MODELING AND CHARACTERIZATION
OF TRAFFIC
IN PUBLIC SAFETY WIRELESS NETWORKS

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Roadmap

- Introduction
- Statistical concepts and analysis tools
- Analysis of traffic data:
  - call inter-arrival times
  - call holding times
- Traffic modeling and characterization
- Conclusions and references
Roadmap

- Introduction
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E-Comm network: coverage and user agencies

RCMP and Police
Fire
Ambulance
Other

TG: Talk group
R: Radio device (user)
E-Comm network architecture

Transmitters/Repeaters

Users

Vancouver

Burnaby

PSTN

PBX

Dispatch console

Network switch

Database server

Data gateway

Management console

Other EDACS systems
Network architecture
Network model

- central switch
- 11 cells
Network characteristics

- **EDACS**: Enhanced Digital Access Communications Systems
- **Simulcast**: repeaters covering one cell use identical frequencies
- **Trunking**: available frequencies in a cell are shared dynamically among mobile users
- **Cell capacity** (number of available frequencies in a cell):
  - one radio channel occupies one frequency
  - one call occupies one radio channel
Call establishment

- Users are organized in talk groups:
  - one-to-many type of conversations
- Push-to-talk (PTT) mechanism for network access:
  - user presses the PTT button
  - system locates other members of the talk group
  - system checks for availability of channels:
    - channel available: call established
    - all channels busy: call queued/dropped
  - user releases PTT:
    - call terminates
Erlang traffic models

Erlang B

\[ P_B = \frac{A^N}{N! \sum_{x=0}^{N} A^x x!} \]

Erlang C

\[ P_C = \frac{A^N}{N! \sum_{x=0}^{N-1} A^x x!} + \frac{N}{N! N - N} \]

- \( P_B \): probability of rejecting a call
- \( P_C \): probability of delaying a call
- \( N \): number of channels/lines
- \( A \): total traffic volume
Erlang models

- Erlang B model assumes:
  - call holding time follows exponential distribution
  - blocked call will be rejected immediately

- Erlang C model assumes:
  - call holding time follows exponential distribution
  - blocked call will be put into a FIFO queue with infinite size
Previous work

- Simulation:
  - OPNET
  - WarnSim
- Traffic prediction based on user clusters
  - Seasonal ARIMA model
- Statistical analysis of traffic
  - three busy hours in 2001


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Statistical concepts

- **Probability distribution:**
  - probability that outcomes of a process are within a given range of values
  - expressed through probability density (pdf) and cumulative distribution (cdf) functions

- **Autocorrelation:**
  - measures the dependence between two outcomes of a process
  - wide-sense stationary processes: autocorrelation depends only on the difference (lag) between the time instances of the outcomes
LRD: definition

- Slow decay of the autocorrelation function $r(k)$ of a (wide-sense) stationary process $X(n)$:

$$\sum_{k=-\infty}^{\infty} r(k) = \infty$$

- Model

$$r(k) = c_r k^{-(2-2H)}, \quad k \to \infty$$

- Corollary

$$f(\nu) = c_f |\nu|^{-\alpha}, \quad \nu \to 0$$

where $f(\nu)$ is the power spectral density of $X(n)$, $c_r$ and $c_f$ are non-zero constants, and $0 < \alpha < 1$

$0.5 < H < 1$ implies LRD

LRD: long-range dependence
Wavelet coefficients

- Discrete wavelet transform of a signal $X(t)$:

$$d(j, k) = \int_{-\infty}^{\infty} X(t) \psi_{j,k}(t) dt$$  

where

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j} t - k)$$

- $\psi(t)$: mother wavelet
  - $j$: octave
  - $k$: translation

- Reconstruction formula:

$$X(t) = \sum_{j=0}^{\infty} \sum_{k} d(j, k) \psi_{j,k}(t)$$
LRD and wavelets

Let $X(t)$ be LRD process (wide-sense stationary)
- its power spectral density:
  $$f(\nu) \sim c_f |\nu|^{-\alpha}, \ \nu \to 0$$

Mean square value of its wavelet coefficients on octave $j$ satisfies:

$$E\{d(j,k)^2\} = 2^{j\alpha} c_f C(\alpha, \psi)$$

where $C(\alpha, \psi) = \int |\nu|^{-\alpha} |\Psi(\nu)|^2 d\nu$ does not depend on $j$

