Main Requirements of the Laser

- Optical Resonator Cavity
- Laser Gain Medium of 2, 3 or 4 level types in the Cavity
- Sufficient means of Excitation (called pumping) eg. light, current, chemical reaction
- Population Inversion in the Gain Medium due to pumping

Laser Types

- Two main types depending on time operation
- Continuous Wave (CW)
- Pulsed operation
- Pulsed is easier, CW more useful

Fig. 2.25 Schematic construction of a low-power gas laser such as the helium–neon laser. The load resistor serves to limit the current once the discharge has been initiated.
Optical Resonator Cavity

- In laser want to confine light: have it bounce back and forth
- Then it will gain most energy from gain medium
- Need several passes to obtain maximum energy from gain medium
- Confine light between two mirrors (Resonator Cavity)
  Also called Fabry Perot Etalon
- Have mirror (M₁) at back end highly reflective
- Front end (M₂) not fully transparent
- Place pumped medium between two mirrors: resonator
- Curved mirror will focus beam approximately at radius
- However is the resonator stable?
- Stability given by g parameters: g₁ back mirror, g₂ front mirror:

\[ g_i = 1 - \frac{L}{r_i} \]

- For two mirrors resonator stable if
  \[ 0 < g_1g_2 < 1 \]
- Unstable if
  \[ g_1g_2 < 0 \quad g_1g_2 > 1 \]
- at the boundary (\( g_1g_2 = 0 \) or 1) marginally stable

![Optical resonator consisting of two spherical mirrors. Radii are defined as having positive values if the mirrors are concave.](image)
Stability of Different Resonators

![Diagram](image)

**Fig. 1.9** Fox and Li stability diagram.

(a) Nearly planar (convex)
- \(-r_1, -r_2 \gg L\)
- \(g_1, g_2 \gg 1\)
- unstable

(b) Planar
- \(r_1 = r_2 = \infty\)
- \(g_1 = g_2 = 1\)
- marginally stable

(c) Nearly planar (concave)
- \(r_1, r_2 \gg L\)
- \(0 < g_1 = g_2 \leq 1\)
- stable

(d) Nearly confocal
- \(r_1, r_2 \gg L\)
- \(g_1 = g_2 \gg 0\)
- stable

(e) Confocal
- \(r_1 = r_2 = L\)
- \(g_1 = g_2 = 0\)
- marginally stable

(f) Nearly concentric
- \(r_1 \gg L/2; r_2 \gg L/2\)
- \(0 > g_1 \geq -1; g_2 \gg L/2\)
- stable

(g) Concentric
- \(r_1 = r_2 = L/2\)
- \(g_1 = g_2 = -1\)
- marginally stable

(h) Nearly concentric
- \(r_1 \ll L/2; r_2 \ll L/2\)
- \(g_1 \ll -1; g_2 \ll -1\)
- unstable

(i) Hemi-concentric
- \(r_1 = L; r_2 = \infty\)
- \(g_1 = 0; g_2 = 1\)
- marginally stable

**Fig. 3.9** Laser cavity mirror configurations. Stability for each of these configurations is indicated.
Polarization and Lasers

- Lasers often need output windows on gain medium in cavity
- Output windows often produce polarized light
- Polarized means light's electric and magnetic vectors at a particular angle
- Normal windows lose light to reflection
- Least loss for windows if light hits glass at Brewster Angle
- Perpendicular polarization reflected
- Parallel polarization transmitted with no loss (laser more efficient)
- Called a Brewster Window & the Brewster Angle is

\[ \theta_b = \tan^{-1} \left( \frac{n_2}{n_1} \right) \]

- \( n_1 \) = index of refraction of air
- \( n_2 \) = index of refraction of glass
- Example: What is Brewster for Glass of \( n_2 = 1.5 \)

\[ \theta_b = \tan^{-1} \left( \frac{1.5}{1} \right) = \tan^{-1} \left( \frac{1.5}{1} \right) = 56.6^\circ \]

![Reflectance as a function of the angle of incidence for light polarized parallel (||) and perpendicular (⊥) to the plane of incidence.](image)

Fig. 2.5 Reflectance as a function of the angle of incidence for light polarized parallel (||) and perpendicular (⊥) to the plane of incidence.
Laser Threshold

