Human Eye (Hecht 5.7.1)

- Human eye is a simple single lens system
- Crystalline lens provide focus
- Cornea: outer surface protection
- Aqueous humor is water like liquid behind cornea
- Iris: control light
- Retina: where image is focused
- Note images are inverted
- Brain's programming inverts the image

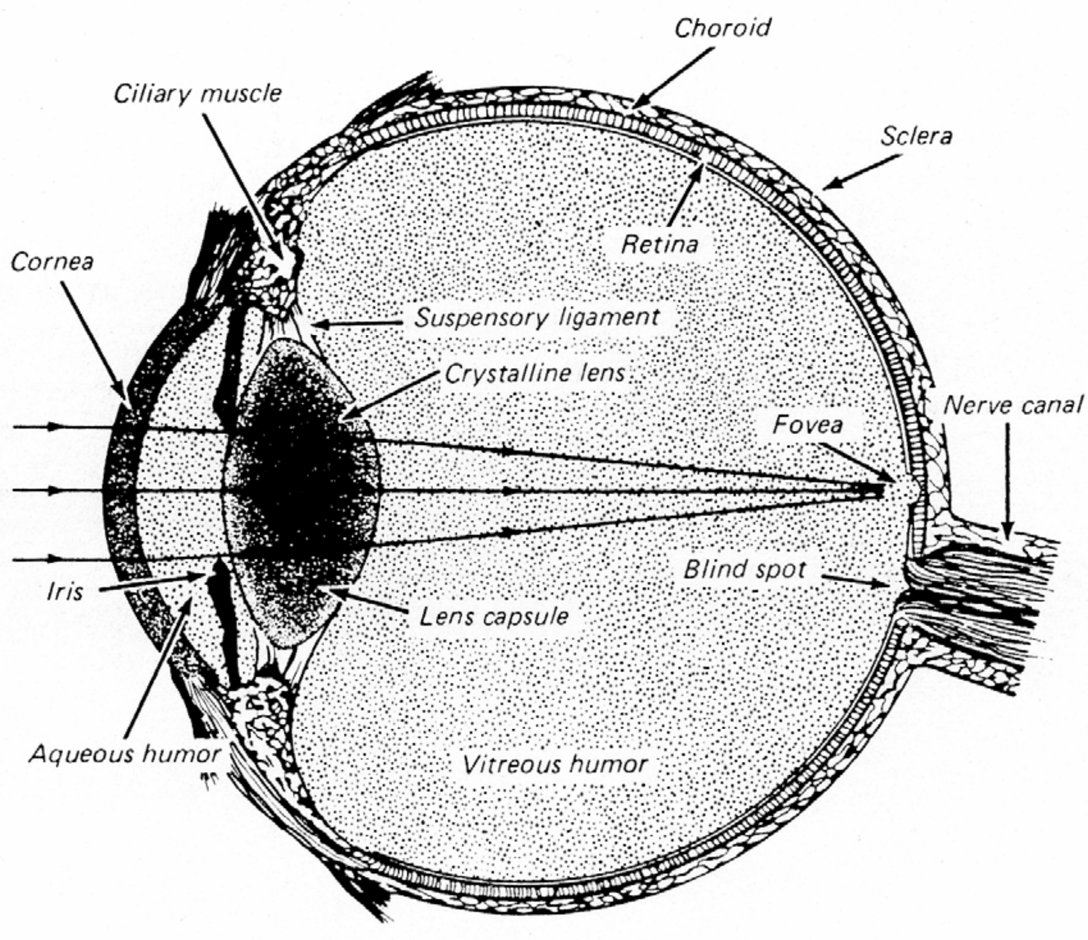


FIGURE 10A
A cross-sectional diagram of a human eye, showing the principal optical components and the retina.

Human Eye Distance

- Crystalline lens to retina distance 24.4 mm
- Eye focuses object up to 25 cm from it
- Called the near point or $D_v = 25$ cm
- Eye muscles to change focal length of lens over $2.22<f<2.44$ cm
- Near sighted: retina to lens distance too long, focused in front
- Infinity object focused in front of retina: out of focus at it
- When bring objects closer focus moves to retina
- Near sighted people can see objects with $D_v < 25$ cm
- Far sighted: eye is too short, focuses behind retina, $D_v > 25$ cm

