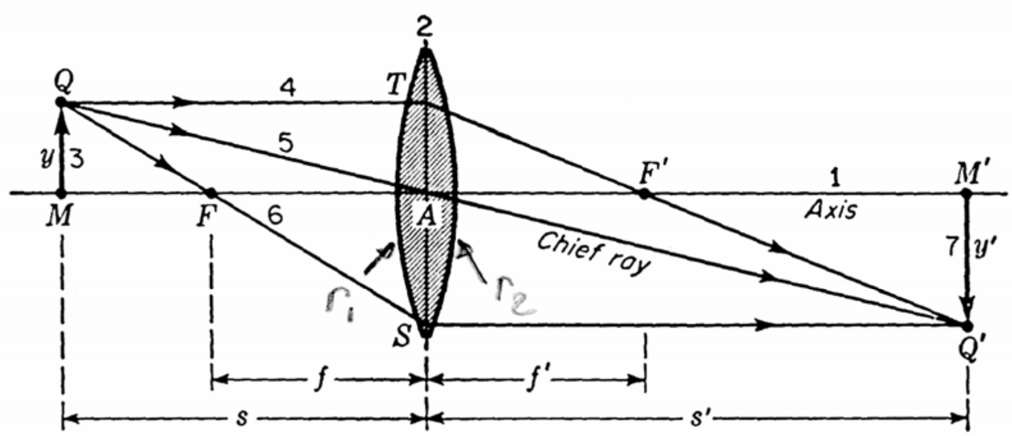


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

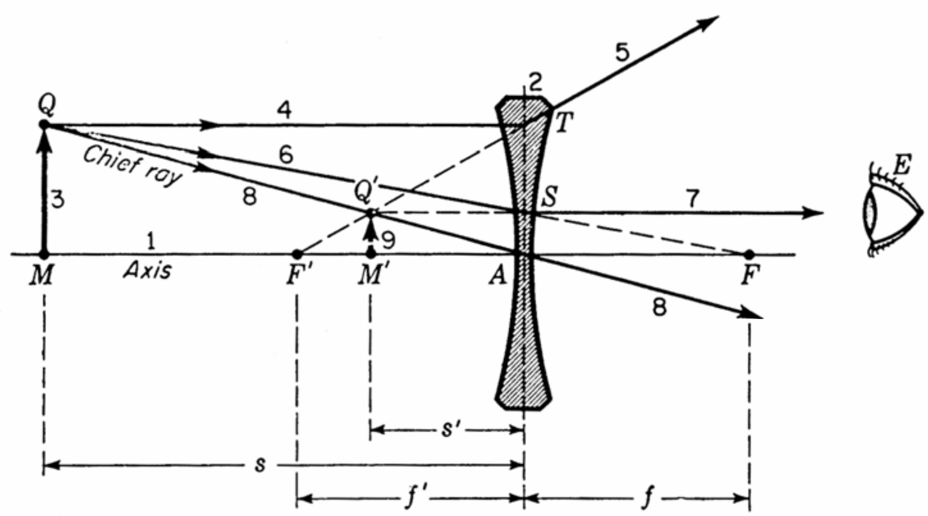


FIGURE 4G
The parallel-ray method for graphically locating the virtual image formed by a negative lens.

Magnification and Thin Lenses

- f positive for convex, negative for concave
- Magnification of a lens is given by

