Lab 3: measurement of Laser Gaussian Beam Profile
v5.1 (Dec. 11, 2013)

- Lab 3: basic experience working with laser
  (1) To create a beam expander for the Argon laser
  (2) To measure the spot size and profile of the Argon laser
    Measure before and after the beam expansion
- Do this by moving a knife edge through the beam
- Have a computer controlled knife that moves through beam
Knife Edge measurement of Gaussian Beam

- Consider a Gaussian shaped beam

\[ I(r) = I_0 \exp\left(-\frac{2r^2}{w^2}\right) = \frac{2P}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \]

Where \( P \) = total power in the beam
\( w = \frac{1}{e^2} \) beam radius at point \( w(z) \)
- This is in cylindrical coordinates
- \( r \) is the radius of the central area

GAUSSIAN IRRADIANCE PROFILE for TEM\(_{00}\) mode, showing definitions of beam radius \( w \).
**Knife Edge and Gaussian**

- Straight knife edge cutting into a Gaussian shaped beam
- Measure the total power seen when knife move in x direction
- Must convert to Cartesian coordinates & integrate
- Assume $-\infty$ is when the knife fully below the beam

\[
I(x) = \frac{2P}{\pi w^2} \int_{-\infty}^{x} \exp\left(-\frac{x^2}{w^2}\right) dx \int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{w^2}\right) dy
\]

Where $P$ is the total power of the beam

$I(x)$ is the intensity measured at position $x$

- In x direction the beam is cut: Integrate from $x$ to $-\infty$
- In y direction get full beam: integrate from $-\infty$ to $+\infty$
- To solve this use the error function or integral of the normal

\[
erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-s^2} ds
\]

- Two ways of fitting this:
  - Fit the power measured: that is the integral
  - Fit the derivative
Fitting the power measured

- The power measured at the detector is the integral
- \( x_0 = \) centre of beam
- Then the power measured is given by for \( x > x_0 \)

\[
I(x) = \frac{P}{2} \text{erf}\left(\frac{(x - x_0)}{w}\right)
\]

For \( x < x_0 \)

\[
I(x) = \frac{P}{2} \left[1 + \text{erf}\left(\frac{(-x + x_0)}{w}\right)\right]
\]

- Must also assume some background light level \( B \)
- In Excel use the Normdist (Normal distribution function)
- This is slightly different from erf function
- Fit with the excel function of the following formula

\[
I(x) = P \cdot \text{normdist}(-x,-x_0,w/1.414,1) + B
\]

Where \( x \) is the position (starting with \( x \) below \( x_0 \))
- \( x_0 \) is the fitted centre point of the beam
- \( w = \frac{1}{e^2} \) size of beam you fit
- \( 1 \) is to make it the integration of the normal distribution
- \( B \) is the background or offset level

- Set up a spreadsheet with initial estimates of each parameter
- Have columns with \( x \), \( I(x) \), fitted \( I(x) \),
- \( I_{\text{fit}}(x) - I(x) = \text{error} \) (called the error of fit or residual)
- \( \text{error}^2 = (I_{\text{fit}}(x) - I(x))^2 \) (called the residual squared in statistics)
- Set a column to sum the \( \text{error}^2 \) (sum of the squares)
Running the Fit

- Want to minimize the sum of the squares
- In stats were shown this gives the best statistical fit to data
- Use the Excel solver function to do this
- Set solver to minimize the sum of squares cell
- Then use solver under tools tab to fit
- Set to minimizing sum of squares cell
- Use sum of squares as fit as minimization,
- Set P, \( x_0 \), \( w \) and \( B \) as variables to be changed for fit
- Solver need initial estimates – these important for getting fit
- Getting good initial values for the fit
  - \( P \) - use the measured \( I \) before the knife edge cuts (start of data)
  - \( x_0 \) – use \( x \) for \( P/2 \) point from the data (nearly right)
  - \( B \) – use background light level
  - \( w \) (spot size) is the difficult one to estimate and the hardest to get.
- See the difference discussion next page to estimate
- Plots help evaluate the fit
  - Plot \( I(x) \) vs \( x \) for both data and fit
  - Suggest put both on the same plot so you tell how good a fit
  - Useful to plot the errors against position (called residuals)
  - Thus plot residuals \( I_{fit}(x)-I(x) \) vs \( x \)
  - Ideal fit residuals should be small
  - Residuals should be on both sides of 0 line (ie + and -)
- See sample excel layout in appendix B
- See Appendix A for running solver
Fit the difference of Power Measured

- The derivative
- Take a derivative of the measurements
- Best if take a simple derivative

\[
\frac{dI_j}{dx} = \frac{[I(x_{j+1}) - I(x_j)]}{[x_{j+1} - x_j]}
\]

- Plot \(dI/dx\) vs \(x\) for your data
- Then the plot is a Gaussian shape with the formula:

\[
\frac{dI(x)}{dx} = \frac{P}{w\sqrt{\pi}} \exp\left(-\frac{(x-x_0)^2}{w^2}\right)
\]

- Note need to be careful with the derivatives units you use
- Suggest you plot the derivative but not fit it
- Derivatives are very prone to errors (small errors magnified)
- Plotting shows if the curve shape you are getting
- Check does it really look like a Gaussian
- This is best way to estimate \(w\)
- Take the plot and find half the peak \(dI/dx\) value
- Width of curve at half point is the FWHM of laser beam
- Then convert FWHM to \(w\) (1/e²)
- FWHM*0.849 = \(w\)
- Gives a good estimate of \(w\) for curve fit
- See the plots in Appendix B
Appendix 1: Solver in excel

Adding solver to excel
For the lab 3 you will probably want to use the excel solver add on in excel (matlab is not friendly for this)
Students who are using MS office 2007/2010 may find that the solver was not loaded into the excel and does not appear in the data menu. In earlier versions a pull down tab showed it still needed to be installed but 2007 (and 2010) does not. Check your data tab ribbon – will show solver on the furthest right if installed.
To get the solver:
The instructions are hard to find in help also. Attached is a screen shot of the help instructions on MS web site and the add-ins list window showing where the solver is once installed.
**Using Solver:**
Excel help is not good in describing this
Here are links to several good sites that give nice examples of how to use the solver for a problem where you are adjusting several variables to minimize one parameter (sum of squares in the lab)
General instructions
http://chandoo.org/wp/2011/05/11/using-solver-to-assign-item/


Using solver in nonlinear fits (as in this lab)

Solver is useful for your work in other labs
Appendix B
Printout of excel sheet for laser profile fitting.

Nd:YAG Glenn

final round. (this is used to roughly profile the laser beam and decide on the resolution for the axis needed)

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