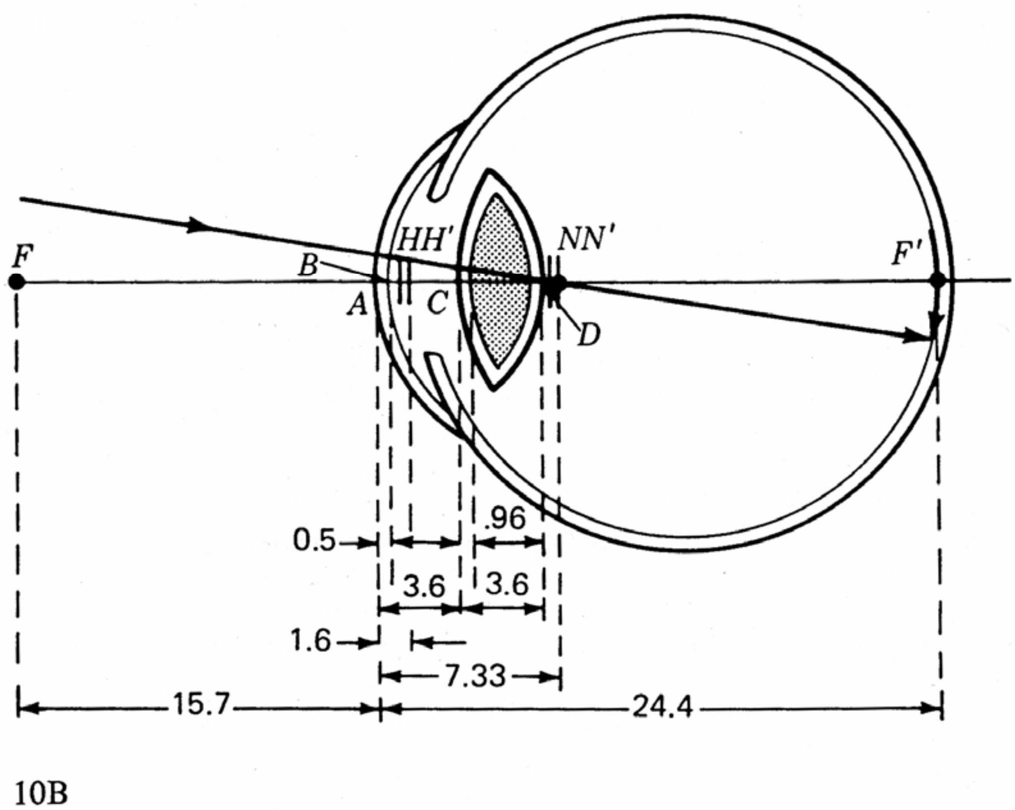


FIGURE 10B
Schematic eye as developed by Gullstrand, showing the real and inverted image on the retina (dimensions are in millimeters).

Magnification of Lens

- Lateral change in distance equals change in image size
- Measures change in apparent image size

$$m = M = \frac{y'}{y} = -\frac{s'}{s}$$

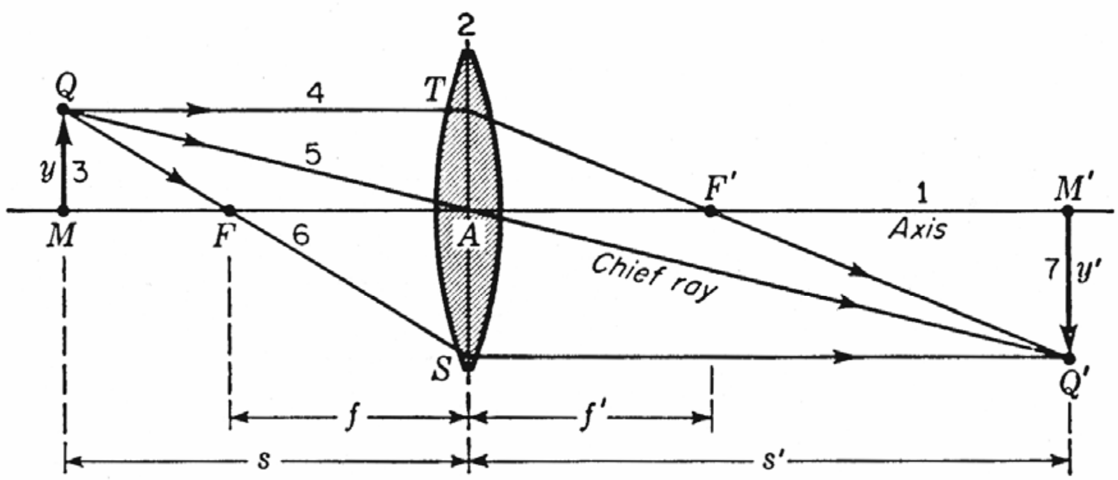


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Magnification with Index Change

- Many different ways of measuring magnification
- With curved index of refraction surface
  measure apparent change in distance to image
- Called Lateral Magnification

$$m = -\frac{s' - r}{s + r}$$

- m is + if image virtual, - if real

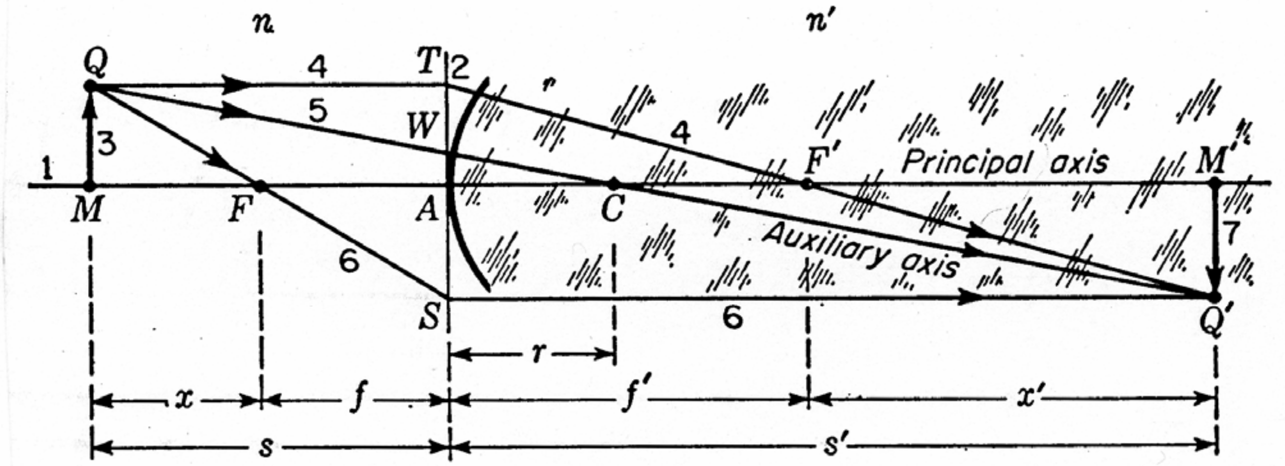


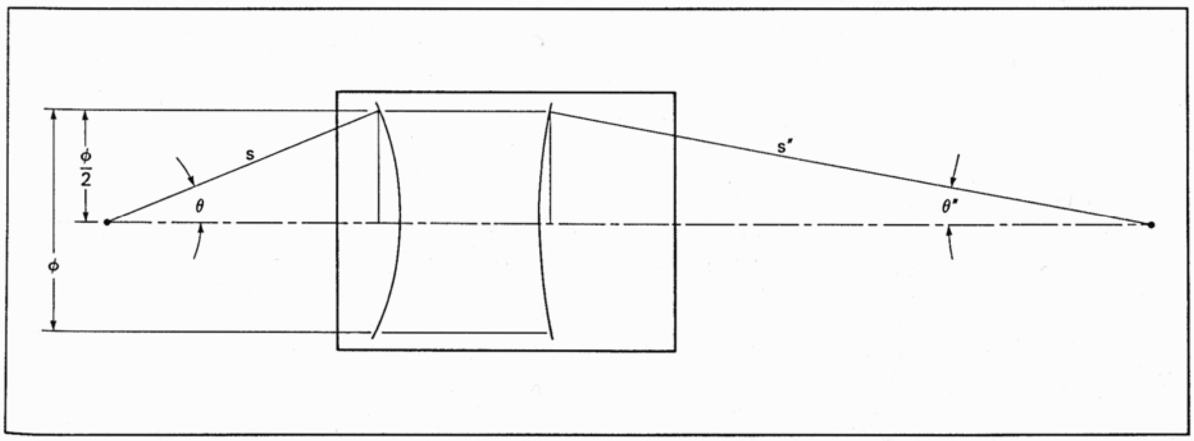
FIGURE 3F
Parallel-ray method for graphically locating the image formed by a single spherical surface.