$$m = -\frac{s'}{s} = \frac{f}{f-s} = \frac{f-s'}{f}$$

- Magnification is negative for convex, positive for concave

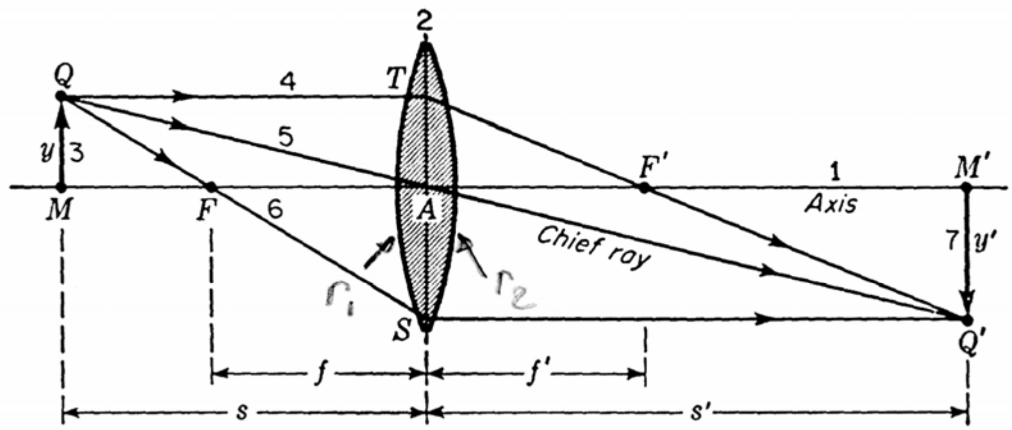


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

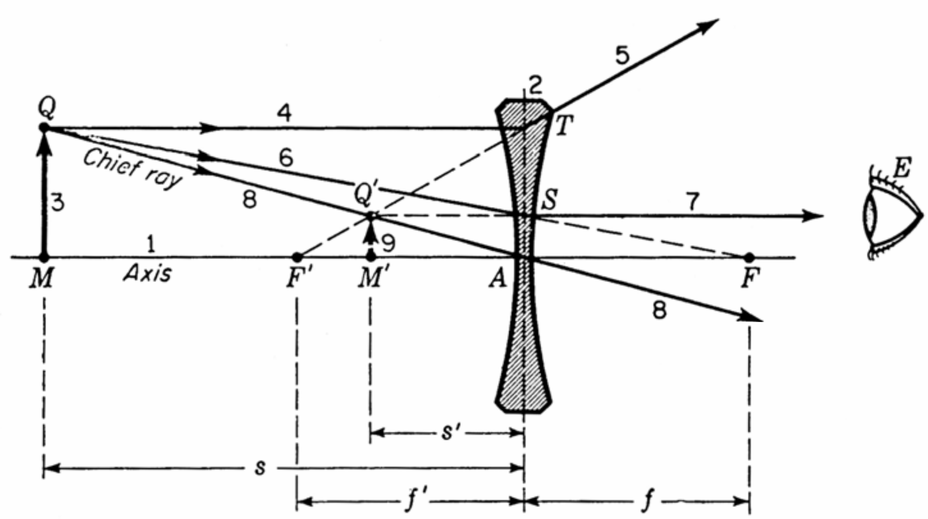


FIGURE 4G
The parallel-ray method for graphically locating the virtual image formed by a negative lens.

Simple Lens Example

- Consider a glass (n=1.5) plano-convex lens radius r_1 = 10 cm
- By the Lens Maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1.5-1)\left(\frac{1}{10} - \frac{1}{\infty}\right) = \frac{0.5}{10} = 0.05$$

$$f = \frac{1}{0.05} = 20\ cm$$

- Now consider a 1 cm candle at s = 60 cm from the vertex
- Where is the image

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{20} - \frac{1}{60} = 0.03333 \qquad s' = \frac{1}{0.0333} = 30\ cm$$

- Magnification $m = \frac{M'}{M} = -\frac{s'}{s} = -\frac{30}{60} = -0.5$
- Image at 30 cm other side of lens inverted and half object size
- What if candle is at 40 cm (twice f)

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{20} - \frac{1}{40} = 0.05 \quad s' = \frac{1}{0.05} = 40\ cm \qquad m = -\frac{s'}{s} = -\frac{40}{40} = -1$$

- Image is at 40 cm other side of lens inverted and same size (1 cm)

