Solid Modeling

25.353 Lecture Series

Prof. Gary Wang
Department of Mechanical and Manufacturing Engineering
The University of Manitoba
Information complete, unambiguous, accurate solid model
Solid Modeling Representation

- A complete geometric data representation of an object is one that enables points in space to be classified relative to the object, if it is inside, outside, or on the object.
- Complete and unambiguous
- Applications
  - Finite-element analysis and fluid flow analysis;
  - NC data programming for computer-aided manufacturing, robotics, assembly;
  - Evaluation of the size, shape, and weight of products.
Solid Modeling Representation

- Constructive Solid Geometry (CSG)
- Boundary Representation (B-Rep)
- Sweep Representation
- Decomposition Schemes
  - Spatial Enumeration / Voxel Representation
  - Cell Decomposition
Constructive Solid Geometry (CSG)

A CSG model is based on the topological notion that *a physical object* can be divided into a set of *primitives* that can be combined in a certain order following a set of *rules* (regularized Boolean operations) to form the object.

- Pre-defined geometric primitives
- Regularized Boolean operations
- CSG tree structure
Primitives (cont'd)

- A set of location data
- A set of geometrical data
- A set of orientation data

(Zeid, 1991)  
(Kunwoo Lee, 1999)
Half Space: Element of Primitives

- Unbounded geometric entities
- Each one of them divides the representation space into infinite portions, one filled with material and the other empty
- Surfaces can be considered half-space boundaries and half spaces can be considered directed surfaces
- An object is defined by the volume space contained within the defined boundary of the object

Introduces the direction into the modeling, thus enables the topological information be stored in a geometric model
Half Spaces

• By specifying different boundary surface, we can have any half-spaces;
• The most commonly used half-spaces are planar, cylindrical, spherical, conical, and toroidal.
• By combining half-spaces (using Boolean operations) in a building block fashion, various solids can be constructed.
Half Spaces

• Planar half-space \( H = \{ (x, y, z) : z < 0 \} \)

• Cylindrical half-space \( H = \{ (x, y, z) : x^2 + y^2 < R^2 \} \)

Block: \( \{(x, y, z) : 0 < x < w, \ 0 < y < H, \ and \ 0 < z < D \} \)

Cylinder: \( \{(x, y, z) : x^2 + y^2 < R^2, \ and \ 0 < z < H \} \)
Half Spaces

(a) Solid S

(b) Half-space representation
Why not use half spaces directly?

- Possible unbounded solid models
- Rely on the user to check the validity
- Modeling with half spaces are not intuitive and cumbersome

Thus, primitives are used, which are bounded and intuitive.
Boolean Operations

• Operations to construct a complex model through geometric primitives

• Common operations
  – Union
  – Intersection
  – Difference
Set Theory

- $P \cup Q = \{x: x \in P \text{ or } x \in Q\}$
- $P \cap Q = \{x: x \in P \text{ and } x \in Q\}$
- $P - Q = \{x: x \in P \text{ and } x \notin Q\}$
Regularized Sets (r-set)

Valid solid objects are bounded, closed, regular subset of 3D space.

Dangling face

Dangling edge

P - Q

i(P – Q)

P -* Q
Boolean Operations (regularized)

Operations to construct a complex model through geometric primitives

- Union
- Intersection
- Difference

(Kunwoo Lee, 1999)
CSG Tree

(Kunwoo Lee, 1999)
Alternative Paths of Modeling

(a) Object

(b) Possible solid model of the solid
(Zeid, 1991)
An Exercise: CSG Tree

Build two CSG trees of different strategies
Advantage

- Easy, concise and requires minimum storage (CSG tree)

Disadvantages

- Not good at complex shape
- Slow to display
A B-rep model is bounded by a set of closed and orientable faces, which are bounded by edges comprised of vertices.

- More suitable for complex designs
- Euler operations to avoid nonsense objects
How B-Rep Knows about a Solid

• The solid is defined by specifying that all points on one side of the boundary are outside and those on the other side are inside of the object. This inside and outside information is represented using the surface normal of all surface patches. Points along outward normal (N > 0) are outside; points for N < 0 are inside.
Primitives of B-rep

- A vertex is a unique point (an ordered triplet) in space.
- An edge is a finite, non-self-intersecting, directed space curve bounded by two vertices.
- A face is defined as a finite, connected, non-self-intersecting region of a closed oriented surface bounded by one or more loops.
Loop and Body

- A loop is an ordered alternating sequence of vertices and edges. A loop defines a non-self-intersecting, piecewise, closed space curve which, in turn, may be a boundary of a face.

