Stability Analysis of RED Gateway with Multiple TCP Reno Connections

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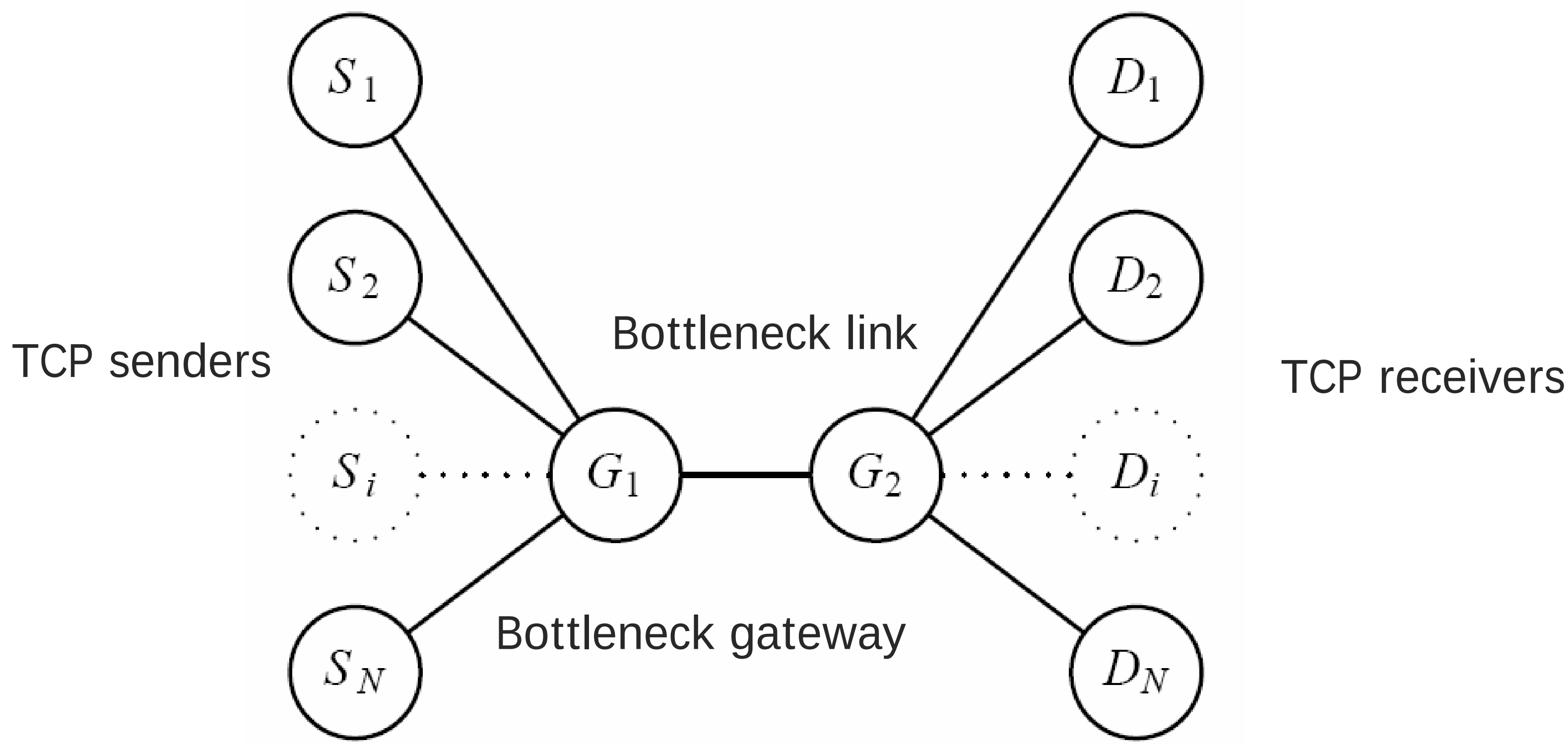
Presented by Xi Chen

Outline

- Overview of TCP-RED (Random Early Detection) systems
- Fluid-flow model of TCP-RED
- Stability boundaries for TCP-RED
- Numerical verification
- Conclusions and future work

Overview



S_1
S_2
S_i
S_N
G_1
G_2
D_1
D_2
D_i
D_N
TCP senders
Bottleneck link
Bottleneck gateway
TCP receivers

