Transmission Latency and Communication over Time-Varying Channels

Dejan V. Djonin
NSERC PostDoctoral Fellow
Dept. of Electrical and Computer Engineering
University of British Columbia

E-mail: ddjonin@ece.ubc.ca
www.ece.ubc.ca/~ddjonin
My Brief Background...

(Sep 2003 - ) Postdoctoral Teaching Fellow, University of British Columbia, Department of Electrical and Computer Engineering

(May 2000- Jun 2003) PhD Studies, University of Victoria, Department of Electrical Engineering

(1996 - 1999), Faculty of Electrical Engineering in Belgrade, M.Sc. studies, M.Sc. Thesis Title: "Application of Non-linear One-dimensional Maps in Generation of Error-Correction Block Codes"
Problem Formulation and Introduction

Cross-layer optimization for transmission delay minimization

Analyzed Transmission Models

(1) Real-Time Traffic (e.g. videophone over wireless)

(2) Constant-Rate playback over time-varying channels

Channel Model: Finite State Markov Model

Mathematical Framework: Stochastic Control and MDP’s (needed for (1))

Solution Techniques

Selected Extensions

Power and rate resource allocation for OFDM schemes

Resource allocation for imperfectly known channel models

Resource allocation for imperfect or delayed channel state information

Multiuser Systems

(2) Constant-Rate playback over time-varying channels
Modern and future wireless networks will support different services with a wide range of quality of service requirements such as delay, rate, BER.

Consideration of Transmission Latency is of crucial interest for some applications (real-time high quality audio, video transmission).

However, time-varying nature of a wireless channel poses a challenging task to delivering a wide variety of services.

- the effect is similar to congestion in wireline networks
- the need for transmission buffer
- transmitted signals are delayed

Does these methods only apply to wireless channels?

The solution is through adaptation of transmission parameters based on the current state and the statistical model of the channel and supported traffic.

Essentially a Cross-layer optimization approach.
A Simple Illustration

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(2) Constant-Rate playback over time-varying channels
**Data Link (Layer 2)** At this layer, data packets are encoded and decoded into **bits**. It furnishes transmission protocol knowledge and management and handles errors in the physical layer, flow control and frame synchronization. The data link layer is divided into two **sublayers**: The **Media Access Control** (MAC) layer and the **Logical Link Control** (LLC) layer. The MAC sublayer controls how a computer on the network gains access to the data and permission to transmit it. The LLC layer controls frame synchronization, flow control and error checking.

**Physical (Layer 1)** This layer conveys the **bit** stream - electrical impulse, light or radio signal -- through the network at the electrical and mechanical level. It provides the **hardware** and software means of sending and receiving data on a carrier.
The conventional approach: Each layer considered separately

Why do we need cross-layer optimization (CLO)?

- Advantages of CLO
- Disadvantages CLO
No Cross-layer optimization

Example: Physical Layer Power allocation (Power Control)

\[
P_{av} = \min_{R(h)} \mathbb{E}[P(R(h), h)] \quad \text{Power Minimization under Average Rate Constraint}
\]

\[
s.t. \quad \mathbb{E}[R(h)] \geq \bar{R} \quad \text{Delay-limited case}
\]

\[
\begin{align*}
P_{av} &= \min_{R(h)} \mathbb{E}[P(R(h), h)] \\
&\quad \text{Power Minimization under Hard Rate Constraint} \\
&\quad \text{Delay-unlimited case (Waterfilling)}
\end{align*}
\]

\[
s.t. \quad R(h) \geq \bar{R}
\]

\[
h \text{ is the flat fading channel state}
\]

Example: Data Layer

MAC protocols such as FIFO

ARQ protocols

Rate control algorithms independent channel condition
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Solution Techniques and Analytical Results

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(2) Constant-Rate playback over time-varying channels
Consider a single user with a finite transmission buffer that is communicating over a fading channel:

Let $A_n$ denote the number of packets arriving at the buffer between time slots $n-1$ and $n$. It is assumed that $\{A_n\}$ forms an ergodic Markov chain.

Transmission adaptation parameters can include power, error-correction or source coding rate.

At the beginning of the $n$-th time slot, the scheduler takes $U_n$ packets from the buffer and maps these into a rate $cU_n$ codeword.
Consider a transmission from an “unlimited” memory bank to a single user equipped with receiver buffer over a time-varying channel.

- At the beginning of the $n$-th time slot, the scheduler takes $U_n$ packets from the buffer and maps these into a rate $cU_n$ codeword.

- In this case the data is first buffered for a fixed amount of time in the Receiver Buffer and then it is read out at a constant rate.

- The goal is to have as few as possible receiver buffer outages.
For example, a slowly varying flat Fading Rayleigh channel can be represented as a Finite State Markov Chain (FSMC) as shown in figure:

Channel can also be modeled as an Auto Regressive (AR) model.
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Solution Techniques and Structural Results

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(2) Constant-Rate playback over time-varying channels
A MDP is a model for sequential decision making when outcomes are uncertain.