Angular Magnification

- For the eye look at angular magnification

$$m = M = \frac{\theta'}{\theta}$$

- Represents the change in apparent angular size



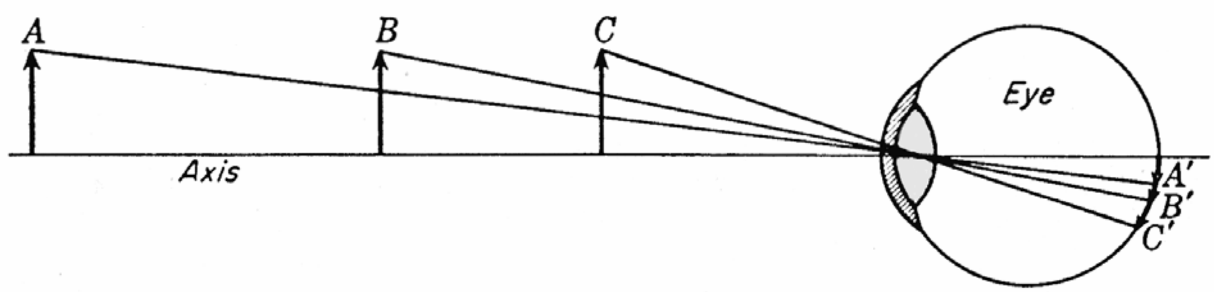


FIGURE 10H
The angle subtended by the object determines the size of the retinal image.

Simple Magnifying Glass

- Human eye focuses near point or D_v = 25 cm
- Magnification of object:
  ratio of angles at eye between unaided and lens
- Angle of Object with lens

$$tan(\theta) = \frac{y}{D_v} = \frac{y}{25} \approx \theta$$

- For maximum magnification place object at lens f (in cm)

$$\theta' = \frac{y}{f}$$

- Thus magnification is (where *f* in cm)

$$m = \frac{\theta'}{\theta} = \frac{25}{f}$$

- e.g. What is the magnification of a lens f = 1 inch = 2.5 cm

$$m = \frac{\theta'}{\theta} = \frac{25}{f} = \frac{25}{2.5} = 10$$

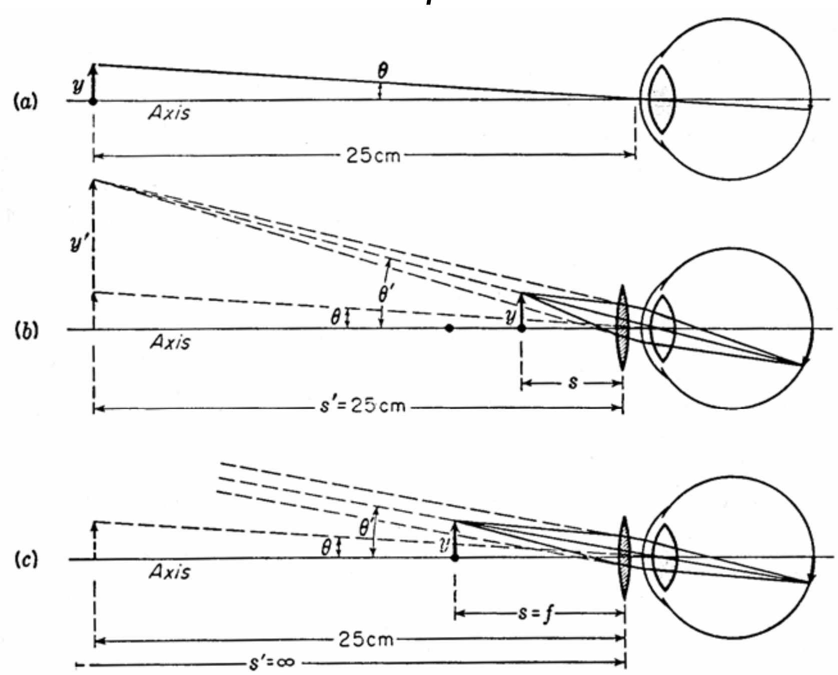


FIGURE 10I
The angle subtended by (*a*) an object at the near point to the naked eye, (*b*) the virtual image of an object inside the focal point, (*c*) the virtual image of an object at the focal point.

Power of a Lens or Surface

- Power: measures the ability to create converging/diverging light by a lens
- Measured in Diopters (D) or 1/m
- For a simple curved surface

$$P = \frac{n' - n}{r}$$

- For a thin lens

$$P = \frac{1}{f}$$

- Converging lens have + D, diverging - D
- eg f = 50 cm, D = +2 D
  f = -20 cm, D = -5 D
- Recall that for multiple lens touching