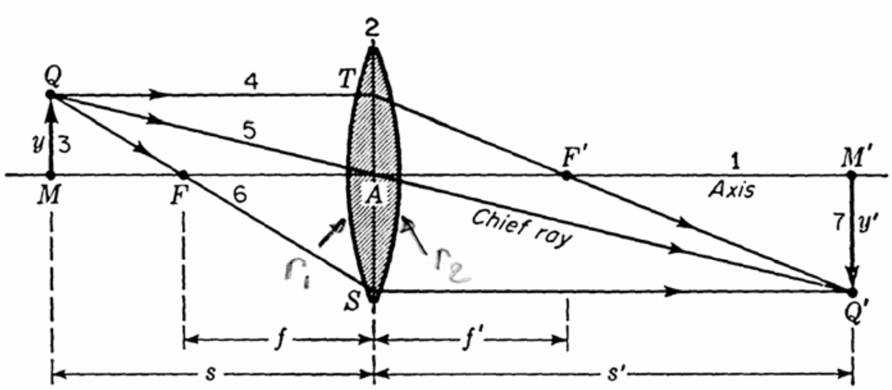


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

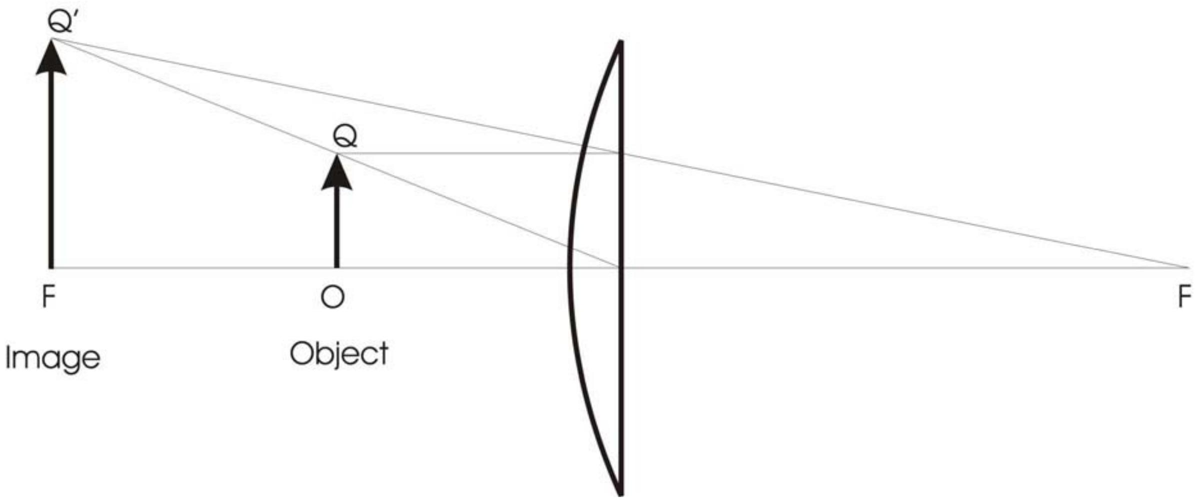
Lens with Object Closer than Focus f

- Now place candle at 10 cm (s <f condition)

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{20} - \frac{1}{10} = -0.05 \quad s' = \frac{1}{0.05} = -20\ cm$$

$$m = -\frac{s'}{s} = -\frac{-20}{10} = 2$$

- Now image is on same side of lens at 20 cm (focal point)
- Image is virtual, erect and 2x object size
- Virtual image means light appears to come from it



Graphic Method of Solving Lens Optics

- Graphic method is why this is called Geometric Optics
- Use some scale (graph paper good)
- Place lens on axis line and mark radius C & focal F points
- Draw line from object top Q to mirror parallel to axis (ray 4)
- Hits vertex line at T
- Then direct ray from T through focus point F and beyond
- Because parallel light from object is focused at f
- Now direct ray from object top Q through lens center (ray 5)
- This intersects ray 4 at image Q' (point 7)
- This correctly shows both position and magnification of object
- This really shows how the light rays are travelling
- Eg Ray through the focal point F (ray 6) becomes parallel
- Intersects ray 5 again at image Q'