A MDP is described by the following ingredients:

- A set of decision epochs or time slots, \( T = \{1, 2, \ldots, m\} \)
- A set of states, \( \Sigma = \{s_1, s_2, \ldots, s_Q\} \)
- A set of actions, \( U = \{u_1, u_2, \ldots, u_U\} \)
- A set of state and action dependent transition probabilities, \( p(s_j|s_i, u_i) \)
- A set of state and action dependent immediate costs, \( g(s_i, u_i) \)

A decision rule \( \mu_n \) prescribes a procedure for action selection in each state at a specified time slot:

\[
\mu_n : S \leftrightarrow U_S
\]

The decision rule to be used at all time slot is called policy \( \pi = \{\mu_1, \mu_2, \ldots, \mu_m\} \).
Markov Decision Processes (MDP)

Markov Chain: Example

\[
\begin{array}{ccc}
S_1 & \xrightarrow[p(S_2|S_1)]{p(S_2|S_1)} & S_2 \\
p(S_1|S_1) & & \quad & \quad & p(S_2|S_2) \\
p(S_1|S_2) & & \quad & \quad & p(S_2|S_2)
\end{array}
\]

Markov Decision Processes: Example for state $S_1$

\[
\begin{array}{ccc}
S_1 & \xrightarrow[p(S_2|S_1,U_1)]{p(S_2|S_1,U_1)} & S_2 \\
p(S_1|S_1,U_1) & & \quad & \quad & p(S_2|S_1,U_2) \\
p(S_1|S_1,U_2) & & \quad & \quad & p(S_2|S_1,U_2)
\end{array}
\]

Action $U_1$

Action $U_2$
The Optimization Criterion

The average cost optimization criteria for Markov Decision Processes

\[ C^* = \inf_{\pi} \mathbb{E} \left[ \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} g(s_n, u_n) \right] \]

This is just an optimization problem over a set of feasible policies \( \pi \)

A relatively simple solution is possible using dynamic programming
Constrained MDPs

What happens if in addition to the immediate costs, \( g(s,u) \), there is another cost \( d(s,u) \) that corresponds to a constraint? I.e. optimization problem is:

\[
C^* = \inf_{\pi} \mathbb{E} \left[ \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} g(s_n, u_n) \right]
\]

s.t. \( \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} d(s_n, u_n) \leq D \)

The answer can be found in the theory of Constrained Markov Decision Processes (CMDP). CMDP can be expressed as equivalent unconstrained MDP using Lagrangian Approach:

\[
C^*(D) = \min_{\pi} \sup_{\lambda > 0} \mathbb{E} \left[ \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left( g(s_n, u_n) + \lambda d(s_n, u_n) \right) \right] - \lambda D
\]

Note that policies do not have to be deterministic in CMDPs. In general optimal policies for CMDPs are randomized.
(1) Real-Time Traffic

How to formulate state space and costs in the real-time traffic model (1)

- State space include: Buffer + Incoming Traffic + Fading Channel
- Immediate cost $g(s,a)$ can be e.g. transmission power
- Constraint cost $d(s,a)$ can be related to buffer delay
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Optimal policy for a MDP model can be found using
- Relative Value Iteration
- Policy Iteration

The details of these algorithms can be found in

“Dynamic Programming and Optimal Control” vol. 1 and 2. by D. Bertsekas

Another advantage of Value Iteration algorithms is that several general structural results on the shape of optimal policies can be derived by just considering the general analytical form of immediate and constrained costs.
As fading rate ↑, the rate of decrease of average power ↑.
As the number of actions ↑, average power ↓.
Example: Physical Layer Power allocation (Power Control)

\[
P_{av} = \min_{R(h)} E[P(R(h), h)]
\]

\[
\text{s.t. } E[R(h)] \geq \bar{R}
\]

Power Minimization under Average Rate Constraint

Delay-limited case

\[
P_{av} = \min_{R(h)} E[P(R(h), h)]
\]

\[
\text{s.t. } R(h) \geq \bar{R}
\]

Power Minimization under Hard Rate Constraint

Delay-unlimited case (Waterfilling)
Sample Results (2)
**Theorem 2:** Let the Assumptions 1, 2, 3 hold. Let the instantaneous Lagrangian cost \( c([h, b, f], a; \lambda) \) (8) be submodular, jointly convex function of \((b, a)\) and increasing in buffer state \(b\). Let \( \Psi(a) \) defined in (10) be concave increasing function of \(a\). Then for cost constraint \( \bar{D} > 0 \), the optimal randomized policy \( \pi^*([h, b, f]) \) is a mixed policy of two pure policies \( \pi^1([h, b, f]) \) and \( \pi^2([h, b, f]) \) that are non-decreasing functions of buffer state \(b\) (see Definition 4) such that \( b < L - G(\bar{F} - 1)^3 \). Furthermore, there exists only one state \( s \in S \) such that \( \pi^1(s) \neq \pi^2(s) \).