- With good Gain Medium in optical cavity can get lasing but only if gain medium is excited enough
- Low pumping levels: mostly spontaneous emission
- At some pumping get population inversion in gain medium
- Beyond inversion get **Threshold** pumping for lasing set by the losses in cavity
- Very sensitive to laser cavity condition eg slight misalignment of mirrors threshold rises significantly
- At threshold gain in one pass = losses in cavity

**Figure 3.5** Laser threshold phenomenon—a laser does not generate significant optical output until the pump energy passes a threshold. At higher pump energies, the output power increases rapidly. In practice, each laser has limits on output, and eventually the output-input curve bends over.
Round Trip Power Gain

• Within medium light intensity I gained in one pass

\[ I(L) = I_0 \exp[(g - \alpha)L] \]

where \( g = \) small signal gain coefficient
\( \alpha = \) the absorption coefficient
\( L = \) length of cavity

• Thus calculate **Round Trip Power** Gain \( G_r \)

• Each mirror has a reflectivity \( R_i \)
  \( R=1 \) for perfect reflection off a mirror

\[ G_r = \frac{I(2L)}{I(0)} = R_1 R_2 \exp[(g - \alpha)2L] \]

• At threshold \( G_r = 1 \)

• Thus threshold gain required for lasing is

\[ g_{th} = \alpha + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

• Some of the loss = laser emission

---

**Fig. 3.3** A bunch of photons bouncing back and forth between the mirrors in a laser cavity.
Transition Lineshape

- Distribution of energy levels creates width of emission
- Usually Gaussian shape
- Width broadened by many mechanisms
- Most more important in gases
- Doppler Broadening (movement of molecules)
- Collision Broadening (atomic/molecular collisions)
- Radiative Lifetime Broadening
  (finite lifetime of transitions)

![Frequency distribution of the radiation emitted from a group of atoms following transitions from energy levels $E_2$ to $E_1$. The precise shape of this distribution, that is the lineshape function $g(\nu)$, depends on the dominant spectral broadening mechanisms.](image)
Axial Modes of laser

- Proper phase relationship only signal in phase in cavity
- Thus cavity must be integer number of half wavelengths

\[ L = \frac{p\lambda}{2} \]

where \( p \) = integer

- Results in frequency separation of peaks

\[ \Delta \nu = \frac{c}{2L} \]

- Emission from the atomic transitions is a wider Gaussian
- Result is axial modes imposed on Gaussian Transition spread

![Graph showing irradiance and frequency distribution](image)

(a) Irradiance distribution with axial modes spaced by \( \frac{c}{2L} \)

(b) Irradiance, \( I \) vs. frequency, \( \nu \)
Axial modes within Transition Gaussian

- Each axial mode is Gaussian shape narrower than transition peak
- eg for L=1 m Argon laser at 514 nm

\[
\Delta \nu = \frac{c}{2L} = \frac{3.00 \times 10^8}{2} = 150 \text{ MHz} \quad \nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{5.14 \times 10^{-7}} = 5.83 \times 10^{14} \text{ Hz}
\]

- Thus emission much smaller than 0.0026% of frequency
  Much narrower than other light sources.

Fig. 3.2 The mode frequency spectrum for a cavity (where \( r_1 = r_2 = 2L \)) superimposed on the laser gain profile. Note that many modes have the same frequency (that is they are "degenerate").

Fig. 1.18 Axial modes formed in a HeNe laser — the mode pattern is repeated (here three times) as the optical frequency analyzer scans through the gain curve of the laser (Photograph courtesy Dr. I. D. Latimer, School of Physics, Newcastle upon Tyne Polytechnic).
Transverse Modes

- Comes from microwave cavities
- Some waves travel off axis, but within cavity
- Result is Phase changes in repeating paths
- These can change shape of output
- Get local minimums (nulls) in the output beam shape
- Reduce these by narrowing the beam
- Called Transverse ElectroMagnetic, TEM
Transverse Modes

- Transverse ElectroMagnetic, TEM depend on cavity type
- In cylindrical geometry two possible phase reversal orientations
  - horizontal (q) and vertical (r)
  - Show these as $\text{TEM}_{qr}$
  - q and r give number of null's in output beam
  - Horizontal (q) gives number of vertical running nulls
  - Vertical (r) gives number of horizontal running nulls
  - Thus $\text{TEM}_{12}$ has 2 vertical, 1 horizontal nulls
- Special mode $\text{TEM}_{01}^*$ or donut mode
  - comes from rapid switching from $\text{TEM}_{01}$ to $\text{TEM}_{10}$