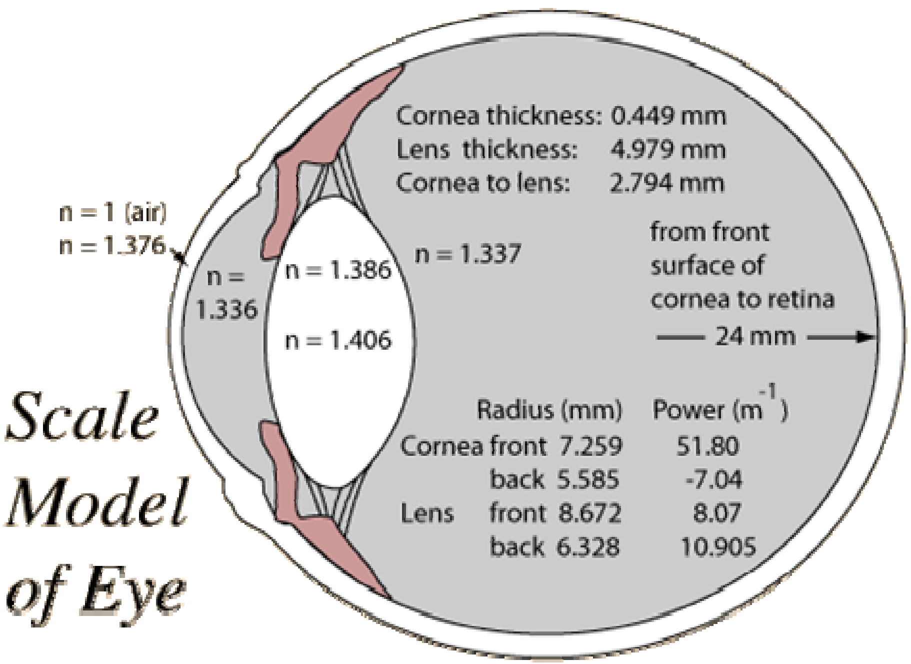
$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots$$

- Hence power in Diopters is additive

$$D = D_1 + D_2 \cdots$$

Human Eye: A two Lens System

- Eye is often treated as single simple lens
- Actually is a two lens system
- Cornea with n=1.376 makes main correction
- Aqueous humor is nearly water index
- Lens n=1.406 relative to aqueous humor Δn causes change
- Eye mussels shape the lens and adjusts focus
- Cornea gives 44.8 D of correction
- Lens gives ~18.9 D of correction
- Cannot see in water because water index 1.33 near cornea
- Thus cornea correction is not there.



Eyeglasses (Hecht 5.7.2)

- Use Diopters in glasses
- Farsighted, Hypermetopia: focus light behind retina
  Use convex lens, +D to correct
- Nearsighted, Myopia: focus in front of retina
  use concave lens, -D to correct
- Normal human eye power is ~58.6 D
- Nearsighted glasses create a reverse Galilean telescope
- Makes objects look smaller.

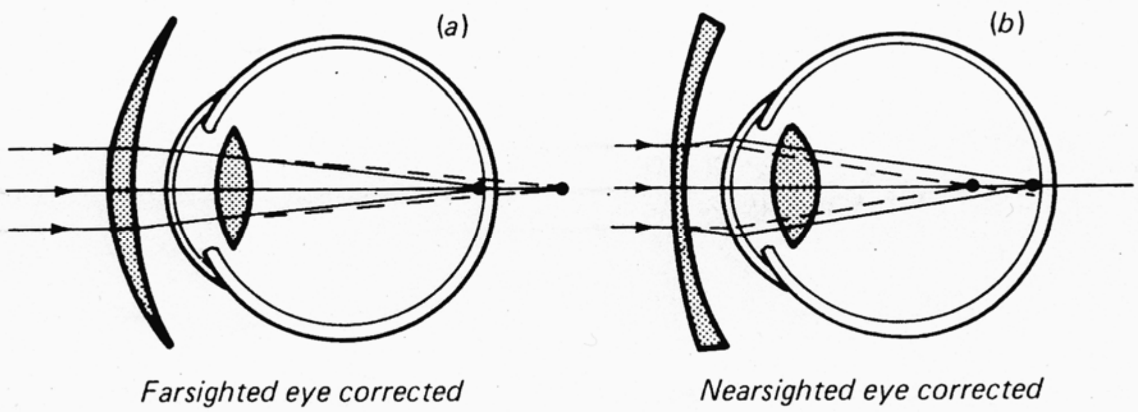
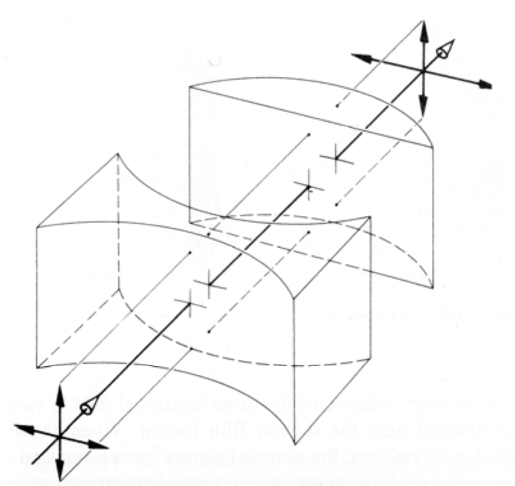
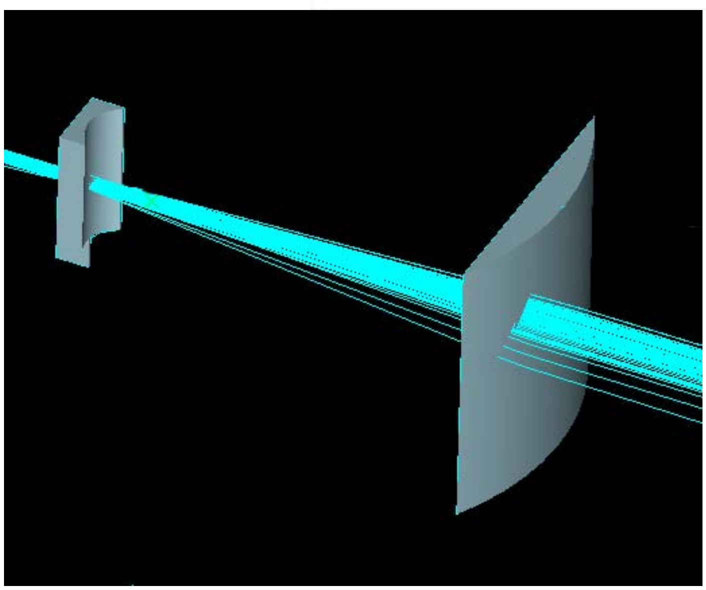


FIGURE 10L
Typical eye defects can be corrected by spectacle lenses.