**Extracted from the paper:**


also to be presented as an invited paper at the Control Decision Conference, Seville 2005.
**Theorem 3**: Let the Assumptions 1, 2, 3 hold.

(1) For any buffer length \( L \in \mathcal{N}_0 \), the optimal average transmission scheduling cost \( C^*(\tilde{D}) \) is a piece-wise linear non-increasing function of \( \tilde{D} \in \mathcal{D} \) that can be expressed as

\[
C^*(\tilde{D}) = \max_{\lambda \in \Lambda} \left( D(\pi^*_\lambda) - \tilde{D} \right) \lambda + C(\pi^*_\lambda)
\]  

(27)

where \( \Lambda \) defined as

\[
\Lambda = \left\{ \lambda_1, \lambda_2, \ldots, \lambda_Q \right\} = \left\{ \arg \sup_{\lambda \geq 0} \min_{\pi \in \Phi_D} \left( J(\pi, \lambda) - \lambda \tilde{D} \right) | \tilde{D} > 0 \right\}
\]  

(28)

is a finite set.

(2) In addition to the above stated assumptions, suppose that \( c([h, b, f], a; \lambda) \) is jointly convex in \( b, a \) and \( \lambda \) and let \( \Psi(a) \) be concave increasing in \( a \). Then \( C^*(\tilde{D}) \) is piece-wise linear convex non-increasing function of \( \tilde{D} \).

\[ \Box \]
**Corollary 6:** Let the Assumptions 1, 2, 3 hold. Let $\mathbf{H}_P(h)$ and $\mathbf{H}_Q(h)$ be second-order stochastically increasing in $h$ and let $\mathbf{H}_P(h)$ be second-order stochastically dominating $\mathbf{H}_Q(h)$ for any $h$. If $c([h, b, f], a)$ is non-increasing and convex function of $h$ for any $b \in \mathcal{B}$, $f \in \mathcal{F}$ and $\lambda \in \mathbb{R}^+$ then

$$C_P^* \leq C_Q^*$$

(41)

for any feasible average buffer cost constraint $\tilde{D} \in \mathcal{D}_P \cap \mathcal{D}_Q$.

**Example:** Channels with less scattering can require less average transmission cost (e.g. power) for the same delay.
Why do we need these structural results?

Structural Results on Optimal Policies can give us some general insights on the shape and qualitative behaviour of optimal costs and policies.

Structural Results can also lead to more efficient algorithms for finding optimal policies. Example:

|                | $\lambda = 0.01, |S| = 500$ | $\lambda = 100, |S| = 500$ | $\lambda = 0.01, |S| = 100$ | $\lambda = 100, |S| = 100$ |
|----------------|-----------------|-----------------|-----------------|-----------------|
| SPI Iterations | 8               | 2               | 8               | 2               |
| NSPI Iterations | 51             | 3               | 22              | 3               |
| Computational Saving | 6.09      | 1.70            | 3.00            | 4.00            |

**TABLE I**

Comparison of Computational Iterations of Structured and Non-Structured Policy Iteration.
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2. Resource allocation for imperfectionlly known channel models
   (Q – learning)

(2) Constant-Rate playback over time-varying channels
These results apply in general for any delay-constrained multi-channel transmission problems.

Major obstacle is the dimensionality of the state and action space.
Fig. 3. Optimal delay and power tradeoffs with different number of carriers (Doppler frequency, $f_d = 100\text{Hz}$).
This a challenging problem as the policy has to be “learned” on-line as the actions are being applied and observations on the incurred cost are collected.

The appropriate framework for the solution of this problem is to consider Q-learning, which is a version of stochastic approximation algorithm.

For details on Q-algorithm and related topics have a look at:

D. Bertsekas and J. Tsitsiklis, “Neuro-Dynamic Programming”
Simulation Settings:

- # Channel States = 6
- # Buffer States = 15
- # Actions = 6

Structured $\mu = 0.3, \mu_2 = 0.02$

Non-Structured $\mu = 0.3$
Consider a transmission from an “unlimited” memory bank to a single user equipped with receiver buffer over a time-varying channel.

The problem can again be formulated as the CMDP with:

- $g(s,a)$ being power cost
- $d(s,a)$ being receiver buffer outage probability
- action representing the transmission rate.
An alternative formulation that does not involve MDPs
This approach falls somewhere in between delay-constrained and no-delay constrained resource allocation problem

\[ P_{av} = \min_{R(h)} E[P(R(h), h)] \]

\[ s.t. \quad E[R(h)] \geq \bar{R} \]
\[ E[(R(h) - \bar{R})^2] \leq \sigma_R^2 \]

Advantages:

- Explicit analytical expression of the rate allocation can be derived
- By changing the parameter \( \sigma_R^2 \) it is possible to adjust the receiver buffer outage probability
Thank You for Your Attention !