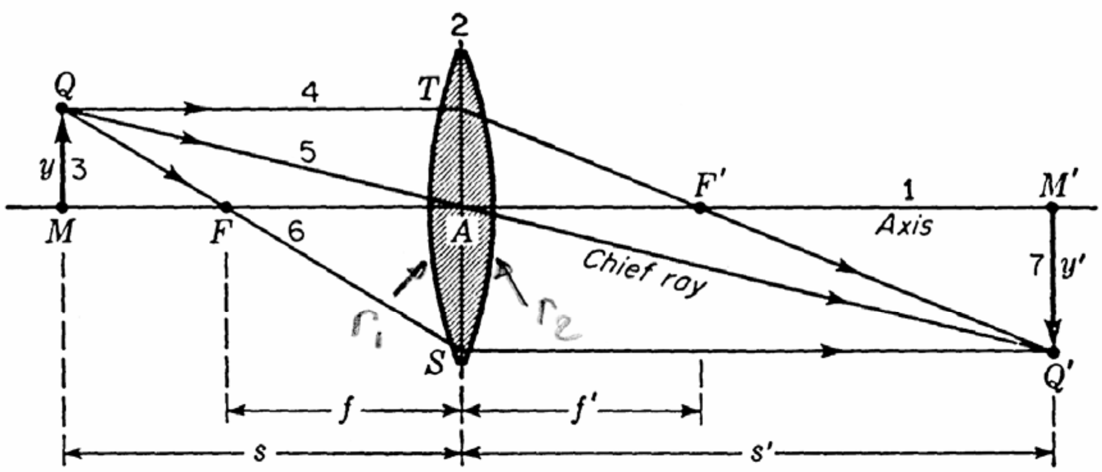
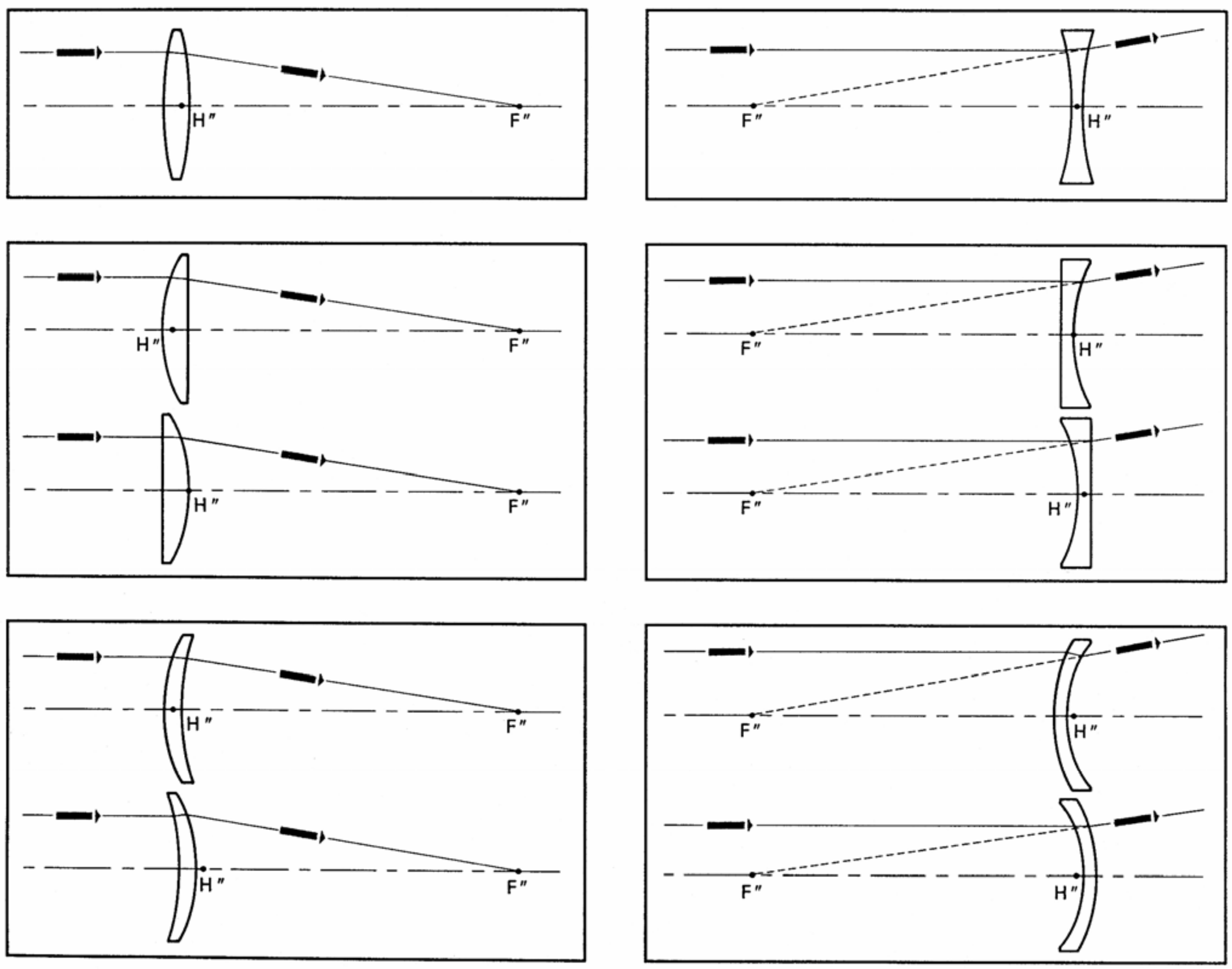