LRD and wavelets

- Logarithm:
  \[
  \log_2 \mathbb{E}\{d(j,k)^2\} = \alpha \times j + c
  \]

- Important property: for given \( j \), \( d(j,k) \) does not exhibit long-range dependence (with respect to \( k \))
  - with appropriately chosen mother wavelet

- Hence:
  - simple estimator for \( \mathbb{E}\{d(j,k)^2\} \) is a sample mean:
    \[
    \mathbb{E}\{d(j,k)^2\} = \frac{1}{n_j} \sum_{k=1}^{n_j} d(j,k)^2
    \]
  - \( n_j \): number of wavelet coefficients at octave \( j \)
Estimation of $\alpha$ and $H$

- Logscale diagram: plot of $\log_2 E\{d(j,k)^2\}$ vs. $j$ (octave)
- Linear relationship between $\log_2 E\{d(j,k)^2\}$ and $j$ on the coarsest octaves indicates LRD
- Estimation of $\alpha$:
  - linear regression of $\log_2 E\{d(j,k)^2\}$ on $j$ in the linear region of the logscale diagram
- $H = 0.5 (\alpha + 1)$
Logscale diagram: example

- call inter-arrival times: 22:00–23:00, 26.03.2003
- $\alpha=0.576$, $H=0.788$ (octaves 4–9)
Test for time constancy of $\alpha$

- $X(n)$: wide-sense stationary process
  - $\alpha$ does not depend on $n$
- Is $\alpha$ constant throughout the time series $X(n)$?

Approach:
- divide $X(n)$ into $m$ blocks of equal lengths
- estimate $\alpha$ for each block
- compare the estimates
- If $\alpha$ varies significantly, estimating $\alpha$ for the entire time series is not meaningful
- In our analysis, $m \in \{3, 4, 5, 6, 7, 8, 10\}$
Test for constancy: example

- Trace is divided into 12 sub-traces of equal lengths
- Variation of the scaling exponent indicates that $\alpha$ is not constant

Star Wars IV (MPEG-4)
Kolmogorov-Smirnov test

- Goodness-of-fit test: quantitative decision whether the empirical cumulative distribution function (ECDF) of a set of observations is consistent with a random sample from an assumed theoretical distribution.

- ECDF is a step function (step size $1/N$) of $N$ ordered data points $Y_1, Y_2, \ldots, Y_N$:

  \[ E_N = \frac{n(i)}{N} \]

  $n(i)$: the number of data samples with values smaller than $Y_i$. 
Parameters

- Hypothesis:
  - null: the candidate distribution fits the empirical data
  - alternative: the candidate distribution does not fit the empirical data
- Input parameters: significance level $\sigma$ and tail
- Output parameters:
  - $p$-value
  - $k$: test statistic
  - $cv$: critical (cut-off) value
Input parameters

- **Significance level $\sigma$:** determines if the null hypothesis is wrongly rejected $\sigma$ percent of times, if it is in fact true
  - default value $\sigma = 0.05$
- $\sigma$ defines sensitivity of the test:
  - smaller $\sigma$ implies larger **critical value** (larger tolerance)
- **tail:** specifies whether the K-S performs two sided test (default) or tests from one or other side of the candidate distribution
Output parameters

- **Test statistic** $k$ is the maximum difference over all data points:
  
  $$k = \max_{1 \leq i \leq N} \left| F(Y_i) - \frac{i}{N} \right|$$

  where $F$ is the CDF of the assumed distribution.

- The null hypothesis is accepted if the value of the test statistic is smaller than the critical value.

- **p-value** is probability level when the difference between distributions (test statistics) becomes significant:
  
  - if $p-value \leq \sigma$: test rejects the null hypothesis.
  