TCP Window Congestion Control algorithm

Sender sends *W* packets at a time

Window size = *W*

- Additive increase (AI):
  window size increases by one per round trip time if no loss
- Multiplicative decrease (MD):
  window size decreases by half on detection of loss

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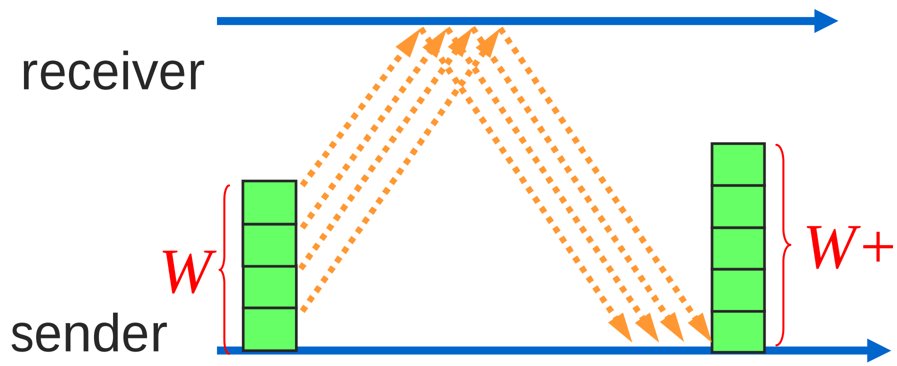

