Criteria for Optical Systems: Optical Path Difference

- How do we determine the quality of a lens system?
- Several criteria used in optical design Computer Aided Design
- Several CAD tools use Ray Tracing (see lesson 4)
- Then measure these criteria using the CAD tools
- Optical Path Difference (OPD) measures quality
- Measures path different from different parts of lens
- Plot OPD difference across the image relative to spherical wave
- Related to the Airy disk creation of a spot

![Figure 4.1: Optical Path Difference (OPD)](image)

![Figure 2: OPD Fan](image)

![OPD Fan Diagram](image)
Point Sources and OPD

- Simplest analysis: what happens to a point source
- Know that point sources should give perfect Airy disc
- Adding the OPD delay creates the distortion
- Little effect at $\lambda/4$
- By OPD $\lambda/2$ get definite distortion
- $\lambda$ OPD point is really distorted

![Figure 4.2](Image)

**Figure 4.2**
Image of a Point Source with Different Amounts of Peak-to-Valley Optical Path Difference Due to Coma
Point Spread Function

- Point Spread Function (PSF) is distribution of point source
- Like the response to an impulse by system in electrical circuits
- Often calculate for a system
- Again distorted by Optical path differences in the system
Wave Front Error

- Measure peak to valley (P-V) OPD
- Measures difference in wave front closest to image and furthest (lagging behind) at image
- Eg. in mirror system a P-V < $\lambda/8$ to meet Rayleigh criteria
- Because P-V is doubled by the reflection in mirrors
- Also measure RMS wave front error
- Difference from best fit of perfect spherical wave front

Figure 4.4
Peak-to-Valley and rms Wavefront Error

RMS OPD = $\sqrt{\frac{\sum OPD^2}{n}}$

difference from best fit of perfect spherical wave front

this wavefront has the same P-V wavefront error as the example at the left, but it has a lower RMS OPD
**Depth of Focus**

- Depth of focus: how much change in position is allowed
- With perfect optical system $<\lambda/4$ wave front difference needed
- Set by the angle $\theta$ of ray from edge of lens
- This sets depth of focus $\delta$ for this OPD $<\lambda/4$

$$\delta = \pm \frac{\lambda}{2n \sin^2 \theta} = \pm 2\lambda (f\#)^2$$

- Thus $f\#$ controls depth of focus
- $f\#:4$ has 16 micron depth
- $f\#:2$ only 2 micron
- Depth of Focus used with microscopes
- Depth of Field is term used in photography
- Depth that objects appear in focus at fixed plan

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**Figure 4.7**

*Depth of Focus*

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![Diagram of Depth of Focus](image)

![Graph of Depth of Focus](image)
Depth of Field in Photography

- Depth of Field is the range over which item stays in focus
- When focusing close get a near and far distance
- When focusing at distance want to use the **Hyperfocal Distance**
- Point where everything is in focus from infinity to a near distance
- Simple cameras with fixed lens always set to Hyperfocal Distance
Depth of Field Formulas

- Every camera has the “circle of confusion” c
- Eg for 35 mm it is 0.033 mm, point & shoot 0.01 mm
- Then Hyperfocal Distance H (in mm)

\[ H = \frac{f^2}{F\#c} + f \]

f is lens focal length in mm
- When focused at closer point distance s in mm
- Then nearest distance for sharp image is \( D_n \)

\[ D_n = \frac{s(H - f)}{H + s - 2f} \]

- Furthers distance for sharp image \( D_f \)

\[ D_f = \frac{s(H - f)}{H + s} \]

As get closer Depth of focus becomes very small
Get good DOF tools at http://www.dofmaster.com/
Modulation Transfer Function

- Modulation Transfer Function or MTF
- Basic measurement of Optical systems
- Look at a periodic target
- Measure Brightest \( I_{\text{max}} \) and darkest \( I_{\text{min}} \)

\[
MTF = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

- Contrast is simply

\[
\text{contrast} = \frac{I_{\text{max}}}{I_{\text{min}}}
\]

- MTF more accurate than contrast
Square Wave vs Sin wave

- Once MTF know for square wave can get sine wave response
- Use fourier components
- If $S(v)$ at frequency $v$ is for square waves
- Then can give response of sine wave

$$M(v) = \frac{\pi}{4} \left[ S(v) + \frac{S(3v)}{3} - \frac{S(5v)}{5} + \frac{S(7v)}{7} - \ldots \right]$$

$$S(v) = \frac{4}{\pi} \left[ M(v) - \frac{M(3v)}{3} + \frac{M(5v)}{5} - \frac{M(7v)}{7} + \ldots \right]$$
Diffraction Limited MTF

- For a perfect optical system

\[
MTF = \frac{2}{\pi} (\phi - \cos(\phi)\sin(\phi))
\]

Where

\[
\phi = \arccos\left(\frac{\lambda v}{2NA}\right)
\]

Maximum or cutoff frequency \(v_0\)

\[
v_0 = \frac{2NA}{\lambda} = \frac{1}{\lambda (f\#)}
\]