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Figure 3.7 Lower-order modes that can be produced by a stable resonator. (Courtesy of Melles Griot, from Optics Guide 4.)
General Laser Types

- Solid State Laser (solid rods): eg ruby
- Gas laser: eg He-Ne
- Dye Lasers
- Semiconductor Laser: GaAs laser diode
- Chemical Lasers
- Free Electron Lasers

Fig. 2. Range of wavelengths for current commercial lasers. First date is date of discovery, the second is of commercialisation (4).
Gas Lasers

- Gas sealed within a tube with brewster windows
- Electric arc in tube causes glowing of gas
- Glow indication of pumping
- Commonest type He-Ne

Fig. 2.25 Schematic construction of a low-power gas laser such as the helium-neon laser. The load resistor serves to limit the current once the discharge has been initiated.

Fig. 2.27 Typical structure of a sealed mirror HeNe laser.
He-Ne Laser Energy levels

- One type of 3 level laser

Fig. 2.26  Energy levels relevant to the operation of the HeNe laser. M indicates a metastable state (see p. 19).
Einstein Coefficients and Lasers

- Recall the Einstein Emission/Absorption Coefficients
- Consider again a 2 level system being pumped by light
- $A_{21}$ is the Einstein spontaneous emission Coefficient
- $B_{12}$ is the Einstein spontaneous absorption coefficient
- $B_{21}$ is the Einstein stimulated emission coefficient
- At equilibrium the absorption from pumping equals the spontaneous and stimulated emission.

$$N_1 \rho B_{12} = N_2 A_{21} + N_2 \rho B_{21}$$

- Now recall the Boltzman distribution

$$\frac{N_1}{N_0} = \exp\left(\frac{[E_1 - E_0]}{KT}\right) = \exp\left(\frac{h\nu}{KT}\right)$$

- $\nu$ = the frequency of the light
- $h\nu$ = energy in photon
- Thus

$$\rho B_{12} \exp\left(\frac{h\nu}{KT}\right) = A_{21} + \rho B_{21}$$
Einstein Coefficients relationships

- Solving for the emitted photon energy density

\[ \rho = \frac{A_{21}}{B_{12} \exp \left( \frac{h \nu}{KT} \right) - B_{21}} \]

- From Planck's law the photons emitted at a given temperature are:

\[ \rho = \frac{8\pi h \nu^3}{c^3 \left[ \exp \left( \frac{h \nu}{KT} \right) - 1 \right]} \]

- From these two can solve noting \( B_{12} = B_{21} \)

\[ A_{21} = \frac{8\pi h \nu^3}{c^3} B_{12} = \frac{8\pi h}{\lambda^3} B_{12} \]

- Note absorption to emission increases rapidly with wavelength
Under Lasing Conditions

- Spontaneous emission is nil so total emission/unite area is
  \[
  \frac{dI}{dx} = (N_2 - N_1)B_{21}\rho h\nu
  \]

- For a linear beam \( I = \rho c \) (energy density times speed) thus
  \[
  \frac{dI}{dx} = \frac{(N_2 - N_1)B_{21}h\nu I}{c}
  \]

- Thus the gain is
  \[
  g = \frac{(N_2 - N_1)B_{21}h\nu}{c}
  \]

- Or substituting for the spontaneous coefficient
  \[
  g = \frac{(N_2 - N_1)A_{21}c^2}{8\pi\nu^2} = \frac{(N_2 - N_1)\lambda^2}{8\pi\tau_{21}}
  \]

- Three important implications because we need \( g > g_{th} \) to lase
  1. For gain \( N_2 > N_1 \) (i.e. population inversion)
  2. Shorter wavelength the lower the gain
     Hence much harder to make UV than IR lasers
  3. Want short lifetime \( \tau_{21} \) for higher gain

- But want metastable long \( \tau_{21} \) to get population inversion!
- Hence tradeoff: \( \tau_{21} \) must be short enough for threshold gain
  but long enough for population inversion
- More difficult to achieve for CW (continuous) lasers