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Thin Lens Principal Points

- Object and image distances are measured from the Principal Points
- Principal point H” Location depends on the lens shape
- H” also depends on a thin lens orientation
- Note if you reverse a lens it often does not focus at the same point
- Need to look at lens specifications for principal points
- Thick lenses have separate Principal points

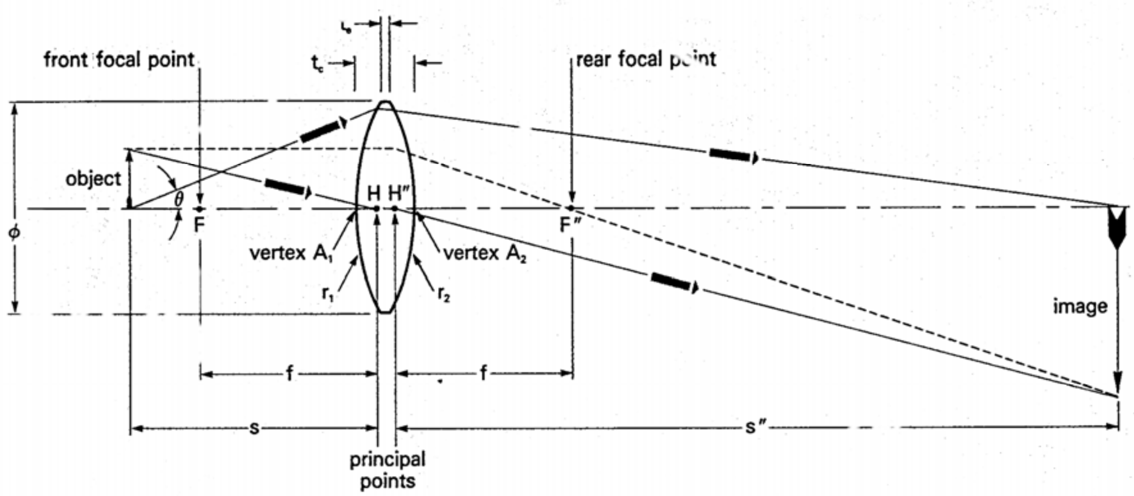


Thick Lens Formula

- As lens gets thicker optical surfaces may be not meet
- Lens thickness t_c (between vertex at the optical axis i.e. centre)
- Now lens formula much more complicated
- Distances measured relative to the principal points
  H" for light coming from the front
  H for light coming from the back of lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2}{n}\left[\frac{t_c}{r_1 r_2}\right]$$

- Note simple lens formula assumes $t_c = 0$ which is never true
- But if f is large then r's large and t_c is small so good approximation
- Note plano-convex $r_2 = \infty$ and $f_{thin} = f_{thick}$ but principal point changes



Note location of object and image relative to front and rear focal points.