In an afocal system or image at infinity then for lens dia \(D\)

\[
v_0 = \frac{D}{\lambda}
\]

**Figure 11.15** The modulation transfer function of an aberration-free system (solid line). Note that frequency is expressed as a fraction of the cutoff frequency. The dashed line is the modulation factor for a square wave (bar) target. Both curves are based on diffraction effects and assume a system with a uniformly transmitting circular aperture.
Defocus in MTF

- Adding defocus decreases MTF
- Defocus MTF

\[
\text{defocus MTF} = \frac{2J_1(x)}{x}
\]

Where \( x \) is

\[
x = 2\pi \delta NA \frac{\nu(v_0 - \nu)}{v_0}
\]

- Max cutoff is 0.017 at \( \nu = v_0/2 \)

---

**Figure 11.16** The effect of defocusing on the modulation transfer function of an aberration-free system.

- (a) In focus \( \text{OPD} = 0.0 \)
- (b) Defocus \( = \lambda/(2n \sin^2 U) \) \( \text{OPD} = \lambda/4 \)
- (c) Defocus \( = \lambda/(n \sin^2 U) \) \( \text{OPD} = \lambda/2 \)
- (d) Defocus \( = 3\lambda/(2n \sin^2 U) \) \( \text{OPD} = 3\lambda/4 \)
- (e) Defocus \( = 2\lambda/(n \sin^2 U) \) \( \text{OPD} = \lambda \)
- (f) Defocus \( = 4\lambda/(n \sin^2 U) \) \( \text{OPD} = 2\lambda \)

(Curves are based on diffraction effects—not on a geometric calculation.)
MTF and Aberrations

- Aberrations degrade MTF
- Eg. 3\textsuperscript{rd} order spherical aberrations
- Effect goes as wavelength defect

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{aberrations.png}
\caption{The effect of third-order spherical aberration on the modulation transfer function.}
\end{figure}

(a) \( L A_m = 0.0 \) \quad \text{OPD} = 0

(b) \( L A_m = \frac{4\lambda}{n \sin^2 U} \) \quad \text{OPD} = \frac{\lambda}{4}

(c) \( L A_m = \frac{8\lambda}{n \sin^2 U} \) \quad \text{OPD} = \frac{\lambda}{2}

(d) \( L A_m = \frac{16\lambda}{n \sin^2 U} \) \quad \text{OPD} = \lambda

These curves are based on diffraction wave-front computations for an image plane midway between the marginal and paraxial foci.
MTF and Filling Lens

- MTF decreases as lens is not filled
- i.e. object blocking part of the lens
- Best result when image fills lens

**Figure 11.19** The effect of a central obscuration on the modulation transfer function of an aberration-free system.

(a) $s_0/s_m = 0.0$
(b) $s_0/s_m = 0.25$
(c) $s_0/s_m = 0.5$
(d) $s_0/s_m = 0.75$
MTF Specifications

• MTF in lenses are specified in lines per millimetre
• Typically 10 and 30 lines
• Specified separately for Saggittal and tangential
• Saggittal – vertical aberrations on focus plane
• Tangential or Meridional: horizontal on focus plane

ASTIGMATISM can be represented by these sectional views.
Reading MTF in Camera Lenses

- Camera lenses often publish MTF charts
- Below example for Nikon 18-55 mm zoom
- Plots show MTF at 10 lines/mm and 30/mm
- Shown with radius in mm from centre of image
- For a 24x15 mm image area
- Usually specified for single aperture (f/5.6 here)
- 10/mm measures lens contrast
- 30/mm lens resolution

<table>
<thead>
<tr>
<th>Spatial Frequencies</th>
<th>S: Sagittal</th>
<th>M: Meridional</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 lines/mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 lines/mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Poor MTF Charts

- Some companies give charts but little info
- Entry level Cannon 18-55 mm lens
- Chart give MTF but does not say lines/mm
- Cannot compare without that
Aerial Image Modulation Curves

- Resolution set in Aerial Image Modulation (AIM)
- Combines the lens and the detector (eg film or digital sensor)
- Measures the smallest resolution detected by sensor
- Sensor can significantly change resolutions

![Graphs showing modulation vs frequency](image)

**Figure 11.11** (a) The image modulation can be plotted as a function of the frequency of the test pattern. When the modulation drops below the minimum that can be detected, the target is not resolved. (b) The system represented by (a) will produce a superior image, although both (a) and (b) have the same limiting resolution.
Film or Sensor MTF

- Film or sensor has MTF measured
- Done with grating directly on sensor
- Eg Fuji fine grain Provia 100 slide film
- 50% MTF frequency (f₅₀) is 42 lp/mm
MTF/AIM and System

- Adding each item degrades system
- Also need to look at f/# for the lens
- Adding digitization degrades image
- This is 4000 dpi digitizing of negative

Line pairs per mm; MTF = 50%, 10% @ 36.8, 68.6/mm; Max MTF = 1.2
MTF and Coherent Light

- MTF is sharpest with coherent light
- Decreases as coherence decreases

**Figure 11.20** (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).
Low Power Laser Applications: Alignment & Measurement
Circularizing Laser Diodes