Anamorphic Lenses

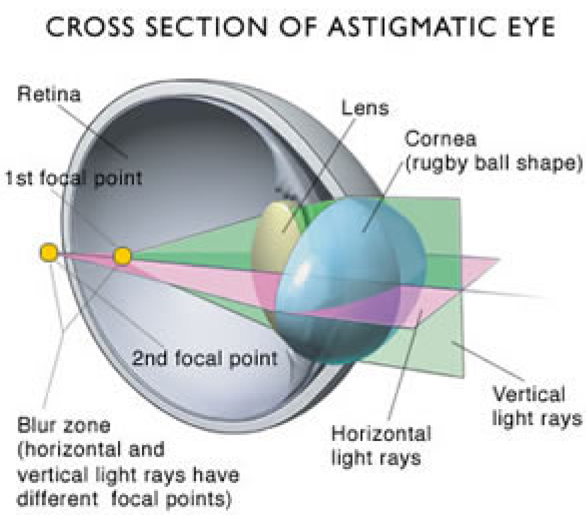
- Lenses & Mirrors do not need to be cylindrically symmetric
- Anamorphic Lenses have different characteristics in each axis
- Sphero-cylinderical most common
- One axis (eg vertical): cylindrical curve just like regular lens
- Other axis (e.g. horizontal): has no curve
- Result light is focused in horizontal axis but not vertical
- Often used to create a line of light



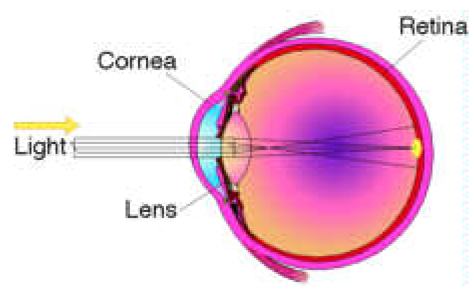


Astigmatism

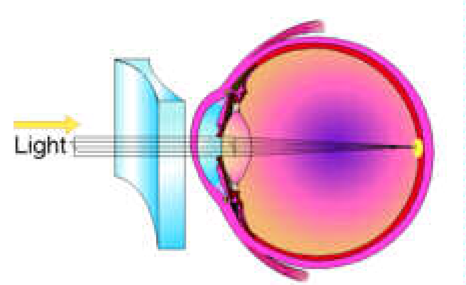
Astigmatism means light is focused in on axis not other
Cylinderical lens cause as Astigmatism: focus in one plan
In eyes astigmatism caused by shape of eye (& lens)
Image is compressed in one axis and out of focus
Typically measure D in both axis
Rotation of astigmatism axis is measured
Then make lens slightly cylindrical
i.e. perpendicular to axis may have higher D in one than other
eg. eyeglass astigmatism prescription gives +D and axis angle
+D is difference between the two axis.



uncorrected corrected



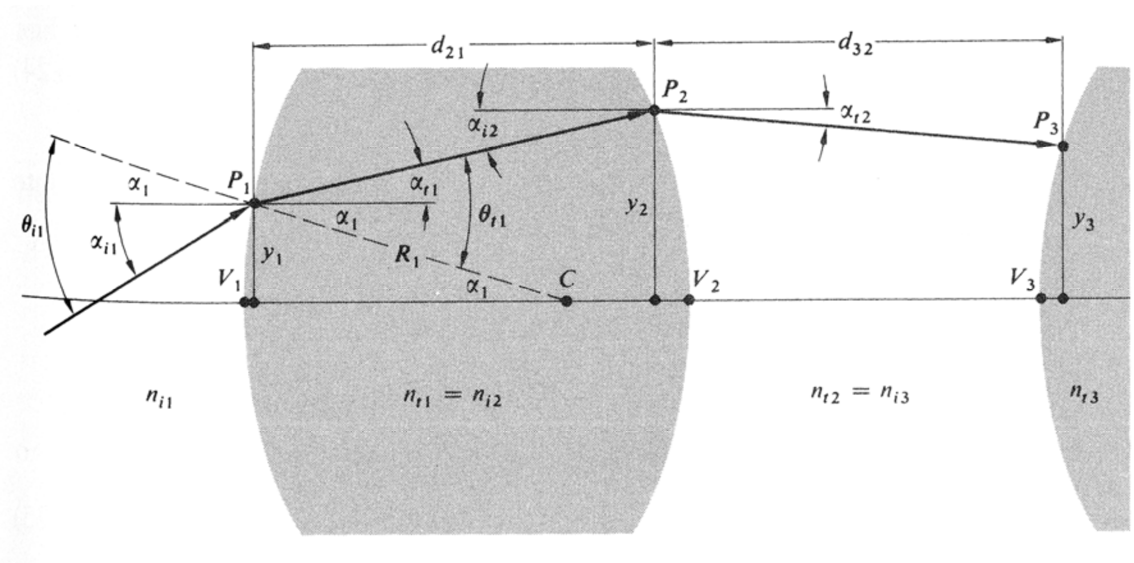
Horizontal line out of focus

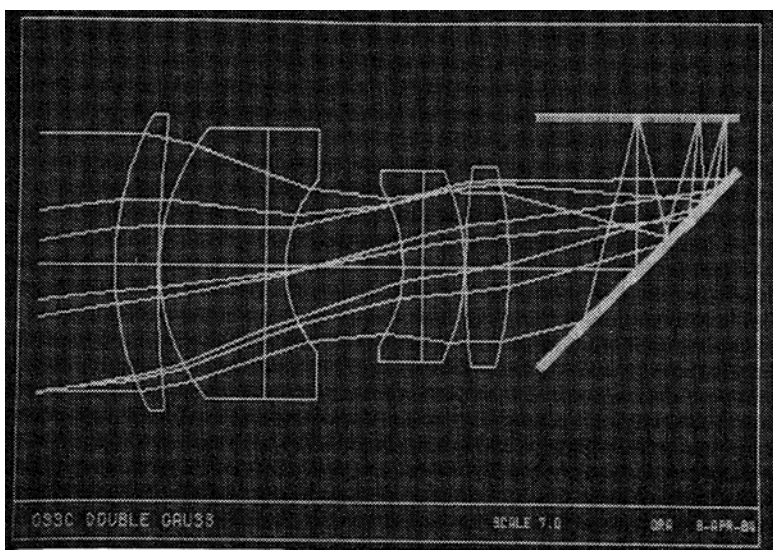


Astigmatism corrected by lens

Ray Tacing (Hecht 6.2)

