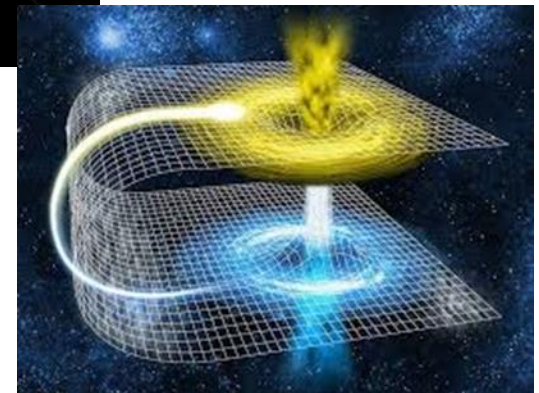
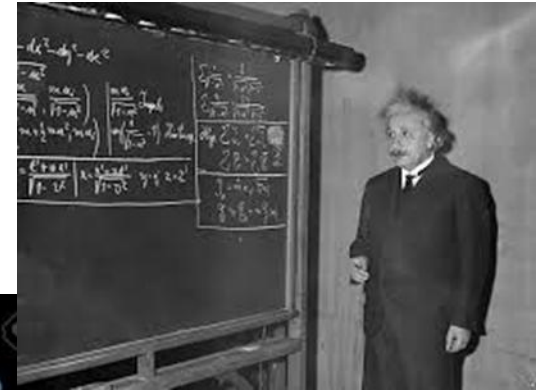
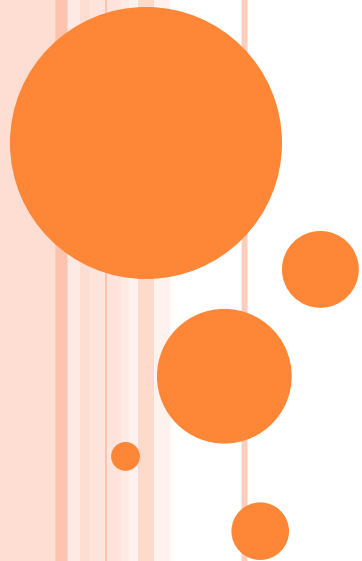


**THE FUNDAMENTAL BUILDING BLOCKS OF A
“THEORY OF EVERYTHING” UNDER A UNIFIED THEORY OF ANALYTICAL
INTEGRATION**



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The basic motivation for this presentation is to provide a more complete generalization of the physical sciences towards a more unified approach to analytical integration.

This would include the Navier-Stokes equations for the case of an incompressible flow with heat transfer and variable viscosity as well as for problems related to linear elasticity and Einstein field theory.



GENERAL OVERVIEW FOR TODAY'S PRESENTATION



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- I will begin with some newly discovered facts about mathematical equations and introduce a very important new mathematical principle related to integral Calculus.
- Describe a universal differential form representation for a class of mathematical equations that would include only algebraic and elementary functions excluding all integrals and difference equations.
- Explain how such a universal representation of these types of functions has uncovered a *unified approach to analytical integration*.
- Introduce the concept of a *numerically controlled system of analytics* (NCSA) table for Physics and Engineering Science as a direct application of this unified approach to analytical integration.



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- Provide a clear demonstration on how such a new mathematical approach can lead to some very fundamental applications to engineering, physics and other sciences as well.
- We will use the Navier-Stokes equations and problems related to linear elasticity as well as the Einstein Field theory to illustrate our new method.



The differential representation of a mathematical equation



Let:

$$f_k(z_i, x_j) = 0 \quad 1 \leq k \leq \infty$$

where z_i for $1 \leq i \leq m$ are the dependent variables, x_j for $1 \leq j \leq n$ are the independent variables and f_k is built up by recursive compositions of algebraic and elementary functions.

Basic functions from Calculus such as the algebraic and elementary functions have their own unique differential representation.

Here are a few well known examples:

$$y = x^n \quad xdy - nydx = 0$$

$$y = e^x \quad dy - ydx = 0$$

$$y = \ln(x) \quad xdy - dx = 0$$

$$y = \text{Tan}(x) \quad dy - (1 + y^2)dx = 0$$

So we ask the question:

“Does a mathematical equation in f_k also have its own unique differential representation ? ”



Example 1:

$$f(x, y) = 0 = a_1 y + a_2 e^{a_3 x} \sin(a_4 x)$$

For simplicity, we use the half angle Tangent formula for expressing the Sine and Cosine function as rational combinations of the Tangent function:

$$\sin(u) = \frac{2 \tan(u/2)}{1 + \tan^2(u/2)} \quad \text{and} \quad \cos(u) = \frac{1 - \tan^2(u/2)}{1 + \tan^2(u/2)}$$

We can introduce new indeterminate functions and their differentials as follow:

$$\begin{aligned} W_1 &= x & dW_1 &= dx \\ W_2 &= y & dW_2 &= dy \\ W_3 &= e^{a_3 x} & dW_3 &= a_3 W_3 dW_1 \\ W_4 &= \tan\left(\frac{a_4 x}{2}\right) & 2dW_4 &= a_4 (1 + W_4^2) dW_1 \end{aligned}$$

so that after multiplying by the least common denominator, our differential transform consists of two basic parts:

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4) = 0 = a_1 W_2 (1 + W_4^2) + 2a_2 W_3 W_4$$

(2). Secondary Expansion:

$$\begin{aligned} dx &= dW_1 \\ dy &= dW_2 \end{aligned}$$

$$a_3 W_3 dx + 0 \cdot dy = dW_3$$

$$a_4 (1 + W_4^2) dx + 0 \cdot dy = 2dW_4$$



Example 2:

$$f(x,y) = 0 = \ln(1 + \sqrt[3]{x+1}) - \sqrt[6]{y+1} - 1$$

$$W_1 = x$$

$$dW_1 = dx$$

$$W_2 = y$$

$$dW_2 = dy$$

$$W_3^3 = x + 1 = W_1 + 1$$

$$3W_3^2 dW_3 = dW_1$$

$$W_4 = \ln(1 + \sqrt[3]{x+1}) = \ln(1 + W_3)$$

$$(1 + W_3)dW_4 = dW_3$$

$$W_5^6 = y + 1 = W_2 + 1$$

$$6W_5^5 dW_5 = dW_2$$

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5) = 0 = W_4 - W_5 - 1$$

(2). Secondary Expansion:

$$dx = dW_1$$

$$dy = dW_2$$

$$dx + 0 \cdot dy = 3W_3^2 dW_3$$

$$dx + 0 \cdot dy = 3W_3^2(1 + W_3)dW_4$$

$$0 \cdot dx + dy = 6W_5^5 dW_5$$



Example 3:

$$f(z, x_1, x_2, x_3) = 0 = 5x_2x_3\sin(zx_1x_2) + (x_1 + x_2)\cos(z + 3x_2 + 2x_3) + 3$$

$$W_1 = z$$

$$W_2 = x_1$$

$$W_3 = x_2$$

$$W_4 = x_3$$

$$W_5 = \tan(zx_1x_2/2)$$

$$W_6 = \tan\left(\frac{z + 3x_2 + 2x_3}{2}\right)$$

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5, W_6) = 0 = 5W_3W_4 \left[\frac{2W_5}{1 + W_5^2} \right] + (W_2 + W_3) \left[\frac{1 - W_6^2}{1 + W_6^2} \right] + 3$$

(2). Secondary Expansion:

$$dz + 0 \cdot dx_1 + 0 \cdot dx_2 + 0 \cdot dx_3 = dW_1$$

$$0 \cdot dz + dx_1 + 0 \cdot dx_2 + 0 \cdot dx_3 = dW_2$$

$$0 \cdot dz + 0 \cdot dx_1 + dx_2 + 0 \cdot dx_3 = dW_3$$

$$0 \cdot dz + 0 \cdot dx_1 + 0 \cdot dx_2 + dx_3 = dW_4$$

$$(1 + W_5^2)W_2W_3dz + (1 + W_5^2)W_1W_3dx_1 + (1 + W_5^2)W_1W_2dx_2 + 0 \cdot dx_3 = 2dW_5$$

$$(1 + W_6^2)dz + 0 \cdot dx_1 + 3(1 + W_6^2)dx_2 + 2(1 + W_6^2)dx_3 = 2dW_6$$



UNIVERSAL DIFFERENTIAL REPRESENTATION FOR

$$f_k(z_i, x_j) = 0 \quad (1 \leq k \leq \infty)$$

where z_i for $1 \leq i \leq m$ are the dependent variables, x_j for $1 \leq j \leq n$ are the independent variables and f_k is built up by recursive compositions of algebraic and elementary functions.



(1). Primary Expansion:

$$F_i(W_1, W_2, \dots, W_{p+q}) = 0 = \sum_t^r a_{i,t} \left(\prod_j^{p+q} W_j^{E_{i,kj}} \right) \quad (1 \leq i \leq k)$$

(2). Secondary Expansion:

$$\begin{aligned} dz_i &= dW_i & (1 \leq i \leq m) \\ dx_j &= dW_{m+j} & (1 \leq j \leq n) \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m N_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n N_{i(m+n+1)-n-1+t} dx_t &= \\ &= N_{i(m+n+1)} dW_j & (1 \leq i \leq p+q-m-n) \\ & & (m+n+1 \leq j \leq p+q) \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m T_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n T_{i(m+n+1)-n-1+t} dx_t &= \\ &= T_{i(m+n+1)} dW_j & (1 \leq i \leq q) \quad (p \leq j \leq p+q) \end{aligned}$$

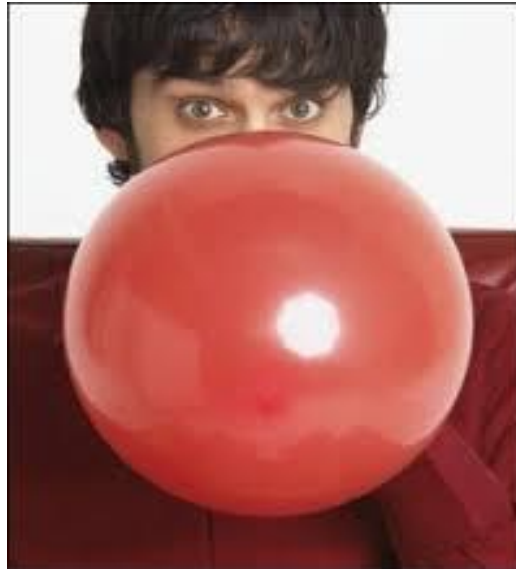
where it is very important to note that when using this expansion for solving DEs and systems of DEs, “q” is the total number of auxiliary variables necessary for defining all functions that can be present in the DE or system of DEs we are attempting to solve for. This will be described in more detail later on.



- We can use this universal differential expansion representation for solving any type of DEs by assigning unknown coefficients to solve for in each of the multivariate polynomials present.
- The last part of the Secondary Expansion (T_u) are multivariate polynomials with known coefficients values that are reserved for representing all basis functions present in a DE or a system of DEs.



IN COMPLETE EXPANDED FORM



(1). Primary Expansion:

$$F_1 = 0 = a_{1,1}W_1^{m_{11}}W_2^{m_{12}} \dots W_{p+q}^{m_{1,p+q}} + a_{1,2}W_1^{m_{1,p+q+1}}W_2^{m_{1,p+q+2}} \dots W_{p+q}^{m_{1,2(p+q)}} \\ + \dots + a_{1,r}W_1^{m_{1,(p+q)(r-1)+1}}W_2^{m_{1,(p+q)(r-1)+2}} \dots W_{p+q}^{m_{1,r(p+q)}}$$

$$F_2 = 0 = a_{2,1}W_1^{m_{21}}W_2^{m_{22}} \dots W_{p+q}^{m_{2,p+q}} + a_{2,2}W_1^{m_{2,p+q+1}}W_2^{m_{2,p+q+2}} \dots W_{p+q}^{m_{2,2(p+q)}} \\ + \dots + a_{2,r}W_1^{m_{2,(p+q)(r-1)+1}}W_2^{m_{2,(p+q)(r-1)+2}} \dots W_{p+q}^{m_{2,r(p+q)}}$$

. . .
. . .
. . .

$$F_k = 0 = a_{k,1}W_1^{m_{k1}}W_2^{m_{k2}} \dots W_{p+q}^{m_{k,p+q}} + a_{k,2}W_1^{m_{k,p+q+1}}W_2^{m_{k,p+q+2}} \dots W_{p+q}^{m_{k,2(p+q)}} \\ + \dots + a_{k,r}W_1^{m_{k,(p+q)(r-1)+1}}W_2^{m_{k,(p+q)(r-1)+2}} \dots W_{p+q}^{m_{k,r(p+q)}}$$



(2). Secondary Expansion:

$$dz_i = dW_i \quad (1 \leq i \leq m)$$

$$dx_i = dW_{m+i} \quad (1 \leq i \leq n)$$

$$[N_1 dz_1 + N_2 dz_2 + \dots + N_m dz_m] + [N_{m+1} dx_1 + N_{m+2} dx_2 + \dots + \dots + N_{m+n} dx_n] = N_{m+n+1} dW_{m+n+1}$$

$$[N_{m+n+2} dz_1 + N_{m+n+3} dz_2 + \dots + N_{2m+n+1} dz_m] + [N_{2m+n+2} dx_1 + \dots + N_{2(m+n+1)-1} dx_n] = N_{2(m+n+1)} dW_{m+n+2}$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$[N_{(p+q-1)(m+n+1)+1} dz_1 + N_{(p+q-1)(m+n+1)+2} dz_2 + \dots + N_{(p+q-1)(m+n+1)+m} dz_m] +$$

$$+ [N_{(p+q-1)(m+n+1)+m+1} dx_1 + N_{(p+q-1)(m+n+1)+m+2} dx_2 + \dots + N_{(p+q)(m+n+1)-1} dx_n]$$

$$= N_{(p+q)(m+n+1)} dW_{p+q}$$

where it is very important to note that when using this expansion for solving DEs and systems of DEs, "q" is the total number of auxiliary variables necessary for defining all functions that can be present in the DE or system of DEs we are attempting to solve for.



- Going from original form to complete differential form is defined as taking the ***Multivariate Polynomial Transform*** of an equation.
- Going back to original form is defined as taking the ***Inverse Multivariate Polynomial Transform*** of an equation.
- The Inverse Multivariate Polynomial Transform always involves a process of *exact* integration that is applied only on the Secondary Expansion.
- This integration process becomes *exact* only if there are *exact* differentials present in the Secondary Expansion.



TAKING THE INVERSE MULTIVARIATE POLYNOMIAL TRANSFORM

“SIMPLE TWO DIMENSIONAL CASE”



(1). Primary Expansion:

$$F_i(W_1, W_2, \dots, W_{p+q}) = 0 = \sum_t^r a_{i,t} \left(\prod_j^{p+q} W_j^{E_{i,kj}} \right) \quad (1 \leq i \leq k)$$

(2). Secondary Expansion:

$$\begin{aligned} dz_i &= dW_i & (1 \leq i \leq m) \\ dx_j &= dW_{m+j} & (1 \leq j \leq n) \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m N_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n N_{i(m+n+1)-n-1+t} dx_t &= \\ &= N_{i(m+n+1)} dW_j & (1 \leq i \leq p+q-m-n) \\ & & (m+n+1 \leq j \leq p+q) \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m T_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n T_{i(m+n+1)-n-1+t} dx_t &= \\ &= T_{i(m+n+1)} dW_j & (1 \leq i \leq q) \quad (p \leq j \leq p+q) \end{aligned}$$

where it is very important to note that “q” is the total number of auxiliary variables necessary just for defining all functions that can be present in the DE or system of DEs we are attempting to solved for.