- Laser diodes are important for low power applications
- But laser diodes have high divergence & asymmetric beams
- Get 5-30° beam divergence
- Start with collimator: high power converging lens: stops expansion
- Then compensate for asymmetry
- Use cylindrical lens beam expander
- Cylindrical lenses: curved in one axis only unlike circular lenses
- Expands/focuses light in one direction only (along curved axis)
- Results in circular collimating beam

Figure 9-1  Optics used to circularize and collimate beam from diode laser. (Courtesy of Melle Griot.)
**Quadrature Detectors for Alignment**

- Often put detector on object being aligned to laser
- Use 4 quadrant detector Silicon photodiode detector
- Expand beam so some light in each quadrant
- Amount of photocurrent in each quadrant proportional to light
- Detect current difference of right/left & top bottom
- Higher current side has more beam
- Perfect alignment null current for both sides

*Figure 9-2* Simplified diagram of a laser alignment system.
Laser Leveling

- Lasers used to project lines of light
- Accuracy is set by the level of the beam source
- Used in construction projects: lines and cross lines
- Get vertical and horizontal
- Laser diodes give low cost levels now
- More complex: reflect light back from object
- Make certain light is reflected along the same path
- Called Autocolation

Figure 9-4  Diagram of a laser scanner system used for leveling.
Laser Size Gauging

- Gauging is measuring the size of objects in the beam
- Simplest expand beam the refocus
- Object (eg sphere) in beam reduces power
- Estimate size based on power reduction
- More accurate: scanning systems
- Scan beam with moving mirror (focused to point)
- Then measure time beam is blocked by object
- Knowing scan range then measure size of object
Laser & Linear Detector Array

- Use laser diode to illuminate a linear or 2D detector array
- Laser diode because creates collimated beam
- Expand beam to fill area
- Image is magnified or shrunk by lens
- Use pixel positions to determine object profile
- Low cost pixel arrays makes this less costly to gage scanners

![Diagram of Laser & Linear Detector Array](image)

**Figure 9-16** Typical linear photodiode array camera system.

![Graph of Focused and Out-of-focus Images](image)

**Figure 9-18** Focused and out-of-focus digitized images produced using a linear photodiode array, collimated light, and a constant image distance.
Laser Scanner to Detect Surface Defects

- Laser beam scanned across surface of reflective (e.g., metal) sheets
- Detect reflected light
- Flaws result in reduce or increase light
- Timing (when scanning) determines defect size
- Instead of spot use cylindrical expander to beam line of light
- Moving sheet (e.g., metal, glass, paper) crosses beam
- Use line or 2D images to detect changes
- Use both reflection and transmission depending on material
- Transmission can detect changes in thickness or quality

Figure 9.8 Laser scanner system designed to detect surface defects.

Figure 9.9 Laser scanner inspection system. (Courtesy of Intec Corp.)
Bar Code Scanners

- Diode laser now widely used in Bar code scanners
- Typically use two axis scanner
- Laser beam reflected from mirror on detector lens
- Bar code reflected light comes back along same path
- Detect rising and falling edge of the pattern
- Note: have the laser beam & return light on same path
- Use small mirror or beam splitter to put beam in path

Figure 9-10  Bar-code scanner/reader.
Laser Triangulation

- Lasers aimed at precise angles depth/profiles using triangulation
- Single spot for depth measurement
- Laser spot focused by lens onto detector array
- Change in laser spot depth position $\Delta z$
- Gives change in position $\Delta z'$ at detector
- Change set by magnification caused by lens
- $\theta$ laser to lens angle
- $\phi$ angle between detector an lens axis
- Resulting equations

$$\Delta z' = m \left( \frac{\sin \theta}{\sin \phi} \right) \Delta z$$

- Get real time measurement of distance changes
Laser Profileometry

- Use cylindrical lens to create line of laser light
- Use 2D detector array (imager) & lens to observe line
- If object is moving get continuous scan of profile
- Problems: Background light eg sunlight
- Changes in surface reflectance makes signal noisy
- Eg log profileometry for precise cutting of logs
- Problem is log surface changes eg dark knots, holes
Lidar

- Laser equivalent of Radar (RAdio Detection And Ranging)
- LIDAR: LIghht Detection And Ranging
- Can use pulses & measure time of flight (like radar)
- But only hard to measure $<10^{-10}$ sec or 3 cm
- Better phase method
- Modulate the laser diode current with frequency $f_m$
- Then detector compares phase of laser to detector signal
- Phase shift for distance $R$ is

$$\phi = \frac{2\pi}{\lambda_m} (2R) \quad \text{and} \quad c = \lambda_m f_m$$

- Then the distance is

$$R = \frac{c}{4\pi f_m} \phi$$

- If $>$ modulation wavelength $\lambda_m$ need to get number of cycles
- In extreme phase changes in the laser light
- That requires a very stable (coherent) laser: HeNe not diode

Figure 9.14  Diagram of laser range finder that uses an amplitude-modulated laser beam.