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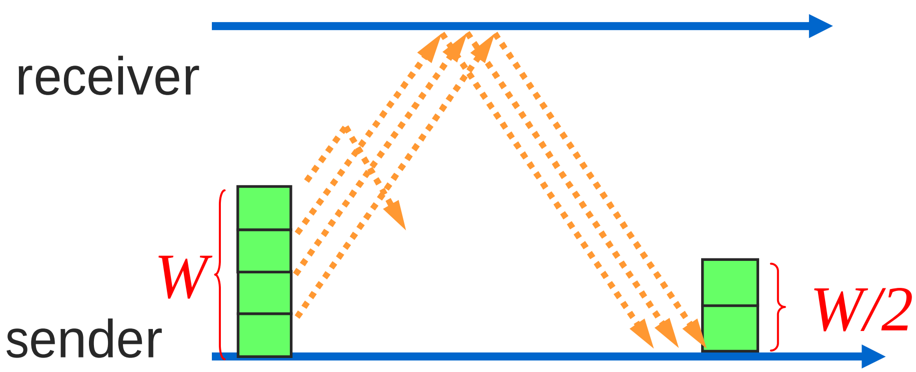
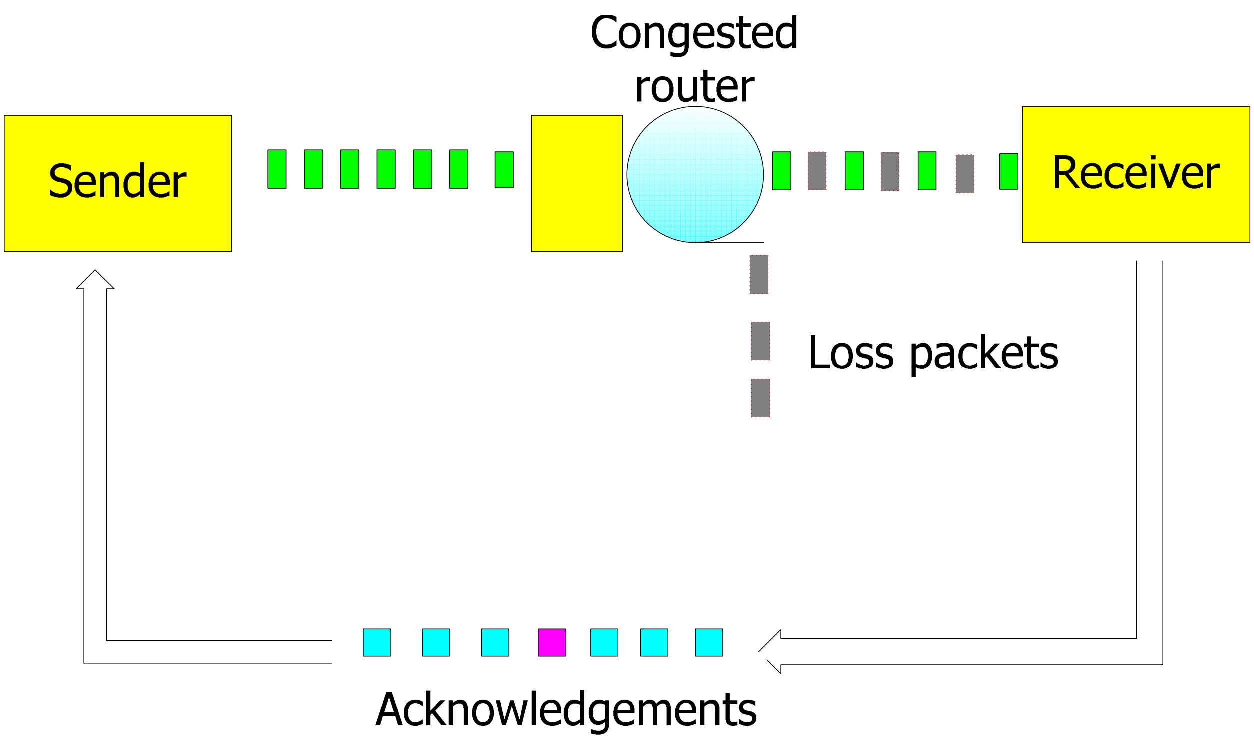
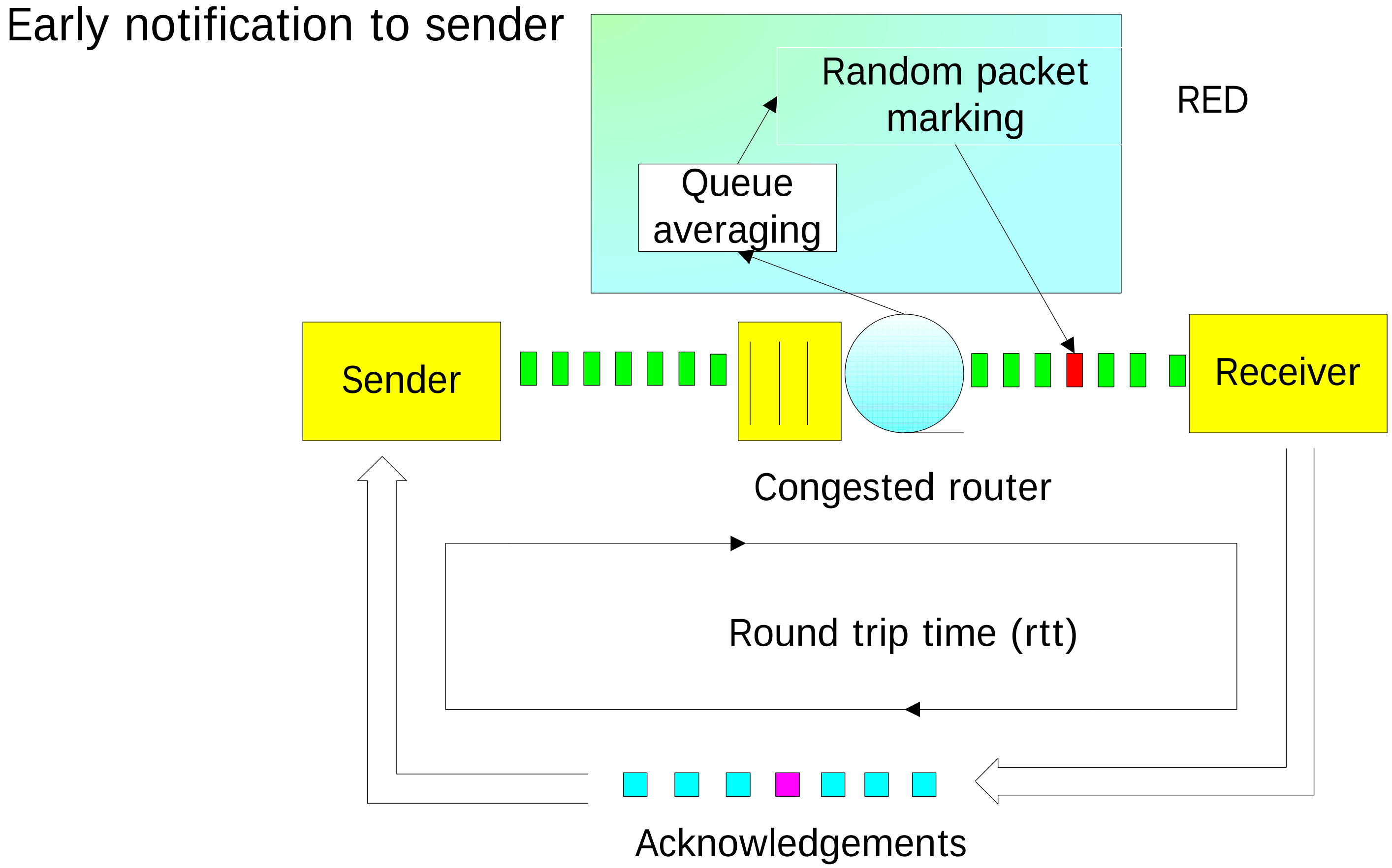