Consider the simplest *two* dimensional case ($k = m = n = 1$) and replace “z” with “y” for the dependent variable:

(1). Primary Expansion:

$$F(W_j) = 0 \quad (1 \leq j \leq p)$$

(2). Secondary Differential Expansion:

$$dx = dW_1$$

$$dy = dW_2$$

$$N_{3i-2}dx + N_{3i-1}dy = N_{3i}dW_j \quad (1 \leq i \leq p-2) \quad (3 \leq j \leq p)$$

General form for the above differential:

$$M(x, y)dx + N(x, y)dy = P(W_j)dW_j$$

From elementary Calculus, the general condition for exactness is:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$



Integral solution using Euler's method:

$$\int Mdx + \int \left(N - \frac{\partial}{\partial y} \int Mdx \right) dy = C$$

or equivalently:

$$\int Ndy + \int \left(M - \frac{\partial}{\partial x} \int Ndy \right) dx = C$$

Integral solution only for the *right hand side* of the general differential form:

$$\int P(W_j)dW_j$$

where $P(W_j)$ will always be defined as a multivariate polynomial.



(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4) = 0 = W_4 + 2W_2$$

(2). Secondary Differential Expansion:

$$\begin{aligned} dx + 0 \cdot dy &= dW_1 \\ 0 \cdot dx + dy &= dW_2 \\ -2W_1 dx + 0 \cdot dy &= W_3 dW_3 \\ 2W_1 dx - W_3 dy &= W_3(W_2 + W_3)dW_4 \end{aligned}$$

We first begin by defining the expression for " $W_1(x) = x$ " and " $W_2(y) = y$ ".

Next, by integration we define the expression for " $W_3(x)$ " as:

$$W_3(x) = \pm \sqrt{C_3 - 2x^2}$$

The differential for " $W_4(x)$ " may now be rewritten as:

$$\frac{2x dx}{W_3(y + W_3)} - \frac{dy}{y + W_3} = dW_4$$

so that:

$$M(x, y) = \frac{2x}{W_3(y + W_3)} \quad \text{and} \quad N(x, y) = \frac{-1}{(y + W_3)}$$



$$M(x, y) = \frac{2x}{W_3(y + W_3)}$$

Since $W_3 = W_3(x)$:

$$\frac{\partial M}{\partial y} = \frac{-2x}{W_3(y + W_3)^2}$$

$$N(x, y) = \frac{-1}{(y + W_3)}$$

$$\frac{\partial N}{\partial x} = \frac{1}{(y + W_3)^2} \frac{dW_3}{dx}$$

From the equation that define the differential of W_3 :

$$\frac{dW_3}{dx} = \frac{-2W_1}{W_3} = \frac{-2x}{W_3}$$

Therefore:

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} = \frac{-2x}{W_3(y + W_3)^2}$$



The differential for “ $W_4(x)$ ” is exact with solution that can be obtained using the first or second integral form.

We use the first integral form:

$$\int Ndy + \int \left(M - \frac{\partial}{\partial x} \int Ndy \right) dx = C$$

where:

$$M(x, y) = \frac{2x}{W_3(y + W_3)}$$

$$N(x, y) = \frac{-1}{(y + W_3)}$$

Starting with the first integral:

$$\int Ndy = \int \frac{-1}{(y + W_3)} dy = -\ln(y + W_3)$$

where “ $W_3 = W_3(x)$ ” is treated as a constant because integration is with respect to the “y” variable only.



$$\int \left(M - \frac{\partial}{\partial x} \int N dy \right) dx = \int \left(M - \frac{\partial}{\partial x} \{-\ln(y + W_3)\} \right) dx$$

$$= \int \left(M + \left(\frac{1}{y + W_3} \right) \frac{dW_3}{dx} \right) dx$$

Again from the differential “ W_3 ” :

$$\frac{dW_3}{dx} = \frac{-2x}{W_3}$$

So that:

$$\int \left(M - \frac{\partial}{\partial x} \int N dy \right) dx = \int \left(M - \frac{2x}{W_3(y + W_3)} \right) dx$$

But:

$$M(x, y) = \frac{2x}{W_3(y + W_3)}$$

Therefore:

$$\int \left(M - \frac{\partial}{\partial x} \int N dy \right) dx = \int (\mathbf{M} - \mathbf{M}) dx = 0$$

So that the exact solution becomes:

$$\int N dy + \int \left(M - \frac{\partial}{\partial x} \int N dy \right) dx = -\ln(y + W_3) + K + 0 = -\ln(y + W_3) + K$$



The **right hand side** of the differential expansion for “ W_4 ” is simply defined as “ dW_4 ” .

After integrating both sides of this differential expansion the complete expression for “ W_4 ” may now be written as:

$$-\ln(y + W_3) = W_4 + C$$

or:

$$W_4 = -\ln(y \pm \sqrt{C_3 - 2x^2}) + C_4$$

The complete inverse *Multivariate Polynomial Transform* is then obtained by substituting each expression for the auxiliary variables into the *Primary Expansion*:

$$f(x, y) = 0 = W_4 + 2W_2 = -\ln(y \pm \sqrt{C_3 - 2x^2}) + 2y + C_{24}$$



TAKING THE INVERSE MULTIVARIATE POLYNOMIAL TRANSFORM

“ THE MOST GENERAL CASE ”



(1). Primary Expansion:

$$F_i(W_1, W_2, \dots, W_{p+q}) = 0 = \sum_t^r a_{i,t} \left(\prod_j^{p+q} W_j^{E_{i,kj}} \right) \quad (1 \leq i \leq k)$$

(2). Secondary Expansion:

$$\begin{aligned} dz_i &= dW_i & (1 \leq i \leq m) \\ dx_j &= dW_{m+j} & (1 \leq j \leq n) \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m N_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n N_{i(m+n+1)-n-1+t} dx_t &= \\ &= N_{i(m+n+1)} dW_j & \begin{aligned} (1 \leq i \leq p+q-m-n) \\ (m+n+1 \leq j \leq p+q) \end{aligned} \end{aligned}$$

$$\begin{aligned} \sum_{t=1}^m T_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n T_{i(m+n+1)-n-1+t} dx_t &= \\ &= T_{i(m+n+1)} dW_j & (1 \leq i \leq q) \quad (p \leq j \leq p+q) \end{aligned}$$

where it is very important to note that when using this expansion for solving DEs and systems of DEs, "q" is the total number of auxiliary variables necessary for defining all functions that can be present in the DE or system of DEs we are attempting to solve for.



General representation of a single equation present in our Secondary Expansion:

$$(M_1 dz_1 + M_2 dz_2 + \dots + M_m dz_m) + (M_{m+1} dx_1 + M_{m+2} dx_2 + \dots + M_{m+n} dx_n) = M_{m+n+1} dW_j \quad (m+n+1 \leq j \leq p)$$

Left hand side:

$$M_k = M_k(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) \quad \begin{array}{l} [k \neq i(m+n+1)] \\ (1 \leq i \leq p+q-m-n) \end{array}$$

Right hand side:

$$M_k = M_k(W_j) \quad \begin{array}{l} [k = i(m+n+1)] \\ (1 \leq i \leq p+q-m-n) \\ (m+n+1 \leq j \leq p) \end{array}$$

We can defined the **left** hand side as:

$$dH_1 = (M_1 dz_1 + M_2 dz_2 + \dots + M_m dz_m) + (M_{m+1} dx_1 + M_{m+2} dx_2 + \dots + \dots + M_{m+n} dx_n)$$

If “ dH_1 ” is an exact differential then from the chain rule it is also true that:

$$dH_1 = \sum_{k=1}^m \frac{\partial H_1}{\partial z_k} dz_k + \sum_{k=1}^n \frac{\partial H_1}{\partial x_k} dx_k$$



By equating both expression defining “ dH_1 ” we arrive at the following conclusion that:

$$M_i = \frac{\partial H_1}{\partial z_i} \quad (1 \leq i \leq m)$$

$$M_{m+j} = \frac{\partial H_1}{\partial x_j} \quad (1 \leq j \leq n)$$



From multivariate calculus, the condition that define an exact differential is of course when:

$$\frac{\partial M_1}{\partial z_2} = \frac{\partial M_2}{\partial z_1}, \quad \frac{\partial M_1}{\partial z_3} = \frac{\partial M_3}{\partial z_1}, \quad \frac{\partial M_1}{\partial z_4} = \frac{\partial M_4}{\partial z_1}, \quad , \dots , \quad \frac{\partial M_1}{\partial z_m} = \frac{\partial M_m}{\partial z_1}$$

$$\frac{\partial M_1}{\partial x_1} = \frac{\partial M_{m+1}}{\partial z_1}, \quad \frac{\partial M_1}{\partial x_2} = \frac{\partial M_{m+2}}{\partial z_1}, \quad , \dots , \quad \frac{\partial M_1}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_1}$$

$$\frac{\partial M_2}{\partial z_3} = \frac{\partial M_3}{\partial z_2}, \quad \frac{\partial M_2}{\partial z_4} = \frac{\partial M_4}{\partial z_2}, \quad \frac{\partial M_2}{\partial z_5} = \frac{\partial M_5}{\partial z_2}, \quad , \dots , \quad \frac{\partial M_2}{\partial z_m} = \frac{\partial M_m}{\partial z_2}$$

$$\frac{\partial M_2}{\partial x_1} = \frac{\partial M_{m+1}}{\partial z_2}, \quad \frac{\partial M_2}{\partial x_2} = \frac{\partial M_{m+2}}{\partial z_2}, \quad , \dots , \quad \frac{\partial M_2}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_2}$$

$$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

$$\frac{\partial M_m}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_m}$$



For an exact differential, the solution is given by:

$$H_1 = \int M_1(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) \partial z_1$$

where in this case, z_i and x_j for $1 < i \leq m$, $1 \leq j \leq n$ and $i \neq 1$ are all treated as constants when evaluating this indefinite integral.

We can also use as another alternative:

$$H_1 = \int M_k(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) \partial z_k$$

where in this case, z_i and x_j for $2 \leq i \leq m$, $1 \leq j \leq n$ and $i \neq k$ are all treated as constants when evaluating this indefinite integral.

Other alternatives for the same expression of " H_1 " can also be obtained from:

$$H_1 = \int M_{m+k}(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) \partial x_k$$

where in this case, z_i and x_j for $1 \leq i \leq m$, $1 \leq j \leq n$ and $j \neq k$ are all treated as constants when evaluating this indefinite integral.



As for the *right hand side* of the equation defining the general format of a *Secondary Expansion* we can define:

$$dH_2 = M_{m+n+1}dW_j$$

and since " $H_2 = H_2(W_j)$ " then:

$$dH_2 = \frac{\partial H_2}{\partial W_j} dW_j$$

By equating each of the expression for " dH_2 " we arrive at the conclusion that:

$$M_{m+n+1} = \frac{\partial H_2}{\partial W_j}$$

The exact expression of " H_2 " can be determined using the following integral :

$$H_2 = H_2(W_j) = \int M_{m+n+1}(W_j) dW_j$$

because " $W_j = W_j(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n)$ " is a multivariate composite function.

The complete exact solution of the first order multivariate ODE that would be present inside a *Secondary Differential Expansion* is:

$$H_1(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) - H_2(W_j) = 0$$

from which " W_j " can be obtained explicitly whenever possible.



- Each auxiliary variable can afterwards be substituted along with each of their initial condition into the *Primary Expansion* for arriving at the complete expression of “ $f_k = 0$ ”.
- The initial condition of each auxiliary variable can be used for satisfying the initial condition of “ $f_k = 0$ ” upon inverting the original *Multivariate Polynomial Transform*.
- Complete detailed examples available in section 1 of “***A better way for managing all of the physical sciences under a single unified theory of analytical integration***” published in the Proceedings of the 6th International Conference on Computational Methods held in New Zealand, July 2015 (Paper ID #845-3475-1-PB, ScienTech Publisher).



One of the example that was presented at that conference :

Secondary Differential Expansion:

$$dx = dW_1$$

$$dy_1 = dW_2$$

$$dy_2 = dW_3$$

$$dy_3 = dW_4$$

$$W_1^2 dx + W_2^2 dy_1 + W_3^2 dy_2 + W_4^2 dy_3 = W_5^2 dW_5$$

$$W_1 dx + W_2 dy_1 + W_3 dy_2 + 0 \cdot dy_3 = W_6 dW_6$$

$$W_1 W_6^{-1} dx + (W_2 W_6^{-1} + 2W_3) dy_1 + (W_3 W_6^{-1} + 2W_2) dy_2 + 0 \cdot dy_3 = \frac{dW_7}{1 + W_7^2}$$

where we were able to determine by using our standard test for exactness the complete inverse of the differential representation for “ W_5 ”, “ W_6 ” and “ W_7 ” as:

$$W_5 = \sqrt[3]{x^3 + y_1^3 + y_2^3 + y_3^3 + c_5}$$

$$W_6 = \sqrt{x^2 + y_1^2 + y_2^2 + c_6}$$

and:

$$W_7 = \tan\left(\sqrt{x^2 + y_1^2 + y_2^2 + c_6} + 2y_1 y_2 + c_7\right)$$



- The general universal differential representation of “ $f_k = 0$ ” can always be converted as an initially assumed differential expansion with unknown coefficients to solve for finding exact analytical solutions to DEs and systems of DEs.
- This would define a “*universal method of analytical integration*”.
- We would refer to this initially assumed universal differential expansion as an “*Initially Assumed Multivariate Polynomial Transform*” or in short as an *IAMPT*.



Numerically controlled system of analytics (NCSA) table for Physics and Engineering Science as a direct application of the unified theory of analytical integration.



General equation for representing all PDEs:

$$G_k = G_k \left[z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n, \frac{\partial z_1}{\partial x_1}, \dots, \frac{\partial z_1}{\partial x_n}, \frac{\partial z_2}{\partial x_1}, \dots, \frac{\partial z_2}{\partial x_n}, \dots, \frac{\partial z_m}{\partial x_1}, \dots, \frac{\partial z_m}{\partial x_n}, \dots, \right. \\ \left. \dots, \frac{\partial^2 z_m}{\partial x_1 \partial x_1}, \dots, \frac{\partial^2 z_m}{\partial x_1 \partial x_n}, \dots, \frac{\partial^2 z_m}{\partial x_2 \partial x_1}, \dots, \frac{\partial^2 z_m}{\partial x_2 \partial x_n}, \dots, \dots, \frac{\partial^2 z_m}{\partial x_n^2}, \dots, \dots, \frac{\partial^r z_m}{\partial x_n^r} \right] = 0$$



NUMERICALLY CONTROLLED SYSTEM OF ANALYTICS TABLE

$G_k = 0$		
Initial Conditions	Coefficient values present in the DE or system of DEs	Exact analytical solution obtained using the Multivariate Polynomial Transform method
$z_{01}, z_{02}, \dots, z_{0m}, x_{01}, \dots, x_{0n} \dots$	a_0, b_0, c_0, \dots	$U_1 = 0$
$z_{11}, z_{12}, \dots, z_{1m}, x_{11}, \dots, x_{1n} \dots$	a_1, b_0, c_0, \dots	$U_2 = 0$
$z_{21}, z_{22}, \dots, z_{2m}, x_{21}, \dots, x_{2n} \dots$	a_0, b_1, c_0, \dots	$U_3 = 0$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮



When solving for DEs and systems of DEs using an initially assumed Multivariate Polynomial Transform, some of the reasons that would account for the existence of an infinite number of numerical solution sets of the corresponding nonlinear simultaneous equations are:

- DEs can satisfy an infinite number of initial conditions.
- Auxiliary variables can be permuted with respect to one another inside an initially assumed Multivariate Polynomial Transform.
- Possible existence of *trivial* algebraic identities such as " $\sin^2(x) + \cos^2(x) = 1$ ".
- Presence of singular solutions.
- Presence of *trivial ratios* consisting of identical multivariate polynomials that can be entirely eliminated from the *Secondary Expansion*.