- For more complicated systems use CAD tools
- Both are based on Ray Tracing concepts
- Solve the optical system by tracing many optical rays
- Use exact surface positions & surface
- Do not make parallex assumption – use Snell's law
- Eg.of programs Z max, Code 5





Matrix Methods in Optics(Hecht 6.2.1)

- Alternative Matrix methods
- Both matrix & CAD are based on Ray Tracing concepts
- Solve the optical system by tracing many optical rays
- In free space a ray has position and angle of direction
  y_1 is radial distance from optical axis
  V_1 is the angle (in radians) of the ray
- Now assume you want to a Translation:
  find the position at a distance t further on
- Then the basic Ray equations are in free space
  making the parallex assumption

$$y_2 = y_1 + V_1 t$$

$$V_2 = V_1$$

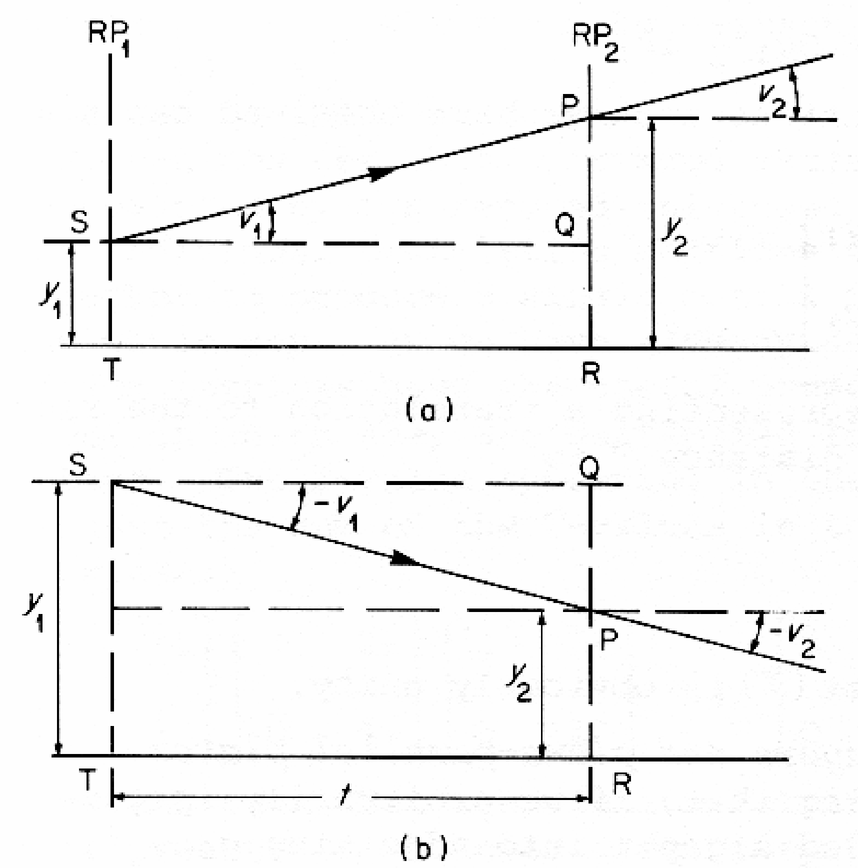


Figure II.2

Matrix Method: Translation Matrix

- Can define a matrix method to obtain the result for any optical process
- Consider a simple translation distance t
- Then the Translation Matrix (or *T* matrix)

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

- The reverse direction uses the inverse matrix

$$\begin{bmatrix} y_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

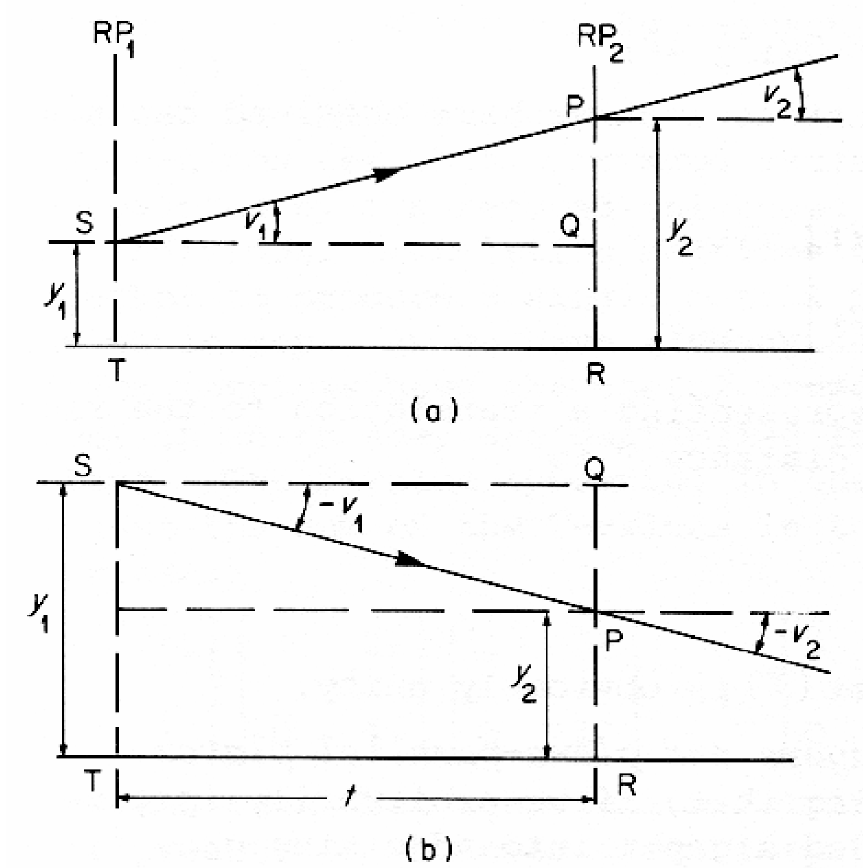
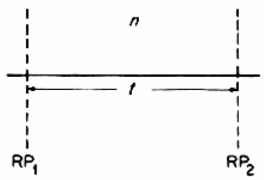
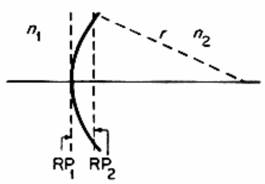
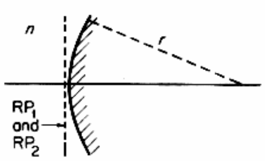
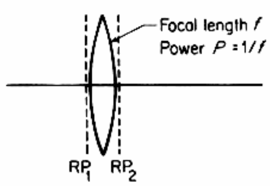