Sender-receiver connection without RED



Sender-receiver connection with RED



Fluid-flow model of TCP-RED (1)

Window size:

$$\frac{dw(t)}{dt} = \frac{1}{r(t)} - \frac{w(t)}{2} \cdot \frac{w(t-r(t))}{r(t-r(t))} p(t-r(t))$$

Additive increase — Multiplicative decrease — Loss arrival rate

Instantaneous queue length:

$$\frac{dq(t)}{dt} = N\frac{w(t)}{r(t)} - C$$

Incoming traffic — Outgoing traffic

Round trip time:

$$r(t) = \frac{q(t)}{C} + R_0$$

Queuing delay — Propagation delay

Fluid-flow model of TCP-RED (2)

Average queue length:

$$\frac{dx}{dt} = \frac{\ln(1-\alpha)}{\delta}\left(x(t) - q(t) \right)$$

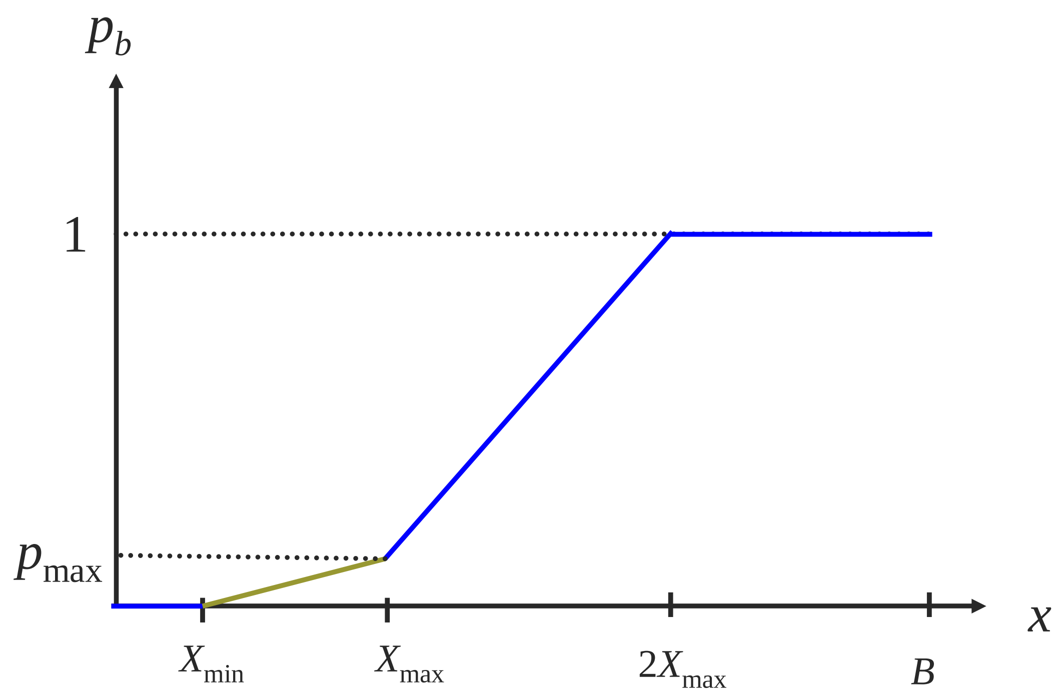
α : queue averaging weight

δ : sampling rate ~1/C

Marking/dropping probability:

$$p_b(t) = \begin{cases} 0 & 0 \le x(t) < X_{\min} \\ \dfrac{x(t) - X_{\min}}{X_{\max} - X_{\min}} p_{\max} & X_{\min} \le x(t) \le X_{\max} \\ p_{\max} - \dfrac{x(t) - X_{\max}}{X_{\max} - X_{\min}}(1 - p_{\max}) & X_{\max} < x(t) \le 2X_{\max} \\ 1 & X_{\max} < x(t) \le B \end{cases}$$