ϕ = Lens diameter

r_1 = Radius of curvature of 1st surface (positive if center of curvature is to right)

r_2 = Radius of curvature of 2nd surface (negative if center of curvature is to left)

r_2 = $-r_1$ for symmetric lens

m = s″/s = magnification = conjugate ratio, said to be infinite if either s″ or s is infinite

θ = Arctan (ϕ/2s)

s = Object distance, positive for object (whether real or virtual) to the left of principal point H

s″ = Image distance (s and s″ are collectively called conjugate distances, with object and image in conjugate planes), positive for image (whether real or virtual) to the right of the principal point H″

t_c = Center thickness

t_e = Edge thickness

f = Effective focal length (EFL), may be positive (as shown or negative. f represents both FH and H″F″, assuming lens to be surrounded by medium of index 1.0

Very Thick Lenses

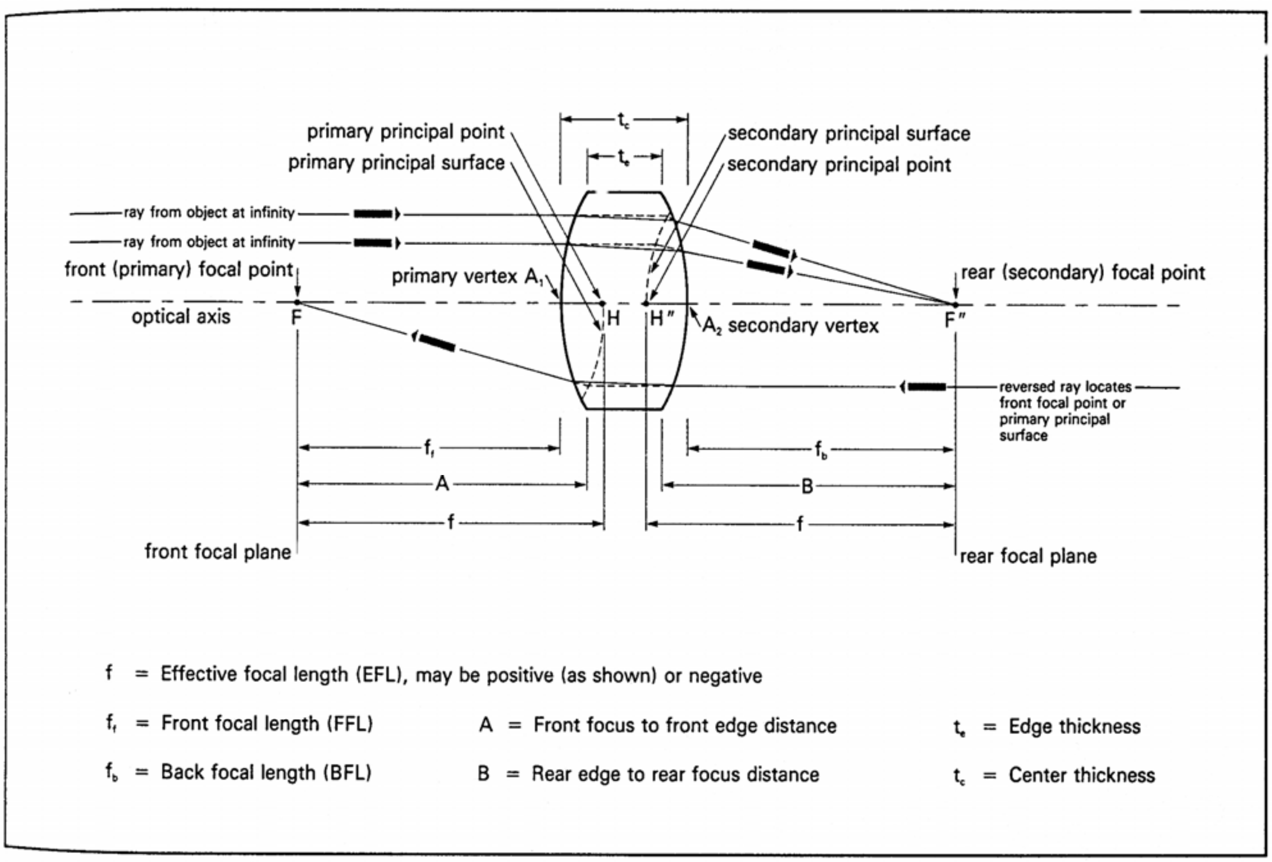
- Now primary and secondary principal points very different
- A_1 = front vertex (optical axis intercept of front surface)
- H = primary (front) principal point
- A_2 = back vertex (optical axis intercept of back surface)
- H" = secondary (back) principal point
- t_c = centre thickness: separation between vertex at optic axis
- Relative to the front surface the primary principal point is