- If test returns critical value $= \text{NaN}$, the decision to accept or reject null hypothesis is based only on p-value.
Best-fitting distributions: CDF

![Graph showing cumulative distribution functions (CDFs) for traffic data, Lognormal model, Exponential model, Gamma model, and Weibull model. The x-axis represents call holding time (s) ranging from 0 to 40, and the y-axis represents cumulative distribution ranging from 0 to 1. The graph compares the actual traffic data with the fitted models, highlighting the best-fitting distributions.](image-url)
Inter-arrival time: complementary CDF
### K-S test: call inter-arrival times 2001

**Significance level \( \sigma = 0.1 \)**

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameter</th>
<th>02.11.2001, 20:00–21:00</th>
<th>02.11.2001, 16:00–17:00</th>
<th>02.11.2001, 15:00–16:00</th>
<th>01.11.2001, 19:00–20:00</th>
<th>01.11.2001, 00:00–01:00</th>
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</thead>
<tbody>
<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<td>0.0001</td>
<td>0.5416</td>
<td>0.0122</td>
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<td>0.0369</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>p</td>
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<td>0.0206</td>
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<td>gamma</td>
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</table>

**Significance level \( \sigma \)**

<table>
<thead>
<tr>
<th>02.11.2001, 16:00–17:00: cv</th>
<th>0.01</th>
<th>0.04</th>
<th>0.05</th>
<th>0.08</th>
<th>0.09</th>
<th>0.1</th>
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<td>0.0275</td>
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<td>0.0230</td>
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<td>0.0211</td>
<td>0.0207</td>
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<tr>
<td>01.11.2001, 00:00–01:00: cv</td>
<td>0.0267</td>
<td>0.0229</td>
<td>0.0223</td>
<td>0.0208</td>
<td>0.0204</td>
<td>0.0201</td>
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</tbody>
</table>
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Traffic data

- Records of network events:
  - established, queued, and dropped calls in the Vancouver cell
- Traffic data span periods during:

<table>
<thead>
<tr>
<th>Trace (dataset)</th>
<th>Time span</th>
<th>No. of established calls</th>
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</thead>
<tbody>
<tr>
<td>2001</td>
<td>November 1–2, 2001</td>
<td>110,348</td>
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<tr>
<td>2002</td>
<td>March 1–7, 2002</td>
<td>370,510</td>
</tr>
</tbody>
</table>
**Hourly traces**

- Call holding and call inter-arrival times from the *five busiest hours* in each dataset (2001, 2002, and 2003)

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th></th>
<th>2002</th>
<th></th>
<th>2003</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Day/hour</td>
<td>No.</td>
<td>Day/hour</td>
<td>No.</td>
<td>Day/hour</td>
<td>No.</td>
<td></td>
</tr>
<tr>
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<td>3,718</td>
<td>01.03.2002 04:00–05:00</td>
<td>4,436</td>
<td>26.03.2003 22:00–23:00</td>
<td>4,919</td>
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<tr>
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<td>3,707</td>
<td>01.03.2002 22:00–23:00</td>
<td>4,314</td>
<td>25.03.2003 23:00–24:00</td>
<td>4,249</td>
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<td>3,492</td>
<td>01.03.2002 23:00–24:00</td>
<td>4,179</td>
<td>26.03.2003 23:00–24:00</td>
<td>4,222</td>
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<td>01.11.2001 19:00–20:00</td>
<td>3,312</td>
<td>01.03.2002 00:00–01:00</td>
<td>3,971</td>
<td>29.03.2003 02:00–03:00</td>
<td>4,150</td>
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<tr>
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<td>3,227</td>
<td>02.03.2002 00:00–01:00</td>
<td>3,939</td>
<td>29.03.2003 01:00–02:00</td>
<td>4,097</td>
<td></td>
</tr>
</tbody>
</table>
Example: March 26, 2003

![Call holding times (s) vs Time (hh:mm:ss)](chart)

- **Call holding times (s)**
- **Time (hh:mm:ss)**

*call inter-arrival time*
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Statistical distributions

- Fourteen candidate distributions:
  - exponential, Weibull, gamma, normal, lognormal, logistic, log-logistic, Nakagami, Rayleigh, Rician, t-location scale, Birnbaum-Saunders, extreme value, inverse Gaussian

- Parameters of the distributions: calculated by performing maximum likelihood estimation

- Best fitting distributions are determined by:
  - visual inspection of the distribution of the trace and the candidate distributions
  - K-S test on potential candidates
Maximum Likelihood Estimation (MLE)