Marking/dropping probability



Steady-state solution and target queue length

Let $N(t) \equiv N$, $C(t) \equiv C$ and $p(t) = \frac{x(t) - X_{\min}}{X_{\max} - X_{\min}} p_{\max}$

Equilibrium point (q_0, r_0, w_0, p_0)

$$q_0 = \frac{X_{\max} + X_{\min}}{\phi}$$

$$r_0 = \frac{q_0}{C} + R_0$$

$$w_0 = \frac{Cr_0}{N}$$

$$p_0 = p_{\max} \frac{1 + (1-\phi)\frac{X_{\min}}{X_{\max}}}{\phi(1 - \frac{X_{\min}}{X_{\max}})} = 2\left(\frac{N}{Cr_0}\right)^2$$

Linearization and characteristic equation

$$\begin{cases} \delta\dot{w}(t) = \dfrac{-N}{r_0{}^2 C}(\delta w(t) + \delta w(t - r_0)) + \dfrac{-1}{r_0{}^2 C}(\delta q(t) - \delta q(t - r_0)) + \dfrac{-r_0 C^2}{2N^2}\delta p(t - r_0) \\ \delta\dot{q}(t) = \dfrac{N}{r_0}\delta w(t) - \dfrac{1}{r_0}\delta q(t) \\ \delta\dot{p}(t) = C\ln(1-\alpha)(\delta p(t) - \beta\delta q(t)) \end{cases}$$

where:

$$\begin{cases} \delta w = w - w_0 \\ \delta q = q - q_0 \\ \delta p = p - p_0 \end{cases}$$

$$\beta = \frac{p_{\max}}{X_{\max} - X_{\min}}$$

Characteristic equation in Laplace domain:

$$\lambda^3 + \left(\frac{1}{r_0} + \frac{N}{r_0{}^2 C} - \alpha_1 C\right)\lambda^2 + \left(\frac{2N}{r_0{}^3 C} - \frac{\alpha_1 C}{r_0} - \frac{\alpha_1 N}{r_0{}^2}\right)\lambda - \frac{2\alpha_1 N}{r_0{}^3} + \left(\frac{N}{r_0{}^2 C}\lambda^2 - \frac{N\alpha_1}{r_0{}^2}\lambda - \frac{C^3\alpha_1\beta}{2N}\right)e^{-\lambda r_0} = 0$$

$$\text{where}: \alpha_1 = \ln(1-\alpha)$$

Characteristic equation

With Padé(1,1) approximation:

$$e^{-\lambda r_0} \approx \frac{1 - \frac{r_0 \lambda}{2}}{1 + \frac{r_0 \lambda}{2}}$$

we obtain:

$$\lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0$$

where:

$$\begin{cases} a_1 = (\frac{3}{\mathrm{K}} - \alpha_1) C \\ a_2 = (\frac{6N}{\mathrm{K}^3} + \frac{2}{\mathrm{K}^2} - \frac{3\alpha_1}{\mathrm{K}}) C^2 \\ a_3 = (\frac{4N}{\mathrm{K}^4} - \frac{6N\alpha_1}{\mathrm{K}^3} - \frac{2\alpha_1}{\mathrm{K}^2} + \frac{2\alpha_1 N}{\Phi \mathrm{K}^2}) C^3 \\ a_4 = -4N\alpha_1 (\frac{1}{\mathrm{K}^4} - \frac{\alpha_1}{\Phi \mathrm{K}}) C^4 \\ K = C r_0 \\ \Phi = X_{\max} - X_{\min} \end{cases}$$