- A body is a set of faces that bound a single connected closed volume. A minimum body is a point. Topologically a point has one face, one vertex, and no edges. It is called a singular body.
Euler’s Law (1752)

\[ F - E + V - L = 2(B - G) \]

- **F**: Number of faces
- **E**: Number of edges
- **V**: Number of vertices
- **L**: Number of faces’ inner loops
- **B**: Number of bodies
- **G**: Genus = Number of handles or through holes

- To create, manipulate, and edit primitives
- To ensure validity, i.e. closeness, no dangling faces and edges
Polyhedral Objects

For polyhedra, $L=G=0$, and $B=1$, thus the Euler's law is simplified to

$$F - E + V = 2$$

$F: 5$

$E: 8$

$V: 5$

$B: 1$

$L: 0$

$G: 0$

$F - E + V = 2$
Curved Objects

(Zeid, 1991)
Curved Objects (cont’d)

\[ F = 16, \ E = 42, \ V = 28, \ L = 2, \ B = 1, \ G = 1 \]

\[ F - E + V - L = 16 - 42 + 28 - 2 = 0, \quad 2(B - G) = 2(1 - 1) = 0 \]

(Zeid, 1991)
Examples

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>F</th>
<th>E</th>
<th>V</th>
<th>L</th>
<th>B</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>10</td>
<td>24</td>
<td>16</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>16</td>
<td>36</td>
<td>24</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>11</td>
<td>24</td>
<td>16</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>12</td>
<td>24</td>
<td>16</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>10</td>
<td>24</td>
<td>16</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>14</td>
<td>36</td>
<td>24</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Through hole
2. Boundary hole
3. Interior hole
4. Through hole

(Zeid, 1991)
An Exercise

F  E  V  L  B  G
11  27  18  0  1  0
**Euler Operator**

**MBFV**: Make body, face, vertex

**MME**: Make multiple edges

**MEF**: Make edge, face

**KEML**: Kill edge, make loop

**KFMLG**: Kill face, make loop, genus

\[
F - E + V - L = 2(B - G)
\]

(Zeid, 1991)
(Zeid, 1991)
Data Structure

**Face Table**
- Face: Edges
  - $F_1$: $E_1, E_5, E_6$
  - $F_2$: $E_2, E_6, E_7$
  - $F_3$: $E_3, E_7, E_8$
  - $F_4$: $E_4, E_8, E_5$
  - $F_5$: $E_1, E_2, E_3, E_4$

**Edge Table**
- Edge: Vertices
  - $E_1$: $V_1, V_2$
  - $E_2$: $V_2, V_3$
  - $E_3$: $V_3, V_4$
  - $E_4$: $V_4, V_1$
  - $E_5$: $V_1, V_5$
  - $E_6$: $V_2, V_5$
  - $E_7$: $V_3, V_5$
  - $E_8$: $V_4, V_5$

**Vertex Table**
- Vertex: Coord.
  - $V_1$: $x_1, y_1, z_1$
  - $V_2$: $x_2, y_2, z_2$
  - $V_3$: $x_3, y_3, z_3$
  - $V_4$: $x_4, y_4, z_4$
  - $V_5$: $x_5, y_5, z_5$

**Surface equation**

**Curve equation**

(Kunwoo Lee, 1999)
Advantages

- Popular
- Appropriate to construct solid models of unusual shapes that are difficult to be built using primitives.
- Simple to convert a B-rep model into a wireframe model

Disadvantages

- Larger storage
- Verbose for input
<table>
<thead>
<tr>
<th></th>
<th>CSG</th>
<th>B-Rep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pros</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cons</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sweep Representation

- Useful in creating 2.5 D objects
- No sweeping-based models
  - Validity condition unknown
- A means of entering object description

Linear sweep (Kunwoo Lee, 1999)
Decomposition Schemes

Spatial enumeration / Voxel representation

- An extension of a raster representation
- Cube cells with a fixed size
- Memory size vs. accuracy

(Kunwoo Lee, 1999)
Decomposition Schemes

Cell Representation

- No restriction on the shape and size of cells
- Applied to the finite element modeling

(Kunwoo Lee, 1999)
Quick Questions

1. CSG uses Euler operators in modeling.
2. CSG needs low storage due to the simple CSG tree structure and primitives.
3. CSG primitives are constructed from the half-space concept.
4. Directed surfaces, Euler operations and Euler’s law fundamentally distinguish the B-rep from wireframe modeling.
5. CSG cannot model sculptured objects and thus is limited in modeling capability.
6. It is easier to convert a CSG model to a wireframe model than to convert a B-rep model to a wireframe model.
7. Because both CSG and B-rep use face direction (half-space or surface normal), they can have a full “body knowledge.”
8. Generally speaking, most high-end CAD tools have the B-rep method while most low-end tools rely heavily on the CSG method.
Review

- CSG
  - Half-space & Primitives
  - Regularized Set Operation
  - CSG Tree
- B-rep
  - Euler’s Law
- Sweep
- Voxel and Cell Representation