COMPLETE EXAMPLE FOR A SECOND ORDER PDE

$$x_2 \left(\frac{\partial^2 z}{\partial x_1 \partial x_2} \right) - \frac{\partial z}{\partial x_1} - x_1 x_2^2 \text{Sin}(x_1 x_2) = 0$$



$$x_2 \left(\frac{\partial^2 z}{\partial x_1 \partial x_2} \right) - \frac{\partial z}{\partial x_1} - x_1 x_2^2 \text{Sin}(x_1 x_2) = 0$$

Only one external input is defined so we use the following trigonometric identity to determine its complete *Multivariate Polynomial Transform*:

$$f(x_1, x_2) = \text{Sin}(x_1 x_2) = \frac{2 \text{Tan}(x_1 x_2 / 2)}{1 + \text{Tan}^2(x_1 x_2 / 2)}$$

$$H(x_1, x_2) = W_{p+1} = \text{Tan}(x_1 x_2 / 2) = \text{Tan}(W_2 W_3 / 2)$$

where "p" is the total number of arbitrarily defined auxiliary variables from within an IAMPT.

(1). Primary Expansion:

$$H(W_{p+1}) = W_{p+1}$$

(2). Secondary Expansion:

$$0 \cdot dz + (1 + W_{p+1}^2) W_3 dx_1 + (1 + W_{p+1}^2) W_2 dx_2 = 2dW_{p+1}$$

where we have selected:

$$\begin{aligned} W_1 &= z \\ W_2 &= x_1 \\ W_3 &= x_2 \end{aligned}$$



For our IAMPT we select the following parameters: “ $p = 8$ ”, “ $u_p = 8$ ” and “ $u_s = 4$ ” where “ $q = 1$ ”.

(1). Primary Expansion:

$$F = 0 = a_1 W_1^{m_1} W_2^{m_2} \dots W_9^{m_9} + a_2 W_1^{m_{10}} W_2^{m_{11}} \dots W_9^{m_{18}} + \dots + a_8 W_1^{m_{64}} W_2^{m_{65}} \dots W_9^{m_{72}}$$

(2). Secondary Expansion:

$$dz = dW_1$$

$$dx_1 = dW_2$$

$$dx_2 = dW_3$$

$$N_1 dz + N_2 dx_1 + N_3 dx_2 = N_4 dW_4$$

$$N_5 dz + N_6 dx_1 + N_7 dx_2 = N_8 dW_5$$

$$N_9 dz + N_{10} dx_1 + N_{11} dx_2 = N_{12} dW_6$$

$$N_{13} dz + N_{14} dx_1 + N_{15} dx_2 = N_{16} dW_7$$

$$N_{17} dz + N_{18} dx_1 + N_{19} dx_2 = N_{20} dW_8$$

$$N_{21} dz + N_{22} dx_1 + N_{23} dx_2 = N_{24} dW_9$$

where :

$$N_1 = b_1 W_1^{m_1} W_2^{m_2} \dots W_9^{m_9} + \dots + b_4 W_1^{m_{28}} W_2^{m_{29}} \dots W_9^{m_{36}}$$

$$N_2 = b_5 W_1^{m_{37}} W_2^{m_{38}} \dots W_9^{m_{45}} + \dots + b_8 W_1^{m_{64}} W_2^{m_{65}} \dots W_9^{m_{72}}$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$N_{20} = b_{77} W_1^{m_{685}} W_2^{m_{686}} \dots W_9^{m_{693}} + \dots + b_{80} W_1^{m_{712}} W_2^{m_{713}} \dots W_9^{m_{720}}$$



To account for the presence of the trigonometric function in the PDE:

$$N_{21} = 0$$

$$N_{22} = (1 + W_{p+1}^2)W_3 = (1 + W_9^2)W_3$$

$$N_{23} = (1 + W_{p+1}^2)W_2 = (1 + W_9^2)W_2$$

$$N_{24} = 2$$

We can compute the total number of unknowns to solve for in our IAMPT using the following general formula with "n = 2", "p = 8", "u_p = 8", "u_s = 4" and "q = 1" :

$$\begin{aligned} N_{Total} &= N_{Primary} + N_{Secondary} \\ &= u_p(p + q + 1) + u_s(p + q + 1)(n + 2)(p - n - 1) \\ &= 8(8 + 1 + 1) + 4(8 + 1 + 1)(2 + 2)(8 - 2 - 1) \\ &= 8(10) + 4(10)(4)(5) = 80 + 800 = 880 \end{aligned}$$

The *Multivariate Polynomial Transform* of the entire PDE:

$$W_3 \left(\frac{\partial^2 Z}{\partial W_2 \partial W_3} \right) - \frac{\partial Z}{\partial W_2} - 2W_2 W_3^2 \left(\frac{W_{p+1}}{1 + W_{p+1}^2} \right) = 0$$

where we have selected:

$$W_1 = z$$

$$W_2 = x_1$$

$$W_3 = x_2$$



Our nonlinear simultaneous equations to solve for is obtained by successively differentiating the PDE in its complete differential representation:

$$G_i = \frac{\partial^{m_1}}{\partial W_2^{m_1}} \frac{\partial^{m_2}}{\partial W_3^{m_2}} \frac{\partial^{m_3}}{\partial W_4^{m_3}} \cdots \frac{\partial^{m_k}}{\partial W_{p+1}^{m_k}} \cdots \left\{ W_3 \left(\frac{\partial^2 Z}{\partial W_2 \partial W_3} \right) - \frac{\partial Z}{\partial W_2} - 2W_2 W_3^2 \left(\frac{W_{p+1}}{1 + W_{p+1}^2} \right) \right\}$$

and then replacing each partial derivative with the one calculated from our IAMPT based on the use of the *Multinomial Expansion Theorem*.

Method of solving for the nonlinear simultaneous equations is by minimizing the following equation:

$$F = \sum_i G_i^2$$

- **Only when “ $F = 0$ ” then an EXACT solution of the PDE is obtained by inverting the corresponding Multivariate Polynomial Transform. Otherwise this would result into defining some approximation of the exact solution.**



THE NCSA TABLE FOR OUR EXAMPLE OF A SECOND ORDER PDE WOULD THEREFORE APPEAR AS FOLLOW:

$$x_2 \left(\frac{\partial^2 z}{\partial x_1 \partial x_2} \right) - \frac{\partial z}{\partial x_1} - x_1 x_2^2 \sin(x_1 x_2) = 0$$

Initial Conditions	Coefficient Values	Exact analytical solution obtained using the Multivariate Polynomial Transform method
$x_{01} = 3.61$ $x_{02} = 1.771$	N/A	$2x_2 x_1^{1.68} + \sin(\ln[x_2^{-1.6}] + x_2^{0.78}) - \sin(x_1 x_2) - z = 0$
$x_{01} = 1.29$ $x_{02} = -1.88$	N/A	$x_2^6 \sqrt{x_1^{0.23} + 1.78} + 1.22 \ln(\sqrt{x_2^2 + 1} + 3.5) - \sin(x_1 x_2) - z = 0$
$x_{01} = 3.555$ $x_{02} = 2.76$	N/A	$0.56 x_2 e^{x_1^{-0.46}} - 4.6 \tan(x_2^{1.86} + \sqrt[4]{x_2^{1.1} - 6.1}) - \sin(x_1 x_2) - z = 0$
$x_{01} = -0.723$ $x_{02} = 1.58$	N/A	$3.06 x_2 \sinh(x_1^2) - 2.45 x_2^{1.46 \sqrt{x_2^{3.1} - 2.3}} - \sin(x_1 x_2) - z = 0$
.	.	.
.	.	.
.	.	.



Simplest model for describing the standard method of analysis for solving DEs and systems of DEs in terms of general analytical solutions based on the concept of an NCSA table.



$$x \frac{dy}{dx} + ay + bx^n y^2 = 0$$



NUMERICALLY CONTROLLED SYSTEM OF ANALYTICS TABLE

$x \frac{dy}{dx} + ay + bx^n y^2 = 0$		
<i>Initial Conditions</i>	<i>Coefficient Values</i>	<i>Exact analytical solution obtained using the Multivariate Polynomial Transform method</i>
$x_0 = 1$ $y_0 = 1$	$a = 1.0$ $b = 1.0$ $n = -1.0$	$(-3x + x^{-1})y + 2 = 0$
$x_0 = 1$ $y_0 = 2$	$a = 1.2$ $b = -1.0$ $n = 2.0$	$(1.4x^{1.2} - x^2)y - 0.80 = 0$
$x_0 = 1$ $y_0 = -1$	$a = 1.2$ $b = 1.5$ $n = -2.0$	$(1.7x^{1.2} + 1.5x^{-2})y + 3.2 = 0$
$x_0 = 1$ $y_0 = 2$	$a = 2.0$ $b = -1.0$ $n = 2.0$	$x^2 y(0.5 - \ln(x)) - 1 = 0$
$x_0 = 1$ $y_0 = -2$	$a = 1.5$ $b = 2.0$ $n = 3.0$	$(-2.75x^{1.5} + 2x^3)y - 1.5 = 0$
$x_0 = 1$ $y_0 = 1$	$a = 1.0$ $b = 1.0$ $n = 1.0$	$xy(1 + \ln(x)) - 1.0 = 0$
.	.	.
.	.	.
.	.	.



From this table we can assume by conjecture the following candidates as being the general analytical solution of the general ODE :

$$f_1(x, y) = 0 = (Ax^B + Cx^D)y + E$$

and:

$$f_2(x, y) = 0 = x^A y(B + C \ln(x)) + D$$

Coefficients "A", "B", "C", "D" and "E" are to be expressed in terms of the coefficients "a", "b", "n" and the initial conditions of the ODE.

General formula used for determining the first derivative of "y":

$$\frac{dy}{dx} = -\frac{\partial f / \partial x}{\partial f / \partial y}$$



Equating like terms to zero after substituting the general derivative of “y” in the ODE for the first assumed general solution:

$$A(a - B) = 0$$

$$C(a - n) - bE = 0$$

$$(Ax_0^a + Cx_0^n)y_0 + E = 0$$

The exact solution of this general system of equations may be expressed as:

$$A \neq 0$$

$$B = a$$

$$C = \frac{-Abx_0^a y_0}{a + bx_0^n y_0 - n}$$

$$E = \frac{(a - n)C}{b} \quad (a \neq n)$$



Equating like terms to zero in the ODE for the second assumed general solution:

$$B(a - n) - C - bD = 0$$

$$C(a - n) = 0$$

$$x_0^n y_0 (B + C \ln(x_0)) + D = 0$$

With exact solution:

$$D \neq 0$$

$$C = -bD$$

$$B = \frac{-D}{x_0^n y_0} - C \ln(x_0) = \frac{-D - C x_0^n y_0 \ln(x_0)}{x_0^n y_0}$$



A typical report that a numerical analyst might be presenting to management would appear as follow:

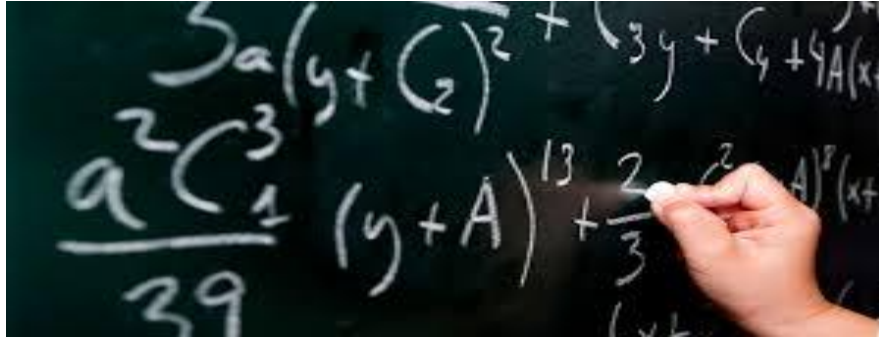
“ ... thus, our empirical findings has indicated to us that for this first order ODE there are two recognizable general exact solutions. The first one is for the case when $n = a$ and the other is when $n \neq a$. The general exact solutions obtained can be expressed as a combination of algebraic and elementary basis functions defined only in explicit form. Furthermore, we have established that there is according to the empirical results presented in our NCSA table an explicit relationship involving the initial condition (x_0, y_0) of the ODE, the coefficients (a, b, n) of the ODE and the coefficients in our two initially assumed general exact solutions. ”



“ It is expected that many such reporting systems applied on a very large variety of DEs and systems of DEs would inevitably lead to the discovery of many new fundamental theorems similar to the superposition theorem !”



Problem solving section



Problem #1:

Prove the *Quadratic Equation* by method of differentials.

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution:

Step 1:

Define “x” as a dependent variable which we will call “z” and all the 3 coefficients A, B and C as the independent variables x_1, x_2 and x_3 respectively.



Step 2:

We thus seek an explicitly defined solution in the form of :

$$z = z(x_1, x_2, x_3)$$

by equating the various partial derivatives of:

$$Az^2 + Bz + C = 0$$

with respect to each independent variables with the following IAMPT that has been converted from *implicit* to *explicit* form for deriving and solving the corresponding system of Nonlinear Simultaneous Equations.



(1). Primary Expansion:

$$z(W_j) = \frac{P(W_j)}{Q(W_j)} \quad (1 \leq j \leq p)$$

where "P" and "Q" are each multivariate polynomials each consisting of a maximum of "p" number of auxiliary variables which would be determined by trial and error and where the exponents of each auxiliary variable are always assumed as floating point numbers.

(2). Secondary Differential Expansion:

$$dz = dW_1$$

$$dx_i = dW_{i+1} \quad (1 \leq i \leq n)$$

$$\sum_{t=1}^1 N_{(i-1)(n+2)+t} dz + \sum_{t=1}^n N_{i(n+2)-n-1+t} dx_t =$$

$$= N_{i(n+2)} dW_j \quad [1 \leq i \leq p-1-n] \quad [n+2 \leq j \leq p]$$

$$N_c(W_j) = \sum_{t=(c-1)r+1}^{cr} b_{c,t} \left(\prod_j^p W_j^{E'_{c,s}} \right) \quad [1 \leq c \leq i(n+2)] \quad [1 \leq i \leq p-1-n]$$



THE NUMERICALLY CONTROLLED SYSTEM OF ANALYTICS TABLE WILL CONFIRM THE EXISTENCE OF AN INFINITE NUMBER OF NUMERICAL SOLUTIONS SETS THEREBY CONFIRMING THE EXACTNESS OF THE FORMULA FOR THE QUADRATIC EQUATION.

$G_k = 0$		
Initial Conditions	Coefficient values present in the DE or system of DEs	Exact analytical solution obtained using the Multivariate Polynomial Transform method
$z_{01}, z_{02}, \dots, z_{0m}, x_{01}, \dots, x_{0n} \dots$	a_0, b_0, c_0, \dots	$U_1 = 0$
$z_{11}, z_{12}, \dots, z_{1m}, x_{11}, \dots, x_{1n} \dots$	a_1, b_0, c_0, \dots	$U_2 = 0$
$z_{21}, z_{22}, \dots, z_{2m}, x_{21}, \dots, x_{2n} \dots$	a_0, b_1, c_0, \dots	$U_3 = 0$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮



Problem #2:

Prove the *Superposition Theorem* by computation which states that for a linear homogeneous ODE, if " $y_1(x)$ " and " $y_2(x)$ " are solutions then so is " $y_1(x) + y_2(x)$ ".

Solution:

Build the same type of *numerically controlled system of analytics table* for linear second order ODEs as we did earlier for the following first order ODE :

$$x \frac{dy}{dx} + ay + bx^n y^2 = 0$$

Such a fundamental theorem could then be easily deduced by just populating this table with a large number of instance analytical solutions satisfying a number of arbitrarily defined initial conditions and values for all the coefficients that are present in the ODE.



Problem #3:

Define a new *measure of composition* for the classification of all Composite Functions using the concept of a *Multivariate Polynomial Transform*.

Solution:

A single composite function to the " n^{th} " degree requires a minimum of " $n+1$ " number of auxiliary variables to define its complete Multivariate Polynomial Transform.

The exponential function " $y = e^x$ " would be considered as a *zeroth* order composite function since it would require a minimum of 1 auxiliary variable for defining its complete Multivariate Polynomial Transform.