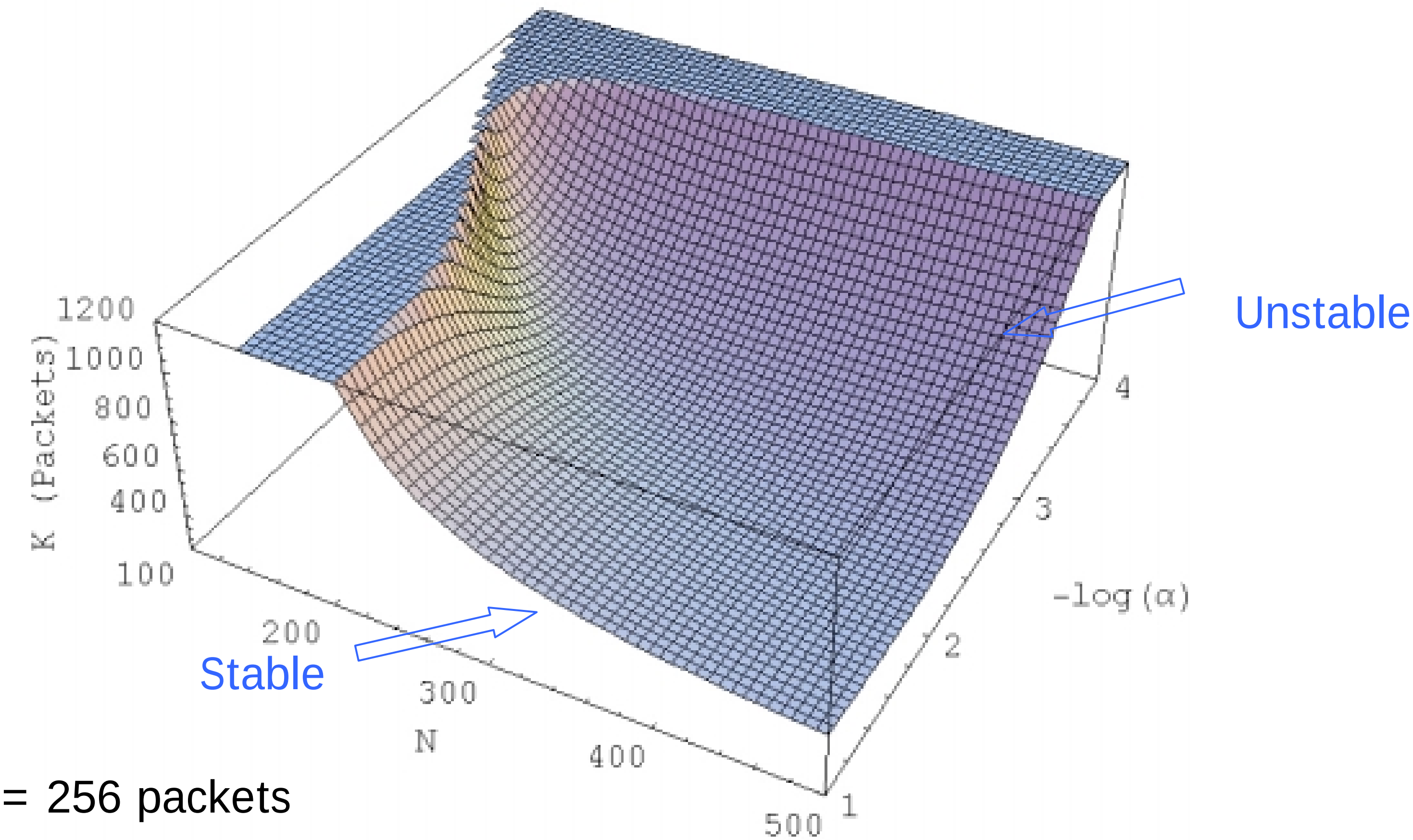
Stability conditions for TCP-RED

The Routh-Hurwitz stability criterion provieds the necessary and sufficient stability conditions for the approximated system in terms of network parameters (N, α, K, Φ):

$$(\frac{4N}{K^2}-\frac{6N\alpha_1}{K}-2\alpha_1+\frac{2\alpha_1 N}{\Phi})[(\frac{3}{K}-\alpha_1)(\frac{6N}{K^2}+\frac{2}{K}-3\alpha_1)$$

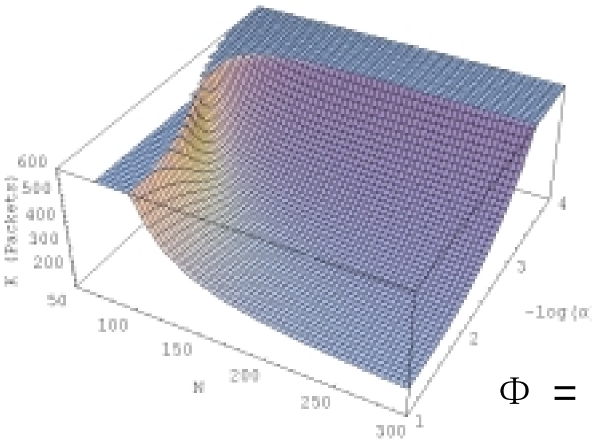
$$-(\frac{4N}{K^3}-\frac{6N\alpha_1}{K^2}-\frac{2\alpha_1}{K}+\frac{2\alpha_1}{\Phi K})]+4N\alpha_1(\frac{3}{K}-\alpha_1)^2(\frac{1}{K}-\frac{1}{\Phi})>0$$