$$A_1 - H = f t_c\left(\frac{n-1}{r_2}\right)$$

- Relative to the back surface the secondary principal point is

$$A_2 - H'' = f t_c\left(\frac{n-1}{r_1}\right)$$

- f_{efl} effective focal length (EFL): usually different for front and back



FRONT AND BACK FOCAL LENGTHS of a lens having spherical surfaces and surrounded by air. Under these conditions, distances labeled f are equal whether or not the lens is symmetric, but distances f_f and f_b are equal *only* if the lens is symmetric. In the paraxial limit (see text), the curvature of the principal surfaces may be neglected.

Numerical Aperture (NA)

- NA is the sine of the angle the largest ray a parallel beam makes when focused

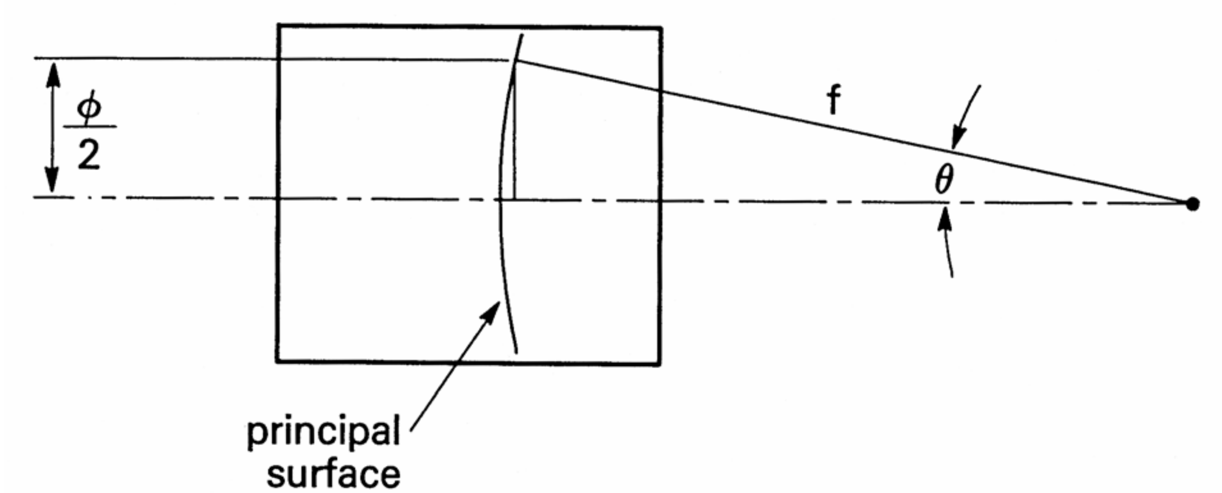
$$NA = sin(\theta) = \frac{\phi}{2f}$$

where θ = angle of the focused beam

ϕ = diameter of the lens

- NA <1 are common
- High NA lenses are faster lenses
- NA is related to the F#

$$F\# = \frac{1}{2NA}$$



Combining Lenses

- Can combine lenses to give Combination Effective Focal Length f_e
- If many thin lenses in contact then

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots$$

- Two lenses f_1 and f_2 separated by distance d
- To completely replace two lens for all calculations
- New image distance for object at infinity (eg laser beam)

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad or \quad f_e = \frac{f_1 f_2}{f_1 + f_2 - d}$$