Figure II.2

General Matrix for Optical Devices

- Optical surfaces however will change angle or location
- Example a lens will keep same location but different angle
- Reference for more lens matrices & operations
  A. Gerrard & J.M. Burch,
  "Introduction to Matrix Methods in Optics", Dover 1994
- Matrix methods equal Ray Trace Programs for simple calculations

Table 1

Number	*Description*	*Optical Diagram*	*Ray-transfer matrix*
1	Translation ($\mathcal{T}$-matrix)		$\begin{bmatrix}1 & t/n\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}$
2	Refraction at single surface ($\mathcal{R}$-matrix)		$\begin{bmatrix}1 & 0\\ -\frac{(n_2 - n_1)}{r} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -P & 1\end{bmatrix}$
3	Reflection at single surface (for convention see section II.11)		$\begin{bmatrix}1 & 0\\ \frac{2n}{r} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -P & 1\end{bmatrix}$
4	Thin lens in air (focal length f, power P)		$\begin{bmatrix}1 & 0\\ -(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -1/f & 1\end{bmatrix}$

General Optical Matrix Operations

- Place Matrix on the left for operation on the right
- Can solve or calculate a single matrix for the system

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \left[M_{image}\right]\left[M_{lens}\right]\left[M_{object}\right]\begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

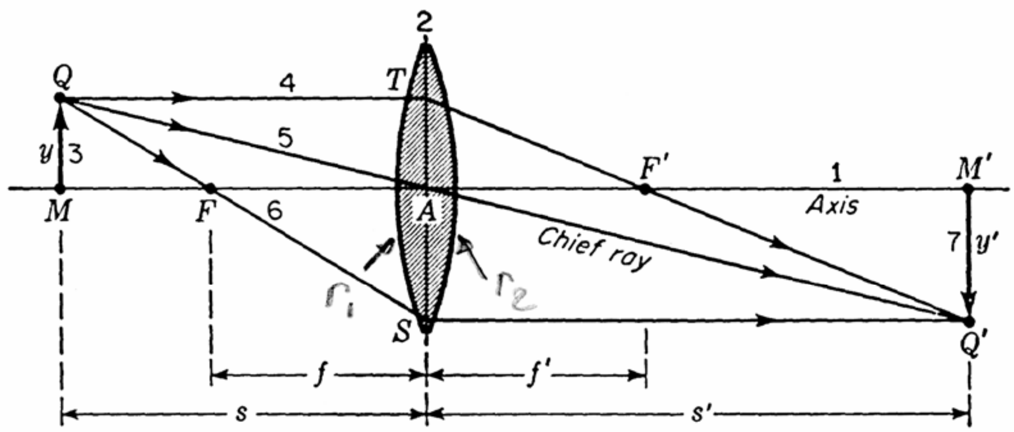


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Solving for image with Optical Matrix Operations

- For any lens system can create an equivalent matrix
- Combine the lens (mirror) and spacing between them
- Create a single matrix

$$[M_n]\cdots[M_2][M_1]=[M_{system}]=\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- Now add the object and image distance translation matrices

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}=[M_{image}][M_{lens}][M_{object}]$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}=\begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix}=\begin{bmatrix} A+s'C & As+B+s'(Cs+D) \\ C & Cs+D \end{bmatrix}$$

- Image distance s' is found by solving for $B_s=0$
- Image magnification is

$$m_s=\frac{1}{D_s}$$

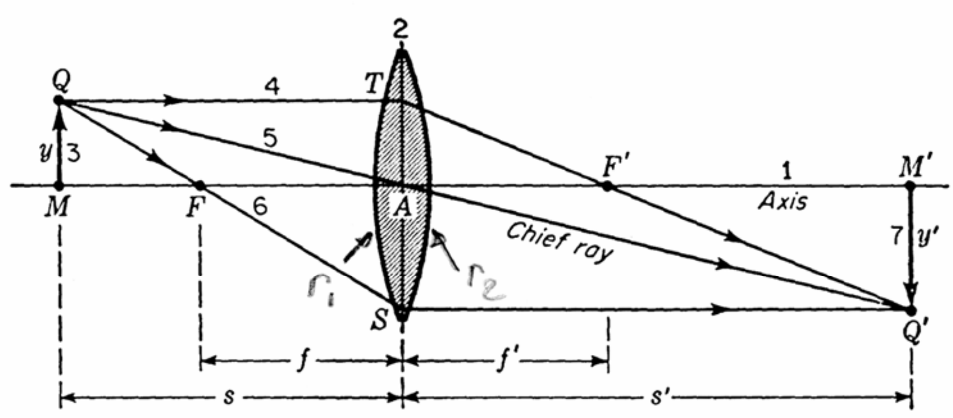


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Example Solving for the Optical Matrix

- Two lens system: solve for image position and size
- Biconvex lens f_1=8 cm located 24 cm from 3 cm tall object
- Second lens biconcave f_2= -12 cm located d=6 cm from first lens
- Then the matrix solution is

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{8} & 1 \end{bmatrix} \begin{bmatrix} 1 & 24 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ \frac{1}{12} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 & 24 \\ -\frac{1}{8} & -2 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_{s1} & B_{s1} \\ C_{s1} & D_{s1} \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 12 \\ -0.1042 & -1 \end{bmatrix}$$