Measure of degree of composition using the Multivariate Polynomial Transform

Degree	Composite Function	Minimum number of auxiliary variables	Secondary Expansion
0	t	$W_1 = t$	$dt = dW_1$
0	e^{at}	$W_1 = e^{at}$	$dt = \frac{dW_1}{aW_1}$
0	$Tan(at)$	$W_1 = Tan(at)$	$dt = \frac{dW_1}{a(1 + W_1^2)}$
1	$ln(at)$	$W_1 = t$ $W_2 = ln(aW_1)$	$dt = dW_1 = W_1 dW_2$
1	$\sqrt[n]{t}$	$W_1 = t$ $W_2^n = W_1$	$dt = dW_1$ $= W_2^{n-1} dW_2$



Measure of degree of composition using the Multivariate Polynomial Transform

Degree	Composite Function	Minimum number of auxiliary variables	Secondary Expansion
1	$e^{\text{Sin}(at)}$	$W_1 = \text{Tan}(at/2)$ $W_2 = e^{P(W_1)}$ $P(W_1) = \frac{2W_1}{1 + W_1^2}$	$dt = \frac{2dW_1}{a(1 + W_1^2)}$ $= \frac{(1 + W_1^2)}{aW_2(1 - W_1^2)}dW_2$
2	$\ln(\ln(at))$	$W_1 = t$ $W_2 = \ln(aW_1)$ $W_3 = \ln(W_2)$	$dt = dW_1 = W_1dW_2$ $= W_2dW_3$



Measure of degree of composition using the Multivariate Polynomial Transform

Degree	Composite Function	Minimum number of auxiliary variables	Secondary Expansion
3	$\sqrt[n]{\ln(\tan(t^2)) + 1}$	$W_1 = t_1$ $W_2 = \tan(W_1^2)$ $W_3 = \ln(W_2)$ $W_4^n = W_3 + 1$	$dt = dW_1$ $= \frac{dW_2}{2W_1(1 + W_2^2)}$ $= \frac{W_2 dW_3}{2W_1(1 + W_2^2)}$ $= \frac{nW_2 W_4^{n-1} dW_4}{2W_1(1 + W_2^2)}$

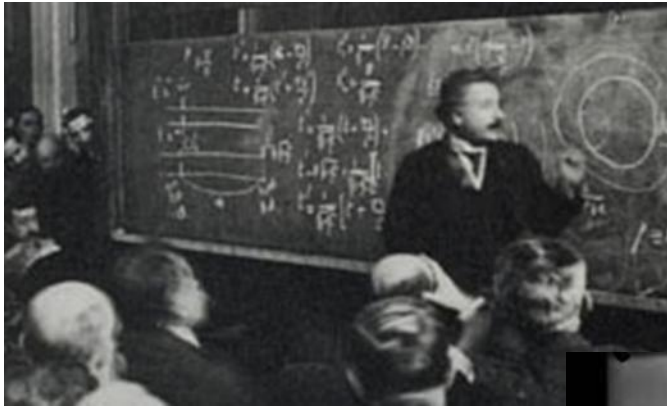


Measure of degree of composition using the Multivariate Polynomial Transform

Degree	Composite Function	Minimum number of auxiliary variables	Secondary Expansion
$k - 1$	$Tan(a_k Tan(a_{k-1} Tan(a_{k-2} \cdots$ $\cdots a_2 Tan(a_1 t))))$	$W_1 = Tan(a_1 t)$ $W_2 = Tan(a_2 W_1)$ $\cdot \quad \cdot \quad \cdot$ $\cdot \quad \cdot \quad \cdot$ $\cdot \quad \cdot \quad \cdot$	$dt = \frac{dW_1}{G_1}$ $= \frac{dW_2}{G_1 G_2}$ $\cdot \quad \cdot \quad \cdot$ $\cdot \quad \cdot \quad \cdot$ $\cdot \quad \cdot \quad \cdot$
		$W_k = Tan(a_k W_{k-1})$	$= \frac{dW_k}{G_1 G_2 \cdots G_k}$ $G_k = a_k(1 + W_k^2)$



The *theory of everything* not just about modern physics anymore



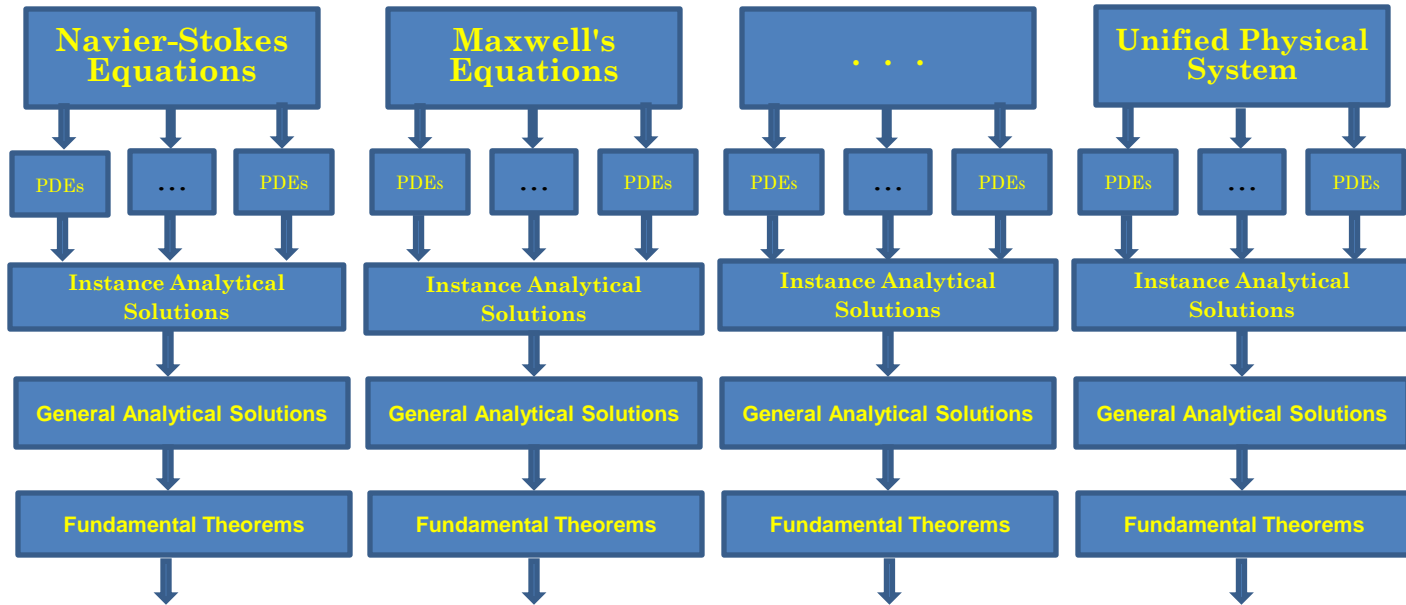
- ❑ Most accepted definition of the *theory of everything* is that it must remain an integral part of modern physics on the principle of defining a unique Space-Time Model that would explain all the basic laws of this universe.
- ❑ One of the primary objective is to unify electromagnetic energy with gravitational energy under a single uniform theory.
- ❑ A *grandiose* physical theory for explaining everything about this Universe would only be possible from the application of an equivalent *grandiose* mathematical theory that would explain everything about the integration of all DEs.
- ❑ DEs are universal and not linked to any specific area of the physical sciences so no evidence to support that modern physics is the only subject by which a complete theory of everything may be entirely constructed from.
- ❑ Instead, only by consolidating the *general* analytical solutions of common key DEs into fundamental theorems can a gigantic theory capable of explaining everything about our physical Universe be constructed.



Here is how *Differential Equations* would now play a central role for establishing a theory of everything.



EACH *UNIFIED PHYSICAL SYSTEM* WOULD HAVE ITS OWN VERY UNIQUE STORY TO TELL THAT WE ALL NEED TO KNOW ABOUT IN THE END.



THEORY OF EVERYTHING



QUESTION FOR THE AUDIENCE :

What lies beyond the theory of everything ?



"My God, you've done it! The unifying Theory of Everything!"



A universal method of differential analysis for solving the Navier-Stokes equations involving incompressible fluid without transformation of variables and for solving linear elastic boundary value problems



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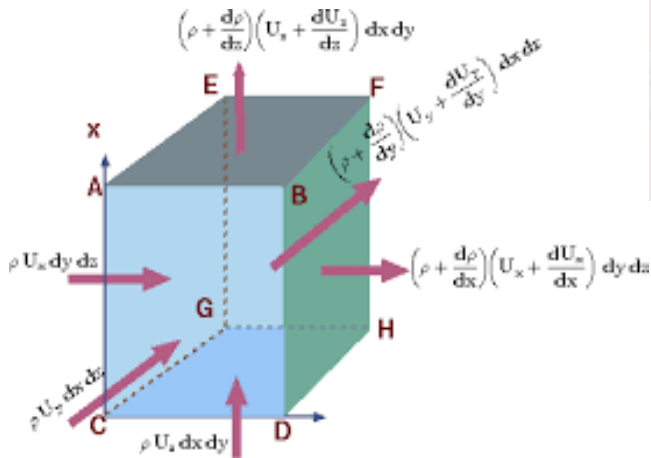
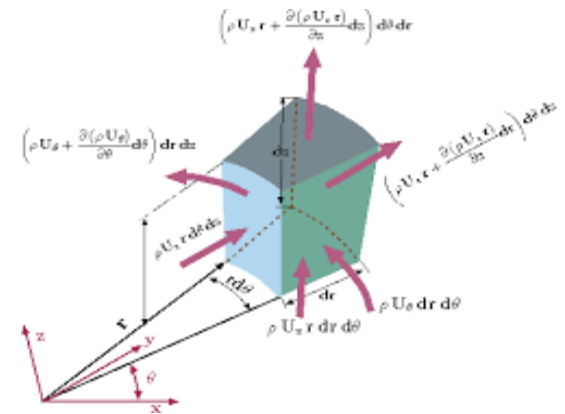
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Simple demonstration for the Navier-Stokes equations



We now compare our method of differential analysis for solving the Navier-Stokes equations with the one described by Muhammad Jamil:

*“Muhammad Jamil , A Class of Exact Solutions to Navier-Stokes Equations for the Given Vorticity, International Journal of Nonlinear Science, Vol.9 (2010) No.3,pp.296-304, * ISSN 1749-3889 (print), 1749-3897 (online)”*

Basic assumptions that were made in his paper:

- Steady plane motion of an incompressible fluid
- Variable viscosity
- Heat transfer
- Reynolds number (type of flow i.e. laminar or turbulent)
- Prandtl number (diffusivity ratio)
- Eckert number (measure of heat dissipation)



Basic governing equations in the absence of any external forces with no heat addition as derived by:

“R. K. Naeem and M. Jamil. On plane steady flows of an incompressible fluid with variable viscosity. Int. J. of Appl. Math. and Mech., 2:(2006), 1-19”.

$$u_x + v_y = 0$$

$$uu_x + vv_y = -P_x + \frac{1}{R_e} \left[(2\mu u_x)_x + (\mu(u_y + v_x))_y \right]$$

$$uv_x + vv_y = -P_y + \frac{1}{R_e} \left[(2\mu v_y)_y + (\mu(u_y + v_x))_x \right]$$

$$uT_x + vT_y = \frac{1}{R_e P_r} (T_{xx} + T_{yy}) + \frac{E_c}{R_e} \mu \left[2(u_x^2 + v_y^2) + (u_y + v_x)^2 \right]$$

where " u " and " v " are the velocity components, " P " is pressure, " T " the temperature, " μ " the viscosity, " R_e ", " P_r " and " E_c " are the Reynolds, Prandtl and Eckert numbers respectively.



- Define the stream function in terms of the velocity component " $u = \psi_y$ " and " $v = -\psi_x$ "
- We assume the vorticity distribution $\nabla^2\psi$ is proportional to the stream function perturbed by an exponential stream of the form: $\nabla^2\psi = K(\psi - Ue^{ax+by})$

The original system of PDEs now transformed directly in terms of the following more integrable system of second order ODEs :

$$\psi_{\xi\xi} - \Lambda\psi = -Ue^\xi$$

$$(\mu\psi)_{\xi\xi} = 0$$

$$T_{\xi\xi} + E_c Pr \Lambda^2 (a^2 + b^2) \mu\psi^2 = 0$$

which led to the following change of coordinates:

$$\xi = ax + by$$

and where:

$$\Lambda = \frac{K}{a^2 + b^2}$$



Exact analytical solution obtained by Jamil in the original coordinates:

Case I: $\Lambda = -n^2$, $n > 0$

$$u = \frac{Ubn^2}{n^2+1} e^{ax+by} - A_1 nb \text{Sin}(n(ax+by) + A_2)$$

$$v = \frac{Uan^2}{n^2+1} e^{ax+by} + A_1 na \text{Sin}(n(ax+by) + A_2)$$

$$P = \frac{Un^2(a^2+b^2)}{2(n^2+1)} e^{ax+by} \left[Ue^{ax+by} - 2A_1 \{ \text{Cos}(n(ax+by) + A_2) + n \text{Sin}(n(ax+by) + A_2) \} \right] -$$

$$- \frac{A_3 n^2 (b^2 - a^2)}{R_e} (bx + ay) + \frac{2abn^2}{R_e} (A_3(ax+by) + A_4) -$$

$$- \frac{n^2(a^2+b^2)}{2(n^2+1)^2} \left[U n e^{ax+by} - A_1 (n^2+1) \text{Sin}(n(ax+by) + A_2) \right]^2 + A_7$$

$$T = \frac{E_c P_r n (a^2 + b^2)}{n^2 + 1} \left[A_3 \left\{ U n^3 e^{ax+by} (ax + by - 2) + A_1 (n^2 + 1) \left\{ n(ax + by) \text{Cos}(n(ax + by) + A_2) - \right. \right. \right.$$

$$\left. \left. - 2 \text{Sin}(n(ax + by) + A_2) \right\} \right\} + n A_4 \left[U n^2 e^{ax+by} + A_1 (n^2 + 1) \text{Cos}(n(ax + by) + A_2) \right] \right] +$$

$$+ A_5 (ax + by) + A_6$$

$$\mu = \frac{A_3(ax+by) + A_4}{A_1 \text{Cos}(n(ax+by) + A_2) - \frac{Ue^{ax+by}}{n^2+1}}$$



Case II: $\Lambda = m^2, m > 0$

$$u = \frac{Ubm^2}{m^2 - 1} e^{ax+by} + B_1 m b e^{m(ax+by)} - B_2 m b e^{-m(ax+by)}$$

$$v = -\frac{Uam^2}{m^2 - 1} e^{ax+by} - B_1 m a e^{m(ax+by)} + B_2 m a e^{-m(ax+by)}$$

$$P = \frac{Um^2(a^2 + b^2)}{2(m^2 - 1)} \left[Ue^{2(ax+by)} + 2B_1(m - 1)e^{(m+1)(ax+by)} - 2B_2(m + 1)e^{(1-m)(ax+by)} \right] +$$

$$+ \frac{B_3 m^2(b^2 - a^2)}{R_e}(bx + ay) - \frac{2abm^2}{R_e}(B_3(ax + by) + B_4) -$$

$$- \frac{m^2(a^2 + b^2)}{(m^2 - 1)^2} e^{-2m(ax+by)} \left[Ume^{(1+m)(ax+by)} + B_1(m^2 - 1)e^{2m(ax+by)} - B_2(m^2 - 1) \right]^2 + B_7$$

$$T = \frac{E_c P_r m(a^2 + b^2)}{1 - m^2} \left[B_3 \left\{ Um^3 e^{m(ax+by)}(ax + by - 2) + B_1(m^2 - 1)(m(ax + by) - 2)e^{m(ax+by)} + \right. \right.$$

$$\left. + B_2(m^2 - 1)(m(ax + by) + 2)e^{-m(ax+by)} \right\} + mB_4 \left\{ Um^2 e^{m(ax+by)} + \right.$$

$$\left. + B_1(m^2 - 1)e^{m(ax+by)} + B_2(m^2 - 1) \right\} \right] + B_5(ax + by) + B_6$$

$$\mu = \frac{B_3(ax + by) + B_4}{B_1 e^{m(ax+by)} + B_2 e^{-m(ax+by)} + \frac{Ue^{ax+by}}{m^2 - 1}}$$



Case III: $\Lambda = 0$

$$u = C_1 b$$

$$v = -C_1 a$$

$$P = -\frac{(a^2 + b^2)C_1^2}{2} + C_7$$

$$T = C_5(ax + by) + C_6$$

$$\mu = \frac{C_3(ax + by) + C_4}{C_1(ax + by) + C_2 - Ue^{(ax+by)}}$$



Here is our general method of solution
for the *Navier-Stokes equations* :



NUMERICALLY CONTROLLED SYSTEM OF ANALYTICS TABLE

$G_k = 0$		
Initial condition of each auxiliary variable from the exact numerical solution sets of the nonlinear simultaneous equations	Coefficient values present in the system of PDEs	Exact analytical solutions obtained based on the universal differential representation of all mathematical equations
$(W_{0,1}, W_{0,2}, \dots, W_{0,12})_1$	$(R_{\theta}, P_r, E_c)_1$	$U_1 = 0$
$(W_{0,1}, W_{0,2}, \dots, W_{0,12})_2$	$(R_{\theta}, P_r, E_c)_2$	$U_2 = 0$
$(W_{0,1}, W_{0,2}, \dots, W_{0,12})_3$	$(R_{\theta}, P_r, E_c)_3$	$U_3 = 0$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮



We first seek to convert using our standard differential form all of the author's solution that satisfy the PDEs in their complete original format without involving any type of transformation processes whatsoever.