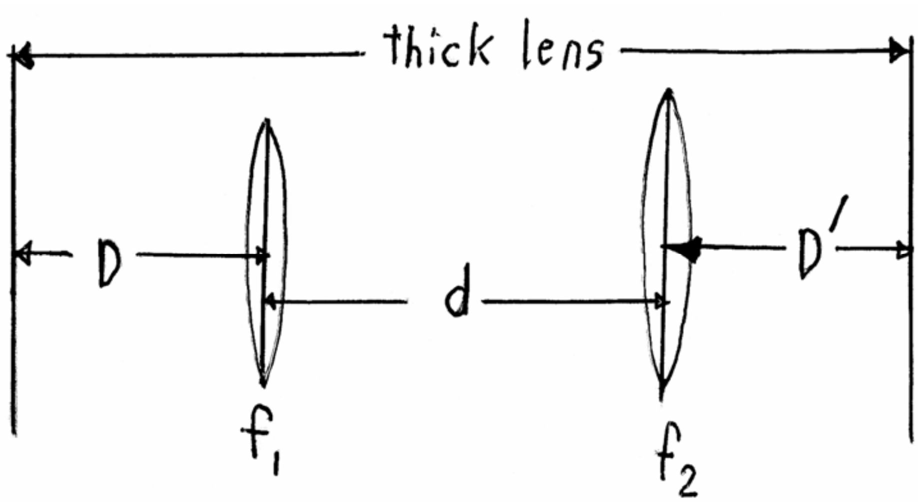
- Distance from first lens primary principal point
  to combined lens primary principal point

$$D = -\frac{d f_e}{f_2}$$

- Distance from second lens secondary principal point
  to combined lens secondary principal point

$$D' = -\frac{d f_e}{f_1}$$

- Combined "thick lens" extends from D to D'



Combining Two Lens Elements

- Combined object distance s_e

$$s_e = s_1 - D$$

- Combined image distance s'_e

$$s'_e = s'_2 - D'$$

- NOTE: Combined object/image distance may change sign
- The thick lens follows the standard formula

$$\frac{1}{s_e} + \frac{1}{s'_e} == \frac{1}{f_e}$$

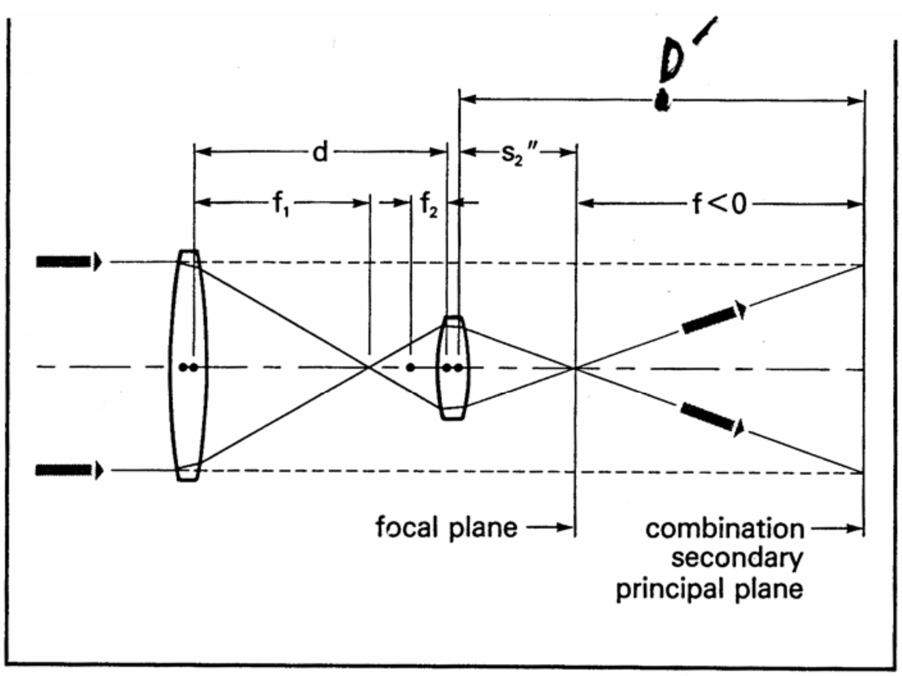
- Combined magnification

$$m_e = -\frac{s'_e}{s_e}$$

- Secondary focus distance relative to 2nd lens vertex is:

$$f = f_e + D$$

- Note some devices (e.g. telescopes) cannot use these formulas



PAIR OF POSITIVE LENSES separated by distance d greater than $f_1 + f_2$.