- Introduced by R. A. Fisher in 1920s
- The most popular method for parameter estimation
- Goal: to find the distribution parameters that make the given distribution that follow the most closely underlying data set
- Conduct an experiment and obtain $N$ independent observations
- $\theta_1, \theta_2, \ldots, \theta_k$ are $k$ unknown constant parameters which

$$L(x_1, x_2, \ldots, x_N | \theta_1, \theta_2, \ldots, \theta_k) = L = \prod_{i=1}^{N} f(x_i; \theta_1, \theta_2, \ldots, \theta_k)$$

$$i = 1, 2, \ldots, N$$
Maximum likelihood estimation

Likelihood Function Surface

- 100.0%
- 90.0%
- 80.0%
- 70.0%
- 60.0%
- 50.0%
- 40.0%
- 30.0%
- 20.0%
- 10.0%
- 0.0%
Call inter-arrival times: pdf candidates

![Graph showing probability density functions for different models]

- Traffic data
- Exponential model
- Lognormal model
- Weibull model
- Gamma model
- Rayleigh model
- Normal model
K-S test results: 2003

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameter</th>
<th>26.03.2003, 22:00–23:00</th>
<th>25.03.2003, 23:00–24:00</th>
<th>26.03.2003, 23:00–24:00</th>
<th>29.03.2003, 02:00–03:00</th>
<th>29.03.2003, 01:00–02:00</th>
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<tbody>
<tr>
<td>Exponential</td>
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<td>1</td>
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<td>Weibull</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td></td>
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<td>0.0133</td>
<td>0.0164</td>
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<td></td>
<td>p</td>
<td>1.015E-20</td>
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<td>4.851E-21</td>
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<td>0.0795</td>
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</table>
Call inter-arrival times, best-fitting distributions: cdf

![Graph showing cumulative distribution function (CDF) of call inter-arrival times with fitted models.]
Call inter-arrival time: autocorrelation

![Graph showing autocorrelation function with 99% and 95% confidence intervals.](image)
Logscale diagram, call inter-arrival times: 26.03.2003, 22:00–23:00

- LRD: $\alpha > 0$ (H>0.5)
- similar logscale diagrams for other traces
**Call inter-arrival times: estimates of H**

- Traces pass the test for time constancy of $\alpha$: estimates of $H$ are reliable

<table>
<thead>
<tr>
<th>Day/hour</th>
<th>2001 $H$</th>
<th>Day/hour</th>
<th>2002 $H$</th>
<th>Day/hour</th>
<th>2003 $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>02.11.2001 15:00–16:00</td>
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<td>01.03.2002 04:00–05:00</td>
<td>0.679</td>
<td>26.03.2003 22:00–23:00</td>
<td>0.788</td>
</tr>
<tr>
<td>01.11.2001 00:00–01:00</td>
<td>0.802</td>
<td>01.03.2002 22:00–23:00</td>
<td>0.757</td>
<td>25.03.2003 23:00–24:00</td>
<td>0.832</td>
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</table>
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Call holding time: pdf candidates

- Traffic data
- Lognormal model
- Gamma model
- Weibull model
- Exponential model
- Normal model
- Rayleigh model
Best-fitting distributions: cdf

Call holding time (s)

Traffic data
Lognormal model
Exponential model
Gamma model
Weibull model
K-S test results: 2003

- No distribution passes the test when the entire trace is tested (significance levels = 0.1 and 0.01)
- Lognormal distribution passes test (significance level = 0.01) for:
  - 5-6 sub-traces from 15 randomly chosen 1,000-sample sub-traces
  - passes the test for almost all 500-sample sub-traces
- Test rejects null hypothesis when the sub-traces are compared with candidate distributions:
  - exponential
  - Weibull
  - gamma
Call holding time: autocorrelation

![Graph showing autocorrelation function with 99% and 95% confidence intervals.](Image)
Logs-scale diagram, call holding times:
26.03.2003, 22:00–23:00

- Independence: $\alpha \approx 0 \ (H \approx 0.5)$
- Similar logscale diagrams for other traces
Call holding times: estimates of $H$

- All (except one) traces pass the test for constancy of $\alpha$
- only one unreliable estimate (*): consistent value