Case I: $\Lambda = -n^2$, $n > 0$

$$\begin{aligned} u &= \frac{Ubn^2}{n^2 + 1} e^{ax+by} - A_1nb\text{Sin}(n(ax + by) + A_2) \\ &= u_1 e^{u_2x+u_3y} + u_4\text{Sin}(u_5x + u_6y + u_7) \end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = u$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^{u_2x}, \quad \frac{dW_4}{W_4} = u_2 dx$$

$$W_5 = e^{u_3y}, \quad \frac{dW_5}{W_5} = u_3 dy$$

$$W_6 = \text{Tan}([u_5x + u_6y + u_7] / 2), \quad \frac{2dW_6}{1 + W_6^2} = u_5 dx + u_6 dy$$

so that:



(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5, W_6) = 0 = W_1 - u_1 W_4 W_5 + \frac{2u_4 W_6}{1 + W_6^2}$$

(2). Secondary Expansion:

$$du + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot du + dx + 0 \cdot dy = dW_2$$

$$0 \cdot du + 0 \cdot dx + dy = dW_3$$

$$0 \cdot du + u_2 dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot du + 0 \cdot dx + u_3 dy = \frac{dW_5}{W_5}$$

$$0 \cdot du + u_5 dx + u_6 dy = \frac{2dW_6}{1 + W_6^2}$$



$$\begin{aligned}
 v &= \frac{Uan^2}{n^2 + 1} e^{ax+by} + A_1 na \sin(n(ax + by) + A_2) \\
 &= v_1 e^{v_2x+v_3y} + v_4 \sin(v_5x + v_6y + v_7)
 \end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = v$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^{v_2x}, \quad \frac{dW_4}{W_4} = v_2 dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$

$$W_6 = \tan([v_5x + v_6y + v_7] / 2), \quad \frac{2dW_6}{1 + W_6^2} = v_5 dx + v_6 dy$$

so that:



(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5, W_6) = 0 = W_1 - v_1 W_4 W_5^{v_3} + \frac{2v_4 W_6}{1 + W_6^2}$$

(2). Secondary Expansion:

$$dv + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dv + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dv + 0 \cdot dx + dy = dW_3$$

$$0 \cdot dv + v_2 dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dv + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

$$0 \cdot dv + v_5 dx + v_6 dy = \frac{2dW_6}{1 + W_6^2}$$



$$\begin{aligned}
P &= \frac{Un^2(a^2 + b^2)}{2(n^2 + 1)} e^{ax+by} \left[Ue^{ax+by} - \{2A_1 \cos(n(ax + by) + A_2) + n\sin(n(ax + by) + A_2)\} \right] - \\
&\quad - \frac{A_3 n^2 (b^2 - a^2)}{R_e} (bx + ay) + \frac{2abn^2}{R_e} (A_3(ax + by) + A_4) - \\
&\quad - \frac{n^2(a^2 + b^2)}{2(n^2 + 1)^2} \left[Ue^{ax+by} - A_1(n^2 + 1)\sin(n(ax + by) + A_2) \right]^2 + A_7 \\
&= e^{P_1x + P_2y} \left[P_3 e^{P_4x + P_5y} + P_6 \cos(P_7x + P_8y + P_9) + P_{10} \sin(P_{11}x + P_{12}y + P_{13}) \right] + \\
&\quad + P_{14}x + P_{15}y + P_{16} + \left[P_{17} e^{P_{18}x + P_{19}y} + P_{20} \sin(P_{21}x + P_{22}y + P_{23}) \right]^2 + P_{24}
\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = P$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x, \quad \frac{dW_4}{W_4} = dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$



$$W_6 = \tan([P_7x + P_8y + P_9] / 2), \quad \frac{2dW_6}{1 + W_6^2} = P_7dx + P_8dy$$

$$W_7 = \tan([P_{11}x + P_{12}y + P_{13}] / 2), \quad \frac{2dW_7}{1 + W_7^2} = P_{11}dx + P_{12}dy$$

$$W_8 = \tan([P_{21}x + P_{22}y + P_{23}] / 2), \quad \frac{2dW_8}{1 + W_8^2} = P_{21}dx + P_{22}dy$$

so that:

(1). Primary Expansion:

$$F(W_1, W_2, \dots, W_8) = 0 = W_1 - W_4^{P_1} W_5^{P_2} \left[P_3 W_4^{P_4} W_5^{P_5} + P_6 \frac{1 - W_6^2}{1 + W_6^2} + \frac{2P_{10}W_7}{1 + W_7^2} \right] + P_{14}W_2 + P_{15}W_3 + P_{16} + \left[P_{17}W_4^{P_{18}} W_5^{P_{19}} + \frac{2P_{20}W_8}{1 + W_8^2} \right]^2 + P_{24}$$

(2). Secondary Expansion:

$$dP + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dP + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dP + 0 \cdot dx + dy = dW_3$$



$$0 \cdot dP + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dP + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

$$0 \cdot dP + P_7 dx + P_8 dy = \frac{2dW_6}{1 + W_6^2}$$

$$0 \cdot dP + P_{11} dx + P_{12} dy = \frac{2dW_7}{1 + W_7^2}$$

$$0 \cdot dP + P_{21} dx + P_{22} dy = \frac{2dW_8}{1 + W_8^2}$$

$$\begin{aligned} \mathbf{T} &= \frac{E_c P_r n (a^2 + b^2)}{n^2 + 1} \left[A_3 \left\{ Un^3 e^{ax+by} (ax + by - 2) + A_1 (n^2 + 1) \left[n(ax + by) \cos(n(ax + by) + A_2) - \right. \right. \right. \\ &\quad \left. \left. \left. - 2 \sin(n(ax + by) + A_2) \right] \right\} + nA_4 \left[Un^2 e^{ax+by} + A_1 (n^2 + 1) \cos(n(ax + by) + A_2) \right] \right] + \\ &\quad + A_5 (ax + by) + A_6 \\ &= \left[\left[T_1 e^{T_2 x + T_3 y} (T_4 x + T_5 y + T_6) + \left\{ (T_7 x + T_8 y) \cos(T_9 x + T_{10} y + T_{11}) + \right. \right. \right. \\ &\quad \left. \left. \left. + T_{12} \sin(T_{13} x + T_{14} y + T_{15}) \right\} \right] + \left[T_{16} e^{T_{17} x + T_{18} y} + T_{19} \cos(T_{20} x + T_{21} y + T_{22}) \right] \right] + \\ &\quad + T_{23} x + T_{24} y + T_{25} \end{aligned}$$



We can select as auxiliary variables the following expressions:

$$W_1 = T$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x, \quad \frac{dW_4}{W_4} = dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$

$$W_6 = \tan([T_9x + T_{10}y + T_{11}] / 2), \quad \frac{2dW_6}{1 + W_6^2} = T_9dx + T_{10}dy$$

$$W_7 = \tan([T_{13}x + T_{14}y + T_{15}] / 2), \quad \frac{2dW_7}{1 + W_7^2} = T_{13}dx + T_{14}dy$$

$$W_8 = \tan([T_{20}x + T_{21}y + T_{22}] / 2), \quad \frac{2dW_8}{1 + W_8^2} = T_{20}dx + T_{21}dy$$

so that:



(1). Primary Expansion:

$$F(W_1, W_2, \dots, W_8) = 0 = W_1 - \left[\left[T_1 W_4^{T_2} W_5^{T_3} (T_4 W_2 + T_5 W_3 + T_6) + \left\{ (T_7 W_2 + T_8 W_3) \frac{1 - W_6^2}{1 + W_6^2} + \frac{2T_{12} W_7}{1 + W_7^2} \right\} \right] + \left[T_{16} W_4^{T_{17}} W_5^{T_{18}} + T_{19} \frac{1 - W_8^2}{1 + W_8^2} \right] \right] + T_{23}x + T_{24}y + T_{25}$$

(2). Secondary Expansion:

$$dT + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dT + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dT + 0 \cdot dx + dy = dW_3$$

$$0 \cdot dT + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dT + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

$$0 \cdot dT + T_7 dx + T_8 dy = \frac{2dW_6}{1 + W_6^2}$$

$$0 \cdot dT + T_{11} dx + T_{12} dy = \frac{2dW_7}{1 + W_7^2}$$

$$0 \cdot dT + T_{21} dx + T_{22} dy = \frac{2dW_8}{1 + W_8^2}$$



$$\begin{aligned}\mu &= \frac{A_3(ax + by) + A_4}{A_1 \cos(n(ax + by) + A_2) - \frac{Ue^{ax+by}}{n^2 + 1}} \\ &= \frac{\mu_1 x + \mu_2 y + \mu_3}{\mu_4 \cos(\mu_5 x + \mu_6 y + \mu_7) + \mu_8 e^{\mu_9 x + \mu_{10} y}}\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = \mu$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x, \quad \frac{dW_4}{W_4} = dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$

$$W_6 = \tan([\mu_5 x + \mu_6 y + \mu_7] / 2), \quad \frac{2dW_6}{1 + W_6^2} = \mu_5 dx + \mu_6 dy$$

so that:



(1). Primary Expansion:

$$F(W_1, W_2, \dots, W_6) = 0 = W_1 - \frac{\mu_1 W_2 + \mu_2 W_3 + \mu_3}{\mu_4 \frac{1 - W_6^2}{1 + W_6^2} + \mu_8 W_4^{\mu_9} W_5^{\mu_{10}}}$$

(2). Secondary Expansion:

$$d\mu + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot d\mu + dx + 0 \cdot dy = dW_2$$

$$0 \cdot d\mu + 0 \cdot dx + dy = dW_3$$

$$0 \cdot d\mu + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot d\mu + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

$$0 \cdot d\mu + \mu_5 dx + \mu_6 dy = \frac{2dW_6}{1 + W_6^2}$$



Case II: $\Lambda = m^2, m > 0$

$$\begin{aligned} \mathbf{u} &= \frac{Ubm^2}{m^2 - 1} e^{ax+by} + B_1 m b e^{m(ax+by)} - B_2 m b e^{-m(ax+by)} \\ &= u_1 e^{u_2 x + u_3 y} + u_4 e^{u_5 x + u_6 y} + u_7 e^{u_8 x + u_9 y} \end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = u$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x$$

$$W_5 = e^y$$

so that:

(1). *Primary Expansion:*

$$F(W_1, W_2, W_3, W_4, W_5) = 0 = W_1 - u_1 W_4^{u_2} W_5^{u_3} + u_4 W_4^{u_5} W_5^{u_6} + u_7 W_4^{u_8} W_5^{u_9}$$



(2). Secondary Expansion:

$$du + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot du + dx + 0 \cdot dy = dW_2$$

$$0 \cdot du + 0 \cdot dx + dy = dW_3$$

$$0 \cdot du + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot du + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

$$v = -\frac{Uam^2}{m^2 - 1} e^{ax+by} - B_1 m a e^{m(ax+by)} + B_2 m a e^{-m(ax+by)}$$

$$= v_1 e^{v_2 x + v_3 y} + v_4 e^{v_5 x + v_6 y} + v_7 e^{v_8 x + v_9 y}$$

We can select as auxiliary variables the following expressions:

$$W_1 = v$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x$$

$$W_5 = e^y$$



(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4) = 0 = W_1 - v_1 W_4^{v_2} W_5^{v_3} + v_4 W_4^{v_5} W_5^{v_6} + v_7 W_4^{v_8} W_5^{v_9}$$

(2). Secondary Expansion:

$$dv + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dv + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dv + 0 \cdot dx + dy = dW_3$$

$$0 \cdot dv + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dv + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$



$$\begin{aligned}
P &= \frac{Um^2(a^2+b^2)}{2(m^2-1)} \left[Ue^{2(ax+by)} + 2B_1(m-1)e^{(m+1)(ax+by)} - 2B_2(m+1)e^{(1-m)(ax+by)} \right] + \\
&+ \frac{B_3m^2(b^2-a^2)}{R_e}(bx+ay) - \frac{2abm^2}{R_e}(B_3(ax+by) + B_4) - \\
&- \frac{m^2(a^2+b^2)}{(m^2-1)^2} e^{-2m(ax+by)} \left[Ume^{(1+m)(ax+by)} + B_1(m^2-1)e^{2m(ax+by)} - B_2(m^2-1) \right]^2 \\
&+ B_7 \\
&= P_1e^{P_2x+P_3y} + P_4e^{P_5x+P_6y} + P_7e^{P_8x+P_9y} + P_{10}x + P_{11}y + P_{12} + \\
&+ e^{P_{13}x+P_{14}y} \left[P_{15}e^{P_{16}x+P_{17}y} + P_{18}e^{P_{19}x+P_{20}y} + P_{21} \right]^2 + P_{22}
\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = P$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x$$

$$W_5 = e^y$$



so that:

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5) = 0 = W_1 - P_1 W_4^{P_2} W_5^{P_3} + P_4 W_4^{P_5} W_5^{P_6} + P_7 W_4^{P_8} W_5^{P_9} + P_{10} W_2 + P_{11} W_3 + \\ + P_{12} + W_4^{P_{13}} W_5^{P_{14}} \left[P_{15} W_4^{P_{16}} W_5^{P_{17}} + P_{18} W_4^{P_{19}} W_5^{P_{20}} + P_{21} \right]^2 + P_{22}$$

(2). Secondary Expansion:

$$dP + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dP + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dP + 0 \cdot dx + dy = dW_3$$

$$0 \cdot dP + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dP + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$



$$\begin{aligned}
\mathbf{T} &= \frac{E_c P_r m(a^2 + b^2)}{1 - m^2} \left[B_3 \left[Um^3 e^{m(ax+by)}(ax + by - 2) + B_1(m^2 - 1)(m(ax + by) - 2)e^{m(ax+by)} + \right. \right. \\
&\quad \left. \left. + B_2(m^2 - 1)(m(ax + by) + 2)e^{-m(ax+by)} \right] + mB_4 \left[Um^2 e^{m(ax+by)} + \right. \right. \\
&\quad \left. \left. + B_1(m^2 - 1)e^{m(ax+by)} + B_2(m^2 - 1) \right] \right] + B_5(ax + by) + B_6 \\
&= (T_1x + T_2y + T_3)e^{T_4x+T_5y} + (T_6x + T_7y + T_8)e^{T_9x+T_{10}y} + \\
&\quad + (T_{11}x + T_{12}y + T_{13})e^{T_{14}x+T_{15}y} + T_{16}e^{T_{17}x+T_{18}y} + (T_{19}x + T_{20}y + T_{21})
\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = T$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x$$

$$W_5 = e^y$$

so that:



(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4, W_5) = 0 = W_1 - (T_1W_2 + T_2W_3 + T_3)W_4^{T_4}W_5^{T_5} + (T_6W_2 + T_7W_3 + T_8)W_4^{T_9}W_5^{T_{10}} + \\ + (T_{11}W_2 + T_{12}W_3 + T_{13})W_4^{T_{14}}W_5^{T_{15}} + T_{16}W_4^{T_{17}}W_5^{T_{18}} + (T_{19}W_2 + T_{20}W_3 + T_{21})$$

(2). Secondary Expansion:

$$dT + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot dT + dx + 0 \cdot dy = dW_2$$

$$0 \cdot dT + 0 \cdot dx + dy = dW_3$$

$$0 \cdot dT + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot dT + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$



$$\begin{aligned}\mu &= \frac{B_3(ax + by) + B_4}{B_1e^{m(ax+by)} + B_2e^{-m(ax+by)} + \frac{Ue^{ax+by}}{m^2 - 1}} \\ &= \frac{\mu_1x + \mu_2y + \mu_3}{\mu_4e^{\mu_5x + \mu_6y} + \mu_7e^{\mu_8x + \mu_9y} + \mu_{10}e^{\mu_{11}x + \mu_{12}y}}\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = \mu$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x, \quad \frac{dW_4}{W_4} = dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$

so that:

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4) = 0 = W_1 - \frac{\mu_1W_1 + \mu_2W_2 + \mu_3}{\mu_4W_3^{\mu_5}W_4^{\mu_6} + \mu_7W_3^{\mu_8}W_4^{\mu_9} + \mu_{10}W_3^{\mu_{11}}W_4^{\mu_{12}}}$$



(2). Secondary Expansion:

$$d\mu + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot d\mu + dx + 0 \cdot dy = dW_2$$

$$0 \cdot d\mu + 0 \cdot dx + dy = dW_3$$

$$0 \cdot d\mu + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot d\mu + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$

Case III: $\Lambda = 0$

$$u = C_1 b$$

$$= u_1$$

We can select as auxiliary variable the following expression:

$$W_1 = u_1$$

so that:

(1). Primary Expansion:

$$F(W_1) = 0 = W_1 - u_1$$

(2). Secondary Expansion:

Undefined



$$\begin{aligned} v &= -C_1 a \\ &= v_1 \end{aligned}$$

We can select as auxiliary variable the following expression:

$$W_1 = v$$

so that:

(1). Primary Expansion:

$$F(W_1) = 0 = W_1 - v_1$$

(2). Secondary Expansion:

Undefined

$$\begin{aligned} P &= -\frac{(a^2 + b^2)C_1^2}{2} + C_7 \\ &= P_1 \end{aligned}$$

We can select as auxiliary variable the following expression:

$$W_1 = P$$

so that:



(1). Primary Expansion:

$$F(W_1) = 0 = W_1 - P_1$$

(2). Secondary Expansion:

Undefined

$$\begin{aligned} T &= C_5(ax + by) + C_6 \\ &= T_1x + T_2y + T_3 \end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = T$$

$$W_2 = x$$

$$W_3 = y$$

so that:

(1). Primary Expansion:

$$F(W_1, W_2) = 0 = W_1 - T_1W_2 + T_2W_3 + T_3$$

(2). Secondary Expansion:

$$dx = dW_1$$

$$dy = dW_2$$



$$\begin{aligned}\mu &= \frac{C_3(ax + by) + C_4}{C_1(ax + by) + C_2 - Ue^{(ax+by)}} \\ &= \frac{\mu_1x + \mu_2y + \mu_3}{\mu_4x + \mu_5y + \mu_6 + \mu_7e^{\mu_8x + \mu_9y}}\end{aligned}$$

We can select as auxiliary variables the following expressions:

$$W_1 = \mu$$

$$W_2 = x$$

$$W_3 = y$$

$$W_4 = e^x, \quad \frac{dW_4}{W_4} = dx$$

$$W_5 = e^y, \quad \frac{dW_5}{W_5} = dy$$

so that:

(1). Primary Expansion:

$$F(W_1, W_2, W_3, W_4) = 0 = W_1 - \frac{\mu_1W_2 + \mu_2W_3 + \mu_3}{\mu_4W_2 + \mu_5W_3 + \mu_6 + \mu_7W_4^{\mu_8}W_5^{\mu_9}}$$



(2). Secondary Expansion:

$$d\mu + 0 \cdot dx + 0 \cdot dy = dW_1$$

$$0 \cdot d\mu + dx + 0 \cdot dy = dW_2$$

$$0 \cdot d\mu + 0 \cdot dx + dy = dW_3$$

$$0 \cdot d\mu + dx + 0 \cdot dy = \frac{dW_4}{W_4}$$

$$0 \cdot d\mu + 0 \cdot dx + dy = \frac{dW_5}{W_5}$$



By visual inspection of the information produced, we would solve this problem using our method of differentials by selecting as a *minimum* " $p = 12$ ", " $u_p = 20$ ", " $u_s = 2$ " and " $q = 0$ " in our IAMPT.

where:

p = Total number of auxiliary variables

u_p = Total number of terms in the Primary Expansion

u_s = Total number of terms in the Secondary Expansion

q = Total number of auxiliary variables required for defining each functional expression that are present in the *original* PDEs.

In doing so, we can begin by assigning the following auxiliary variables to each dependent and independent variables as:

$u = W_1$

$v = W_2$

$P = W_3$

$T = W_4$

$\mu = W_5$

$x = W_6$

$y = W_7$



**COMPLETE IAMPT IN
EXPANDED FORM**



(1). Primary Expansion:

$$F_1 = 0 = a_{1,1}W_1^{m_{1,1}}W_6^{m_{1,2}} \dots W_{12}^{m_{1,8}} + a_{1,2}W_1^{m_{1,9}}W_6^{m_{1,10}} \dots W_{12}^{m_{1,16}} + \dots + \\ + \dots + a_{1,20}W_1^{m_{1,153}}W_6^{m_{1,154}} \dots W_{12}^{m_{1,160}}$$

$$F_2 = 0 = a_{2,1}W_2^{m_{2,1}}W_6^{m_{2,2}} \dots W_{12}^{m_{2,8}} + a_{2,2}W_2^{m_{2,9}}W_6^{m_{2,10}} \dots W_{12}^{m_{2,16}} + \dots + \\ + \dots + a_{2,20}W_2^{m_{2,153}}W_6^{m_{2,154}} \dots W_{12}^{m_{2,160}}$$

$$F_3 = 0 = a_{3,1}W_3^{m_{3,1}}W_6^{m_{3,2}} \dots W_{12}^{m_{3,8}} + a_{3,2}W_3^{m_{3,9}}W_6^{m_{3,10}} \dots W_{12}^{m_{3,16}} + \dots + \\ + \dots + a_{3,20}W_3^{m_{3,153}}W_6^{m_{3,154}} \dots W_{12}^{m_{3,160}}$$

$$F_4 = 0 = a_{4,1}W_4^{m_{4,1}}W_6^{m_{4,2}} \dots W_{12}^{m_{4,8}} + a_{4,2}W_4^{m_{4,9}}W_6^{m_{4,10}} \dots W_{12}^{m_{4,16}} + \dots + \\ + \dots + a_{4,20}W_4^{m_{4,153}}W_6^{m_{4,154}} \dots W_{12}^{m_{4,160}}$$

$$F_5 = 0 = a_{5,1}W_5^{m_{5,1}}W_6^{m_{5,2}} \dots W_{12}^{m_{5,8}} + a_{5,2}W_5^{m_{5,9}}W_6^{m_{5,10}} \dots W_{12}^{m_{5,16}} + \dots + \\ + \dots + a_{5,20}W_5^{m_{5,153}}W_6^{m_{5,154}} \dots W_{12}^{m_{5,160}}$$



(2). Secondary Expansion:

$$du = dW_1$$

$$dv = dW_2$$

$$dP = dW_3$$

$$dT = dW_4$$

$$d\mu = dW_5$$

$$dx = dW_6$$

$$dy = dW_7$$

$$N_1 dx + N_2 dy = N_3 dW_8$$

$$N_4 dx + N_5 dy = N_6 dW_9$$

$$N_7 dx + N_8 dy = N_9 dW_{10}$$

$$N_{10} dx + N_{11} dy = N_{12} dW_{11}$$

$$N_{13} dx + N_{14} dy = N_{15} dW_{12}$$

where :

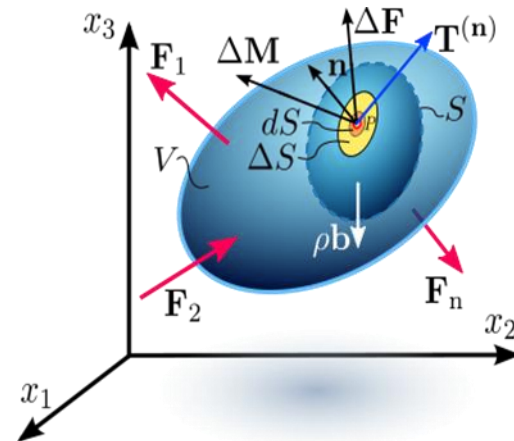
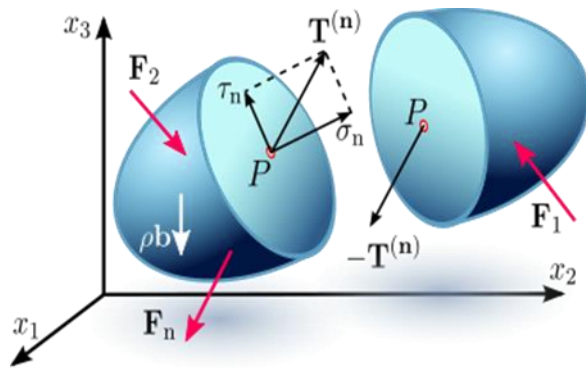


- In the *Secondary Expansion* of our Initially Assumed Universal Differential Form, the first set of auxiliary variables will be selected on the basis of representing the dependent and independent variables in that order.
- This will be followed by a series of other initially assumed auxiliary variables used for representing all basis functions in complete differential form that will be present in the final analytical solution of the system of PDEs.
- Regardless of the type of coordinate system used in our physical analysis, our Initially Assumed Universal Differential Expansion will be selected on the basis of solving the system of PDEs in terms of a system of **implicitly** defined equations that would consist only of the algebraic and elementary basis functions.
- In order to maximize our numerical solution rate of the *Nonlinear Simultaneous Equations*, we can set all the **initial values of each auxiliary variable** which would define the complete **Boundary Conditions** of the system of PDEs as part of the unknowns to solve for.
- Other unknowns to solve for are the *variable* coefficients from our system of PDEs defined in our NCSA table which would include the Reynolds number " R_e ", the Prandtl number " P_r " and the Eckert number " E_c ".
- Over time, the NCSA table should eventually succeed in capturing from the infinite number of possible numerical solution sets of the *Nonlinear Simultaneous Equations*, all those **exact Instance Analytical Solutions** that would conform with experimental results obtained under controlled laboratory conditions.
- Only through the gathering of this type of information over a span of say many years or even many decades that a large number of **generalized** analytical solutions may potentially be uncovered.
- This would lead to a far better understanding of general fluid behavior than having to depend solely on the use of experimental method of analysis or the numerical solutions of PDEs.

The unified theory of analytical integration can be converted into a **single major universal software** by which all DEs may be resolved under a single **common ideology**. Such a universal software development would be referred to as a **Numerically Controlled Analytics Software** or **NCAS** and would operate on the principle of determining the existence of **general analytical solutions** to DEs by method of **conjecture** that would be entirely driven by computational analysis. A far better alternative than having to maintain a large number of highly **dispersed** mathematical theories all of which could never be consolidated in terms of a **single universal software**.



Problem formulation for Linear Elastic Boundary Value Problems



In this section ...

- We will fundamentally illustrate the complete universality of our differential method of analysis beyond the Navier-Stokes equations.
- We will expand our method into the mechanics of materials.
- We will provide a more *universal* approach for attempting to solving *linear elastic boundary value problems*.
- These are governed by a system of PDEs defined in the three most popular coordinate systems.



Cartesian coordinates:

In cartesian coordinates the equations of motion are according to Slaughter, W. S., (2002), "The linearized theory of elasticity", Birkhauser :

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + P_x = \rho \frac{\partial^2 u_x}{\partial t^2}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + P_y = \rho \frac{\partial^2 u_y}{\partial t^2}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + P_z = \rho \frac{\partial^2 u_z}{\partial t^2}$$

where " P_i " are the external body forces, " ρ " the mass density and " u_i " the displacement.

The strain-displacement relations are:

$$\epsilon_x = \frac{\partial u_x}{\partial x} \quad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

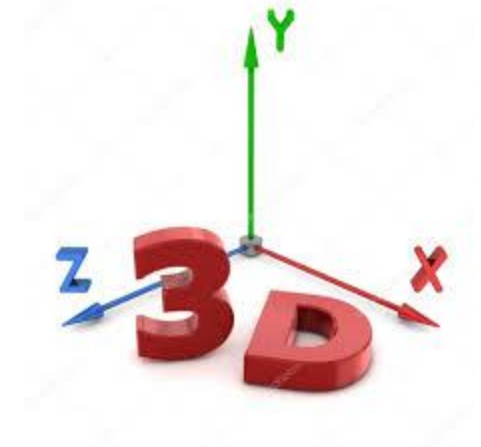
$$\epsilon_y = \frac{\partial u_y}{\partial y} \quad \gamma_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}$$

$$\epsilon_z = \frac{\partial u_z}{\partial z} \quad \gamma_{zx} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}$$

The constitutive equations are defined from Hooke's law which in tensor form is:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}$$

where " C_{ijkl} " is the stiffness tensor.



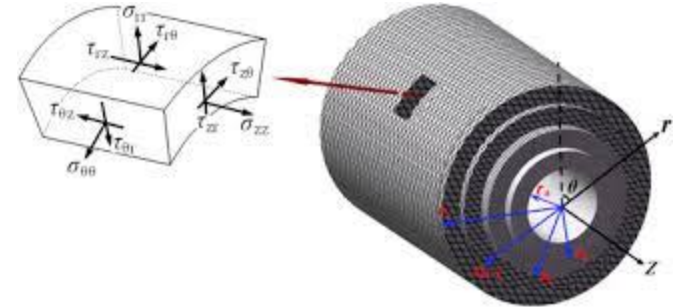
Cylindrical coordinates:

In cylindrical coordinates the equations of motion are:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) + P_r = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2}{r} \sigma_{r\theta} + P_\theta = \rho \frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r} \sigma_{rz} + P_z = \rho \frac{\partial^2 u_z}{\partial t^2} \varepsilon_{rr} = \frac{\partial u_r}{\partial r}$$



The strain-displacement relations are:

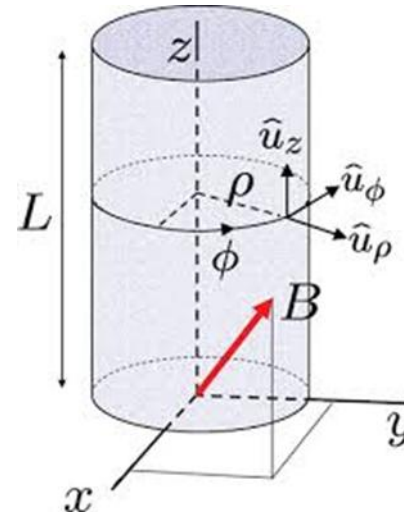
$$\varepsilon_{\theta\theta} = \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right)$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{r\theta} = \frac{1}{2} \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right)$$

$$\varepsilon_{\theta z} = \frac{1}{2} \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right)$$

$$\varepsilon_{zr} = \frac{1}{2} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right)$$



and the constitutive relations with a change of indices are the same as in the Cartesian coordinate system.