- Solving for the image position using the s1 matrix & X matrix:

$$B_s = B_{s1} + XD_{s1} = 0 \quad or \quad X = \frac{-B_{s1}}{D_{s1}} = \frac{-12}{-1} = 12\ cm$$

- Then the magnification is

$$m = \frac{1}{D_s} = \frac{1}{D_{s1}} = \frac{1}{-1} = -1$$

- Thus the object is at 12 cm from 2^{nd} lens, -3 cm high

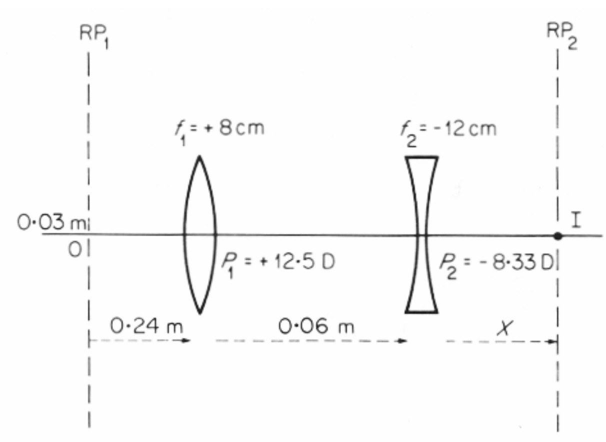


Figure II.14

Matrix Method and Spread Sheets

- Easy to use matrix method in Excel or matlab or maple
- Use mmult array function in excel
- Select array output cells (eg. matrix) and enter =mmult(
- Select space 1 cells then comma
- Select lens 1 cells (eg =mmult(G5:H6,I5:J6))
- Then do control+shift+enter (very important)
- Here is example from previous page

E460 example lesson 6			Distances in cm						
Lens Matrix		Lens 2		Matrx 1		Space 1		Lens 1	
		f2	-12			d	6	f1	8
0.25	6	1	0	0.25	6	1	6	1	0
-0.104167	1.5	0.083333	1	-0.125	1	0	1	-0.125	1
second focal length		-1/C	9.6						
second focal point		-A/C	2.4						
Image		System Matrix S1		Lens Matrix		Object			
						d	24		
1	X	0.25	12	0.25	6	1	24		
0	1	-0.104167	-1	-0.104167	1.5	0	1		
Object size		y	3						
image distance		=-Bs1/Ds1	12						
Magnification		=1/Ds1	-1						
Object size		=y/Ds1	-3						

Optical Matrix Equivalent Lens

- For any lens system can create an equivalent matrix & lens
- Combine all the matrices for the lens and spaces
- The for the combined matrix

where RP_1 = first lens left vertex

RP_2 = last lens right most vertex

n_1=index of refraction before 1st lens

n_2=index of refraction after last lens

System parameter described	Measured From	Measured To	Function of matrix elements	Special case $n_1 = n_2 = 1$
First focal point	RP_1	F_1	n_1D/C	D/C
First focal length	F_1	H_1	$-n_1/C$	$-1/C$
First principal point	RP_1	H_1	$n_1(D-1)/C$	$(D-1)/C$
First nodal point	RP_1	L_1	$(Dn_1-n_2)/C$	$(D-1)/C$
Second focal point	RP_2	F_2	$-n_2A/C$	$-A/C$
Second focal length	H_2	F_2	$-n_2/C$	$-1/C$
Second principal point	RP_2	H_2	$n_2(1-A)/C$	$(1-A)/C$
Second nodal point	RP_2	L_2	$(n_1-An_2)/C$	$(1-A)/C$

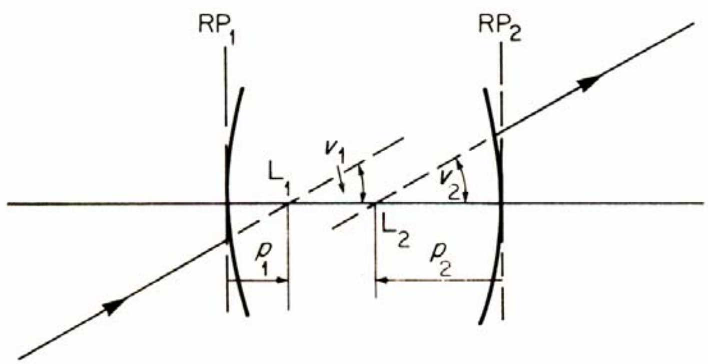


Figure II.17c

Example Combined Optical Matrix

- Using Two lens system from before
- Biconvex lens f_1=8 cm
- Second lens biconcave f_2= -12 cm located 6 cm from f_1
- Then the system matrix is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{8} & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 6 \\ -0.1042 & 1.5 \end{bmatrix}$$

- Second focal length (relative to H_2) is

$$f_{s2} = -\frac{1}{C} = -\frac{1}{-0.1042} = 9.766\ cm$$

- Second focal point, relative to RP_2 (second vertex)

$$f_{rP2} = -\frac{A}{C} = -\frac{0.25}{-0.1042} = 2.400\ cm$$

- Second principal point, relative to RP_2 (second vertex)

$$H_{s2} = \frac{1-A}{C} = \frac{1-0.25}{-0.1024} = -7.198\ cm$$

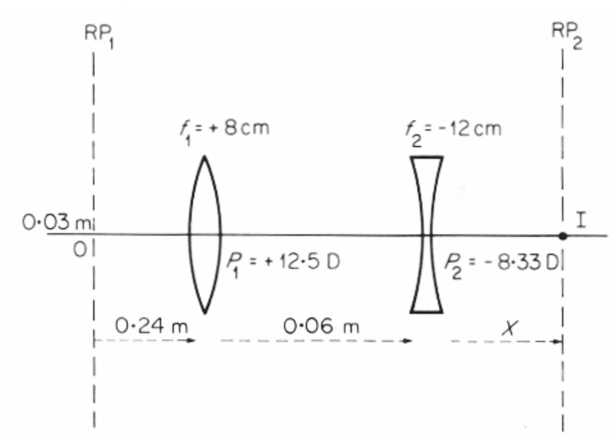


Figure II.14