<table>
<thead>
<tr>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
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<td>Day/hour</td>
<td>$H$</td>
<td>Day/hour</td>
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<td>04:00–05:00</td>
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<td>00:00–01:00</td>
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<tr>
<td>02.11.2001</td>
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<td>01.03.2002</td>
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<td>23:00–24:00</td>
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<tr>
<td>20:00–21:00</td>
<td>00:00–01:00</td>
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</tbody>
</table>
Roadmap

- Introduction
- Statistical concepts and analysis tools
- Analysis of traffic data:
  - call inter-arrival times
  - call holding times
- Traffic modeling and characterization
- Conclusions and references
### Call inter-arrival and call holding times

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day/hour</td>
<td>Avg. (s)</td>
<td>Day/hour</td>
</tr>
<tr>
<td><strong>inter-arrival</strong></td>
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<td></td>
</tr>
<tr>
<td>02.11.2001</td>
<td>02:00–03:00</td>
<td>1.08</td>
<td>25.03.2003</td>
</tr>
<tr>
<td>15:00–16:00</td>
<td>02:00–03:00</td>
<td>1.08</td>
<td>25.03.2003</td>
</tr>
<tr>
<td><strong>holding</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>02.11.2001</td>
<td>02:00–03:00</td>
<td>3.78</td>
<td>25.03.2003</td>
</tr>
<tr>
<td>15:00–16:00</td>
<td>02:00–03:00</td>
<td>3.78</td>
<td>25.03.2003</td>
</tr>
<tr>
<td><strong>inter-arrival</strong></td>
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<td></td>
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<tr>
<td>01.11.2001</td>
<td>02:00–03:00</td>
<td>1.09</td>
<td>25.03.2003</td>
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<tr>
<td>00:00–01:00</td>
<td>02:00–03:00</td>
<td>1.09</td>
<td>25.03.2003</td>
</tr>
<tr>
<td><strong>holding</strong></td>
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<td>01.11.2001</td>
<td>02:00–03:00</td>
<td>3.97</td>
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<td>00:00–01:00</td>
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<td>02.11.2001</td>
<td>02:00–03:00</td>
<td>1.12</td>
<td>29.03.2003</td>
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<td>02:00–03:00</td>
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<td>02:00–03:00</td>
<td>3.84</td>
<td>29.03.2003</td>
</tr>
</tbody>
</table>

Avg. call inter-arrival times: 1.08 s (2001), 0.86 s (2002), 0.84 s (2003)
## Distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Expression</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponential</td>
<td>( f(x) = \frac{e^{-x/\mu}}{\mu} )</td>
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<tr>
<td>Weibull</td>
<td>( f(x) = ba^{-b}x^{b-1}e^{-(x/a)^b}I_{(0,\infty)}(x) )</td>
<td>( I_{(0,\infty)}(x) ): incomplete beta function</td>
</tr>
<tr>
<td>gamma</td>
<td>( f(x) = \frac{x^{a-1}e^{-(x/b)}}{b^a\Gamma(a)} )</td>
<td>( \Gamma(a) ): gamma function</td>
</tr>
<tr>
<td>lognormal</td>
<td>( f(x) = \frac{e^{-(\ln x - \mu)^2/2\sigma^2}}{x\sigma\sqrt{2\pi}} )</td>
<td></td>
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</tbody>
</table>
## Best fitting distributions

<table>
<thead>
<tr>
<th>Busy hour</th>
<th>Weibull</th>
<th>Gamma</th>
<th>Lognormal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>μ</td>
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<td>0.8579</td>
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</tbody>
</table>

- **Distribution**
  - Call inter-arrival times
  - Call holding times
Estimates of $H$

- Call inter-arrival times: $H \approx 0.7–0.8$
- Call holding times: $H \approx 0.5$
Conclusions

- We analyzed voice traffic from a public safety wireless network in Vancouver, BC
  - call inter-arrival and call holding times during five busy hours from each year (2001, 2002, 2003)
- Statistical distribution and the autocorrelation function of the traffic traces:
  - Kolmogorov-Smirnov goodness-of-fit test
  - autocorrelation functions
  - wavelet-based estimation of the Hurst parameter
Conclusions

- Call inter-arrival times:
  - best fit: Weibull and gamma distributions
  - long-range dependent: $H \approx 0.7–0.8$

- Call holding times:
  - best fit: lognormal distribution
  - uncorrelated
References