Spherical coordinates:

In spherical coordinates the equations of motion are :

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{r\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi} + \sigma_{r\theta} \cot(\theta) \right] + P_r = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\theta\phi}}{\partial \phi} + \frac{1}{r} \left[(\sigma_{\theta\theta} - \sigma_{\phi\phi}) \cot(\theta) + 3\sigma_{r\theta} \right] + P_\theta = \rho \frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{r\phi}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{\theta\phi} \cot(\theta) + 3\sigma_{r\phi} \right] + P_\phi = \rho \frac{\partial^2 u_\phi}{\partial t^2}$$

The strain-displacement relations are:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}$$

$$\varepsilon_{\theta\theta} = \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right)$$

$$\varepsilon_{\phi\phi} = \frac{1}{r \sin(\theta)} \left(\frac{\partial u_\phi}{\partial \phi} + u_r \sin(\theta) + u_\theta \cos(\theta) \right)$$

$$\varepsilon_{r\theta} = \frac{1}{2} \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right)$$

$$\varepsilon_{\theta\phi} = \frac{1}{2r} \left[\frac{1}{\sin(\theta)} \frac{\partial u_\theta}{\partial \phi} + \left(\frac{\partial u_\phi}{\partial \theta} - u_\phi \cot(\theta) \right) \right]$$

$$\varepsilon_{r\phi} = \frac{1}{2} \left[\frac{1}{r \sin(\theta)} \frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \right]$$



Here is our *general* method of solution for all *linear elastic boundary value problems*



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We will select the *Spherical Coordinate System* as the standard model for establishing a universal process that is based entirely on our method of differential analysis for attempting to solve PDEs in terms of *generalized* analytical solutions.



This would involve the extensive application of an *NCSA* table for the gathering of information that can eventually lead us to complete generalized analytical solutions satisfying an extremely wide range of boundary conditions.



Spherical coordinates:

In spherical coordinates the equations of motion are :

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{r\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi} + \sigma_{r\theta} \cot(\theta) \right] + P_r = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\theta\phi}}{\partial \phi} + \frac{1}{r} \left[(\sigma_{\theta\theta} - \sigma_{\phi\phi}) \cot(\theta) + 3\sigma_{r\theta} \right] + P_\theta = \rho \frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{r\phi}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{\theta\phi} \cot(\theta) + 3\sigma_{r\phi} \right] + P_\phi = \rho \frac{\partial^2 u_\phi}{\partial t^2}$$

The strain-displacement relations are:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}$$

$$\varepsilon_{\theta\theta} = \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right)$$

$$\varepsilon_{\phi\phi} = \frac{1}{r \sin(\theta)} \left(\frac{\partial u_\phi}{\partial \phi} + u_r \sin(\theta) + u_\theta \cos(\theta) \right)$$

$$\varepsilon_{r\theta} = \frac{1}{2} \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right)$$

$$\varepsilon_{\theta\phi} = \frac{1}{2r} \left[\frac{1}{\sin(\theta)} \frac{\partial u_\theta}{\partial \phi} + \left(\frac{\partial u_\phi}{\partial \theta} - u_\phi \cot(\theta) \right) \right]$$

$$\varepsilon_{r\phi} = \frac{1}{2} \left[\frac{1}{r \sin(\theta)} \frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \right]$$



We can begin by assigning the following auxiliary variables to each dependent and independent variables :

$$W_1 = z_1 = \sigma_{rr}$$

$$W_2 = z_2 = \sigma_{r\theta}$$

$$W_3 = z_3 = \sigma_{r\phi}$$

$$W_4 = z_4 = \sigma_{\theta\theta}$$

$$W_5 = z_5 = \sigma_{\theta\phi}$$

$$W_6 = z_6 = \sigma_{\phi\phi}$$

$$W_7 = z_7 = u_r$$

$$W_8 = z_8 = u_\theta$$

$$W_9 = z_9 = u_\phi$$

$$W_{10} = x_1 = r$$

$$W_{11} = x_2 = \theta$$

$$W_{12} = x_3 = \phi$$

$$W_{13} = x_4 = t$$

Because of the presence of the Cotangent function in the PDEs we can add the following new auxiliary variable to our IAMPT :

$$W_{p+1} = \text{Tan}(\theta)$$

where "p" is the total number of auxiliary variables defined from our IAMPT.



We can also define :

$$h_1 = \text{Cot}(\theta) = \frac{1}{W_{p+1}}$$

so that its *Multivariate Polynomial Transform* can be written as :

(1). Primary Expansion:

$$h_1 = \frac{1}{W_{p+1}}$$

(2). Secondary Differential Expansion:

$$(1 + W_{p+1}^2)dx_2 = dW_{p+1}$$

Now since :

$$\text{Sin}(\theta) = \frac{2 \text{Tan}(\theta/2)}{1 + \text{Tan}^2(\theta/2)}$$

then we can add the following second auxiliary variable to account for the presence of the "Sin(θ)" function in the PDEs :

$$W_{p+2} = \text{Tan}(\theta/2)$$



We can let:

$$h_2 = \sin(\theta) = \frac{2 W_{p+2}}{1 + W_{p+2}^2}$$

so that its *Multivariate Polynomial Transform* can be written as :

(1). Primary Expansion:

$$h_2 = \frac{2 W_{p+2}}{1 + W_{p+2}^2}$$

(2). Secondary Differential Expansion:

$$(1 + W_{p+2}^2) dx_2 = 2 dW_{p+2}$$

We will use the following notation for defining the multivariate polynomials that corresponds to the various partial derivatives of each dependent variable with respect to the independent variables under the application of the chain rule for $(i \leq 9)$ and $(1 \leq j \leq 4)$:

$$\frac{P_{nij}}{Q_{nij}} = \frac{\partial^n z_i}{\partial x_j^n}$$



Spherical coordinates:

In spherical coordinates the equations of motion are :

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{r\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi} + \sigma_{r\theta} \cot(\theta) \right] + P_r = \rho \frac{\partial^2 u_r}{\partial t^2}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\theta\phi}}{\partial \phi} + \frac{1}{r} \left[(\sigma_{\theta\theta} - \sigma_{\phi\phi}) \cot(\theta) + 3\sigma_{r\theta} \right] + P_\theta = \rho \frac{\partial^2 u_\theta}{\partial t^2}$$

$$\frac{\partial \sigma_{r\phi}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin(\theta)} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{1}{r} \left[2\sigma_{\theta\phi} \cot(\theta) + 3\sigma_{r\phi} \right] + P_\phi = \rho \frac{\partial^2 u_\phi}{\partial t^2}$$

The Primary Expansion of the entire system of second order PDEs may now be completely defined as :

$$\frac{P_{111}}{Q_{111}} + \frac{1}{W_{10}} \frac{P_{122}}{Q_{122}} + \frac{(1 + W_{p+2}^2)}{2 W_{p+2} W_{10}} \frac{P_{133}}{Q_{133}} + \frac{1}{W_{10}} \left[2W_1 - W_4 - W_6 + W_2 \frac{1}{W_{p+1}} \right] + P_r^\wedge = \rho \frac{P_{274}}{Q_{274}}$$

$$\frac{P_{121}}{Q_{121}} + \frac{1}{W_{10}} \frac{P_{142}}{Q_{142}} + \frac{(1 + W_{p+2}^2)}{2 W_{p+2} W_{10}} \frac{P_{153}}{Q_{153}} + \frac{1}{W_{10}} \left[(W_4 - W_6) \frac{1}{W_{p+1}} + 3W_2 \right] + P_\theta^\wedge = \rho \frac{P_{284}}{Q_{284}}$$

$$\frac{P_{131}}{Q_{131}} + \frac{1}{W_{10}} \frac{P_{152}}{Q_{152}} + \frac{(1 + W_{p+2}^2)}{2 W_{p+2} W_{10}} \frac{P_{163}}{Q_{163}} + \frac{1}{W_{10}} \left[2W_5 \frac{1}{W_{p+1}} + 3W_3 \right] + P_\phi^\wedge = \rho \frac{P_{294}}{Q_{294}}$$

where " P_r^\wedge ", " P_θ^\wedge " and " P_ϕ^\wedge " each represent the Primary Expansion of the Multivariate Polynomial Transform corresponding to each of the body forces " P_r ", " P_θ " and " P_ϕ " respectively.



The Secondary Expansion of the PDEs is exactly identical to the one defined in our IAMPT where "q = 2" to account for the presence of both trigonometric functions :

(2). Secondary Expansion:

$$d\sigma_{rr} = dz_1 = dW_1$$

$$d\sigma_{r\theta} = dz_2 = dW_2$$

$$d\sigma_{r\phi} = dz_3 = dW_3$$

$$d\sigma_{\theta\theta} = dz_4 = dW_4$$

$$d\sigma_{\theta\phi} = dz_5 = dW_5$$

$$d\sigma_{\phi\phi} = dz_6 = dW_6$$

$$du_r = dz_7 = dW_7$$

$$du_\theta = dz_8 = dW_8$$

$$du_\phi = dz_9 = dW_9$$

$$dr = dx_1 = dW_{10}$$

$$d\theta = dx_2 = dW_{11}$$

$$d\phi = dx_3 = dW_{12}$$

$$dt = dx_4 = dW_{13}$$



$$\sum_{t=1}^m N_{(i-1)(m+n+1)+t} dz_t + \sum_{t=1}^n N_{i(m+n+1)-n-1+t} dx_t =$$

$$= N_{i(m+n+1)} dW_j \quad \begin{array}{l} (1 \leq i \leq p+2-m-n) \\ (m+n+1 \leq j \leq p+2) \end{array}$$

$$(1 + W_{p+1}^2) dx_2 = dW_{p+1}$$

$$(1 + W_{p+2}^2) dx_2 = 2 dW_{p+2}$$

Having defined the complete Multivariate Polynomial Transform of the PDEs, we are now in the position of defining the complete IAMPT that will be used for solving the PDEs.

(1). Primary Expansion:

$$F_i(W_1, W_2, \dots, W_p, W_{p+1}, W_{p+2}) = 0 = \sum_t^r a_{i,t} \left(\prod_j^{p+2} W_j^{E_{i,kj}} \right) \quad (1 \leq i \leq 9)$$

(2). Secondary Expansion:

Exactly the same as above



CORRESPONDING NCSA TABLE

$G_k = 0$		
Initial condition of each auxiliary variable from the exact numerical solution set of the nonlinear simultaneous equations	Coefficient values present in the system of PDEs	Exact analytical solutions obtained based on the universal differential representation of all mathematical equations
$(W_{0,1}, W_{0,2}, \dots, W_{0,13})_1$	ρ_1	$U_1 = 0$
$(W_{0,1}, W_{0,2}, \dots, W_{0,13})_2$	ρ_2	$U_2 = 0$
$(W_{0,1}, W_{0,2}, \dots, W_{0,13})_3$	ρ_3	$U_3 = 0$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮



Our NCSA table must be sufficiently populated with a fairly large number of exact expressions for "U_i".

Afterwards, the entire computational process becomes entirely transformed into a purely deductive method of reasoning by attempting to establish a general analytical solution of the PDEs satisfying a wide range of general boundary conditions.





Simple demonstration in Cosmology for Einstein Field Equations



What are the *Einstein Field Equations* used for ?

- Space and time is know to be entirely influenced by mass and energy so *Einstein Field Equations* describes its fundamental interaction that exists with gravitation.
- We are interested in determining locally the space-time geometry being the results of the presence of mass-energy and linear momentum
- Very similar to determining the field in the form of electromagnetic energy from charges and currents using Maxwell's equations.
- The original equations are formulated in terms of tensors that results into 16 coupled hyperbolic-elliptic nonlinear partial differential equations (ignoring the effect of symmetry).
- When velocities are much less than the speed of light the *Einstein Field Equations* reduce to Newton's law of gravitation



In simple layman's term how are they derived ?

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

MATTER:

Starting with the right hand side of this tensor equation, the energy-momentum tensor " $T_{\mu\nu}$ " being multiplied by some set of fundamental constants " $\frac{8\pi G}{c^4}$ " encodes exactly how matter is distributed in this universe

GEOMETRY:

On the left hand side of the equation is purely geometry for describing space-time based on the use of a metric tensor (from differential geometry) for defining the complete geometry of our manifold. We should be able to extract all the relevant information about the curvature of the manifold just from the metric tensor. This is done by constructing the Riemann (curvature) tensor which holds pretty much all the information about the curvature of the manifold



Why are *Einstein Field Equations* so important in physics ?

- Special classes of exact solutions to these PDEs can lead to various models of gravitational phenomena, such as rotating black holes and the expanding universe and even gravitational waves.

Existing well known analytical solutions correspond to the following special cases of *Einstein Field Equations* :

- **Vacuum solutions**: these describe regions in which no matter or no gravitational fields are present,
- **Electro vacuum solutions**: must arise entirely from an electromagnetic field which solves the *source-free* Maxwell equations on the given curved Lorentzian manifold; this means that the only source for the gravitational field is the field energy (and momentum) of the electromagnetic field,
- **Null dust solutions**: must correspond to a stress–energy tensor which can be interpreted as arising from incoherent electromagnetic radiation, without necessarily solving the Maxwell field equations on the given Lorentzian manifold,
- **Fluid solutions**: must arise entirely from the stress–energy tensor of a fluid (often taken to be a perfect fluid); the only source for the gravitational field is the energy, momentum, and stress (pressure and shear stress) of the matter comprising the fluid.
- In addition to such well established phenomena as fluids or electromagnetic waves, one can contemplate models in which the gravitational field is produced entirely by the field energy of various exotic hypothetical fields:
- **Scalar field solutions**: must arise entirely from a scalar field (often a massless scalar field); these can arise in classical field theory treatments of meson beams, or as quintessence,
- **Lambda vacuum solutions** (not a standard term, but a standard concept for which no name yet exists): arises entirely from a nonzero cosmological constant.



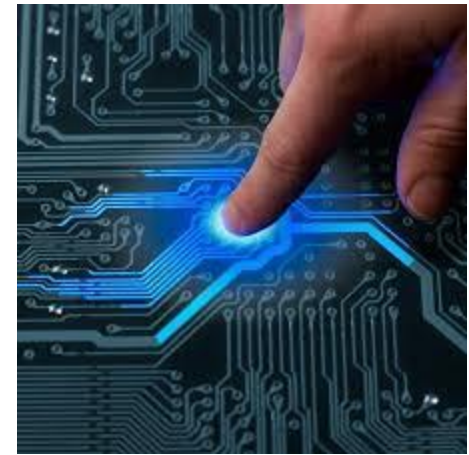
Just as with the Navier-Stokes equations and the basic equations of elasticity, there are just an incredible number of physical scenarios that can be constructed just by solving for all the corresponding PDEs.

In the case of *Einstein Field Equations* they are all important for understanding the basic physical properties of our universe which even makes it more imperative for solving them only in their complete original form without the use of any type of transformation processes whatsoever !

$G_k = 0$		
Initial Conditions	Coefficient values present in the DE or system of DEs	Exact analytical solution obtained using the Multivariate Polynomial Transform method
$z_{01}, z_{02}, \dots, z_{0m}, x_{01}, \dots, x_{0n} \dots$	a_0, b_0, c_0, \dots	$U_1 = 0$
$z_{11}, z_{12}, \dots, z_{1m}, x_{11}, \dots, x_{1n} \dots$	a_1, b_0, c_0, \dots	$U_2 = 0$
$z_{21}, z_{22}, \dots, z_{2m}, x_{21}, \dots, x_{2n} \dots$	a_0, b_1, c_0, \dots	$U_3 = 0$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮



Exact computational method for calculating the various derivatives and partial derivatives of an Initially Assumed Multivariate Polynomial Transform



- Determining the various derivatives of a product of several expressions is similar to algebraically expanding to some exponent value the sum of several terms.
- Only major difference between the two is that in the case of differentiation, exponentiation becomes treated as an order of differentiation while all other algebraic operations remain completely identical.

Simple case of a product of two functions.

$$\frac{d^n}{dx^n} fg = \sum_{k=0}^n \binom{n}{k} f^{(k)} g^{(n-k)}$$

where:

$$\binom{n}{k} = B_{n,k} = \frac{n!}{k!(n-k)!}$$

is defined as the Binomial Coefficients

In complete expanded form using a special set of notations to symbolize differentiation:

$$[f + g]^{(n)} = f^{(0)} g^{(n)} + B_{n-1,1} f^{(1)} g^{(n-1)} + B_{n-2,2} f^{(2)} g^{(n-2)} + \dots + f^{(n)} g^{(0)}$$



When there are more than two functional expressions involved, we instead use the *Multinomial Expansion Theorem* :

$$(a_1 + a_2 + \dots + a_k)^n = \sum_{\substack{n_1, n_2, \dots, n_k \geq 0 \\ n_1 + n_2 + \dots + n_k = n}} \frac{n!}{n_1! n_2! \dots n_k!} a_1^{n_1} a_2^{n_2} \dots a_k^{n_k}$$

where “ $n = n_1 + n_2 + \dots + n_k$ ”

In terms of differentiation and using our special notations:

$$\begin{aligned} \frac{d^n}{dx^n} (f_1 f_2 \dots f_k) &= [f_1 + f_2 + \dots + f_k]^{(n)} \\ &= \sum_{\substack{n_1, n_2, \dots, n_k \geq 0 \\ n_1 + n_2 + \dots + n_k = n}} \frac{n!}{n_1! n_2! \dots n_k!} f_1^{(n_1)} f_2^{(n_2)} \dots f_k^{(n_k)} \end{aligned}$$



$$y = e^{2x} = e^{-x} e^{0.5x} e^{2.5x}$$

Now define:

$$f_1 = e^{-x}, \quad f_2 = e^{0.5x} \quad \text{and} \quad f_3 = e^{2.5x}$$

so that each of their individual derivative at "x=0.5" up to a maximum order of 5 is calculated as follow:

$$f_1^{(0)} = e^{-x}, \quad f_2^{(0)} = e^{0.5x} \quad \text{and} \quad f_3^{(0)} = e^{2.5x}$$

$$f_1^{(1)} = -e^{-x}, \quad f_2^{(1)} = 0.5e^{0.5x} \quad \text{and} \quad f_3^{(1)} = 2.5e^{2.5x}$$

$$f_1^{(2)} = e^{-x}, \quad f_2^{(2)} = 0.25e^{0.5x} \quad \text{and} \quad f_3^{(2)} = 6.25e^{2.5x}$$

$$f_1^{(3)} = -e^{-x}, \quad f_2^{(3)} = 0.125e^{0.5x} \quad \text{and} \quad f_3^{(3)} = 15.625e^{2.5x}$$

$$f_1^{(4)} = e^{-x}, \quad f_2^{(4)} = 0.0625e^{0.5x} \quad \text{and} \quad f_3^{(4)} = 39.0625e^{2.5x}$$

$$f_1^{(5)} = -e^{-x}, \quad f_2^{(5)} = 0.03125e^{0.5x} \quad \text{and} \quad f_3^{(5)} = 97.65625e^{2.5x}$$

At "x = 0.5" we thus have:

$$f_1^{(0)} = e^{-0.5} = 0.607, \quad f_2^{(0)} = e^{0.25} = 1.284 \quad \text{and} \quad f_3^{(0)} = e^{1.25} = 3.490$$

$$f_1^{(1)} = -e^{-0.5} = -0.607, \quad f_2^{(1)} = 0.5e^{0.25} = 0.642 \quad \text{and} \quad f_3^{(1)} = 2.5e^{1.25} = 8.726$$

$$f_1^{(2)} = e^{-0.5} = 0.607, \quad f_2^{(2)} = 0.25e^{0.25} = 0.321 \quad \text{and} \quad f_3^{(2)} = 6.25e^{1.25} = 21.815$$

$$f_1^{(3)} = -e^{-0.5} = -0.607, \quad f_2^{(3)} = 0.125e^{0.25} = 0.161 \quad \text{and} \quad f_3^{(3)} = 15.625e^{1.25} = 54.537$$

$$f_1^{(4)} = e^{-0.5} = 0.607, \quad f_2^{(4)} = 0.0625e^{0.25} = 0.080 \quad \text{and} \quad f_3^{(4)} = 39.0625e^{1.25} = 136.342$$

$$f_1^{(5)} = -e^{-0.5} = -0.607, \quad f_2^{(5)} = 0.03125e^{0.25} = 0.040 \quad \text{and} \quad f_3^{(5)} = 97.65625e^{1.25} = 340.854$$



Using our special notation:

$$\frac{d^5 y}{dx^5} = [f_1 + f_2 + f_3]^{(5)} = \sum_{\substack{n_1, n_2, n_3 \geq 0 \\ n_1 + n_2 + n_3 = 5}} \frac{n!}{n_1! n_2! n_3!} f_1^{(n_1)} f_2^{(n_2)} f_3^{(n_3)}$$

$$= (1)(-0.607)(1.284)(3.490) + (5)(0.607)(0.642)(3.490) + (10)(-0.607)(0.321)(3.490) + (10)(0.607)(0.161)(3.490) +$$

$$(5)(-0.607)(0.080)(3.490) + (1)(0.607)(0.040)(3.490) + (5)(0.607)(1.284)(8.726) + (20)(-0.607)(0.642)(8.726) +$$

$$(30)(0.607)(0.321)(8.726) + (20)(-0.607)(0.161)(8.726) + (5)(0.607)(0.080)(8.726) + (10)(-0.607)(1.284)(21.815) +$$

$$(30)(0.607)(0.642)(21.815) + (30)(-0.607)(0.321)(21.815) + (10)(0.607)(0.161)(21.815) + (10)(0.607)(1.284)(54.537) +$$

$$(20)(-0.607)(0.642)(54.537) + (10)(0.607)(0.321)(54.537) + (5)(-0.607)(1.284)(136.342) + (5)(0.607)(0.642)(136.342) +$$

$$(1)(0.607)(1.284)(340.854)$$

where there are a total number of 21 terms satisfying the criteria that " $n_1, n_2, n_3 \geq 0$ " and " $n_1 + n_2 + n_3 = 5$ "

By writing a short computer program for performing the above arithmetical operation but with higher precision, the value obtained based on the *Multinomial Expansion Theorem* was determined as **"86.985019"**.

The 5th derivative of " e^{2x} " is " $2^5 e^{2x}$ " so that at " $x = 0.5$ " this value becomes $32e^{2(0.5)} = 32e = 86.98501851$ which is the same value as the one computed using the *Multinomial Expansion Theorem* above.



We would like to expand our set of notations for calculating the various partial derivatives of a product of several multivariate expressions.

$$\frac{\partial^{m_1}}{\partial x_1^{m_1}} \frac{\partial^{m_2}}{\partial x_2^{m_2}} \frac{\partial^{m_3}}{\partial x_3^{m_3}} \cdots \frac{\partial^{m_k}}{\partial x_j^{m_k}} [f_1(x_1, x_2, \dots, x_j) \cdot f_2(x_1, x_2, \dots, x_j) \cdots f_i(x_1, x_2, \dots, x_j)]$$

Expanding our set of notations for including the general multivariate case:

$$\begin{aligned} & \left[f_1^{(0)} + f_2^{(0)} + \dots + f_i^{(0)} \right]_{1(m_1)}^{(m_1)} \Delta \left[f_1^{(0)} + f_2^{(0)} + \dots + f_i^{(0)} \right]_{2(m_2)}^{(m_2)} \Delta \cdots \Delta \\ & \Delta \cdots \Delta \left[f_1^{(0)} + f_2^{(0)} + \dots + f_i^{(0)} \right]_{j(m_k)}^{(m_k)} \end{aligned}$$

In terms of the *Multinomial Expansion Theorem* this can be rewritten as:

$$\begin{aligned} & \left[\sum_{\substack{n_1, n_2, \dots, n_i \geq 0 \\ n_1 + n_2 + \dots + n_i = m_1}} \frac{n!}{n_1! n_2! \cdots n_i!} f_{1,1}^{(n_1)} f_{2,1}^{(n_2)} \cdots f_{i,1}^{(n_i)} \right] \Delta \\ & \left[\sum_{\substack{n_1, n_2, \dots, n_i \geq 0 \\ n_1 + n_2 + \dots + n_i = m_2}} \frac{n!}{n_1! n_2! \cdots n_i!} f_{1,2}^{(n_1)} f_{2,2}^{(n_2)} \cdots f_{i,2}^{(n_i)} \right] \Delta \cdots \Delta \\ & \left[\sum_{\substack{n_1, n_2, \dots, n_i \geq 0 \\ n_1 + n_2 + \dots + n_i = m_k}} \frac{n!}{n_1! n_2! \cdots n_i!} f_{1,j}^{(n_1)} f_{2,j}^{(n_2)} \cdots f_{i,j}^{(n_i)} \right] \end{aligned}$$



$$\begin{aligned}\frac{\partial^3 f_1 f_2}{\partial x_1 \partial x_2^2} &= [f_1 + f_2]_{1(1)}^{(1)} \Delta [f_1 + f_2]_{2(2)}^{(2)} \\ &= [f_{1,1(1)}^{(1)} + f_{2,1(1)}^{(1)}] \Delta [f_{1,2(2)}^{(2)} + 2f_{1,2(1)}^{(1)}f_{2,2(1)}^{(1)} + f_{2,2(2)}^{(2)}]\end{aligned}$$

Algebraically performing a term by term symbolic multiplication by treating all exponent values as order of differentiation.

$$\begin{aligned} &= f_{1,1(1)}^{(1)}f_{1,2(2)}^{(2)} + 2f_{1,1(1)}^{(1)}f_{1,2(1)}^{(1)}f_{2,2(1)}^{(1)} + f_{1,1(1)}^{(1)}f_{2,2(2)}^{(2)} + \\ &\quad + f_{2,1(1)}^{(1)}f_{1,2(2)}^{(2)} + 2f_{2,1(1)}^{(1)}f_{1,2(1)}^{(1)}f_{2,2(1)}^{(1)} + f_{2,1(1)}^{(1)}f_{2,2(2)}^{(2)}\end{aligned}$$

which in the conventional symbolic form may be translated as:

$$= \frac{\partial^3 f_1}{\partial x_1 \partial x_2^2} + 2 \frac{\partial^2 f_1}{\partial x_1 \partial x_2} \frac{\partial f_2}{\partial x_2} + \frac{\partial f_1}{\partial x_1} \frac{\partial^2 f_2}{\partial x_2^2} + \frac{\partial^2 f_1}{\partial x_2^2} \frac{\partial f_2}{\partial x_1} + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \frac{\partial^3 f_2}{\partial x_1 \partial x_2^2}$$

Each term in the expansion must always contain the two functions that is being differentiated

$$= \frac{\partial^3 f_1}{\partial x_1 \partial x_2^2} \mathbf{f_2} + 2 \frac{\partial^2 f_1}{\partial x_1 \partial x_2} \frac{\partial f_2}{\partial x_2} + \frac{\partial f_1}{\partial x_1} \frac{\partial^2 f_2}{\partial x_2^2} + \frac{\partial^2 f_1}{\partial x_2^2} \frac{\partial f_2}{\partial x_1} + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \mathbf{f_1} \frac{\partial^3 f_2}{\partial x_1 \partial x_2^2}$$



$$= \frac{\partial^3 f_1}{\partial x_1 \partial x_2^2} f_2 + 2 \frac{\partial^2 f_1}{\partial x_1 \partial x_2} \frac{\partial f_2}{\partial x_2} + \frac{\partial f_1}{\partial x_1} \frac{\partial^2 f_2}{\partial x_2^2} + \frac{\partial^2 f_1}{\partial x_2^2} \frac{\partial f_2}{\partial x_1} + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + f_1 \frac{\partial^3 f_2}{\partial x_1 \partial x_2^2}$$

We can validate the use of our symbolic notations by performing the same operation manually and compare the results with the one obtained in the above equation:

$$\frac{\partial^2 f_1 f_2}{\partial x_2^2} = \frac{\partial}{\partial x_2} \left(\frac{\partial f_1}{\partial x_2} f_2 + f_1 \frac{\partial f_2}{\partial x_2} \right) = \frac{\partial^2 f_1}{\partial x_2^2} f_2 + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial f_2}{\partial x_2} + f_1 \frac{\partial^2 f_2}{\partial x_2^2}$$

$$\frac{\partial^3 f_1 f_2}{\partial x_1 \partial x_2^2} = \frac{\partial}{\partial x_1} \left(\frac{\partial^2 f_1 f_2}{\partial x_2^2} \right) = \frac{\partial}{\partial x_1} \left(\frac{\partial^2 f_1}{\partial x_2^2} f_2 + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial f_2}{\partial x_2} + f_1 \frac{\partial^2 f_2}{\partial x_2^2} \right)$$

$$= \frac{\partial^3 f_1}{\partial x_1 \partial x_2^2} f_2 + \frac{\partial^2 f_1}{\partial x_2^2} \frac{\partial f_2}{\partial x_1} + 2 \frac{\partial^2 f_1}{\partial x_1 \partial x_2} \frac{\partial f_2}{\partial x_2} + 2 \frac{\partial f_1}{\partial x_2} \frac{\partial^2 f_2}{\partial x_1 \partial x_2} + \frac{\partial f_1}{\partial x_1} \frac{\partial^2 f_2}{\partial x_2^2} + f_1 \frac{\partial^3 f_2}{\partial x_1 \partial x_2^2}$$

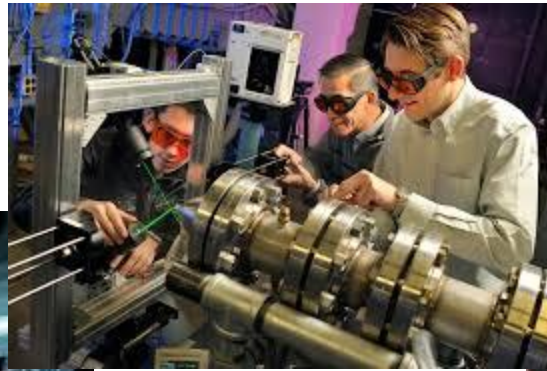
- It is by the use of these special notations that the entire process of calculating the various EXACT partial derivatives of a product involving any number of multivariate functions can be reduced entirely to a computational level.



- The highly computational nature of the universal differential expansion makes it very difficult for conducting any real meaningful numerical experimentations even for solving the simplest type of DEs.
- Super computers are by far more suitable for this type of high level and very advanced form of computational analysis.
- The advent of Quantum computers in the near future could significantly improve the performance of handling even the most complex systems of PDEs.
- They would by far exceed the capabilities of even our most powerful super computer of our time because they would operate entirely on the fundamental principles of Quantum theory which is based on the study of energy at the atomic and subatomic level.
- Such advanced computer technology would allow for the capability of performing multiple tasks in parallel thereby resulting in a significant increase in the billion-fold when compared to conventional computer systems.



Major importance of a unified theory of integration for the physical sciences



- Unified theory of analytical integration can be converted into a *single major universal software* by which all DEs may be resolved under a single *common ideology*.
- Such a universal software development would be referred to as a "*Numerically Controlled Analytics Software*" or *NCAS*.
- It would operate on the principle of determining the existence of *general analytical solutions* to DEs using the method of *conjecture* that would be *entirely driven* by computational analysis.
- Much better alternative than having to maintain a large number of highly *dispersed* mathematical theories all of which could never be consolidated in terms of a *single universal software*.



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