Stable region in (K, N, α)

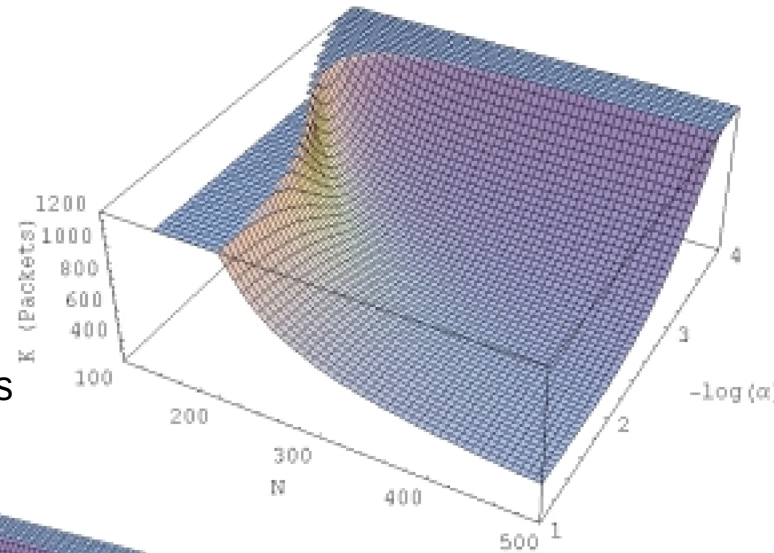


Φ = 256 packets

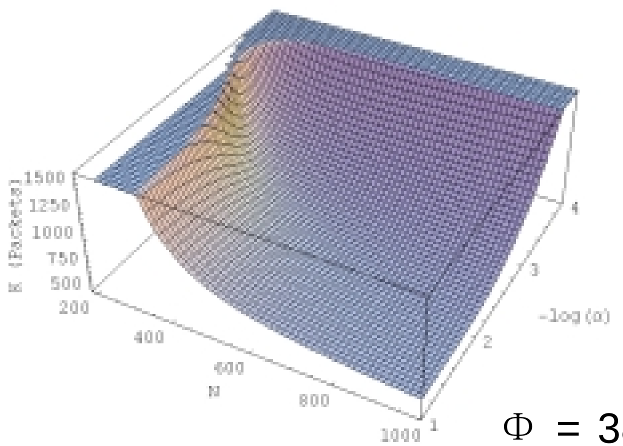
Stable region in (K, N, α)



Φ = 128 packets



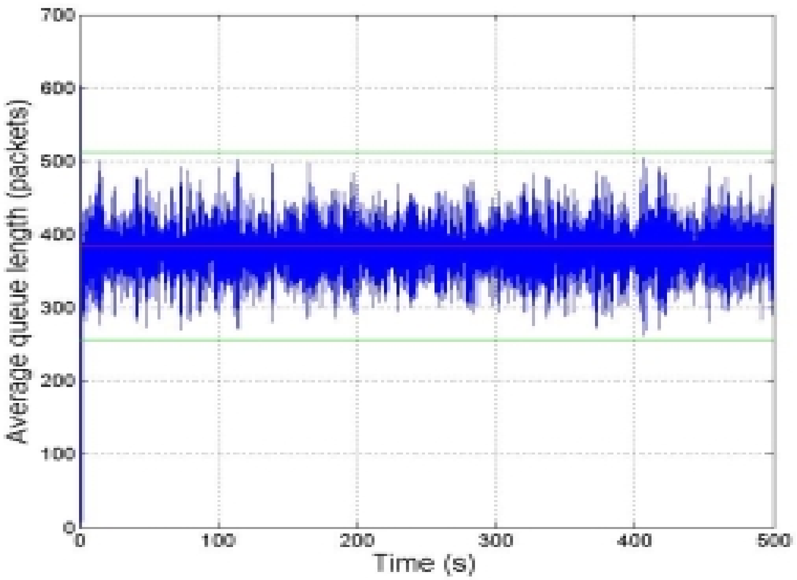
Φ = 256 packets



Φ = 384 packets

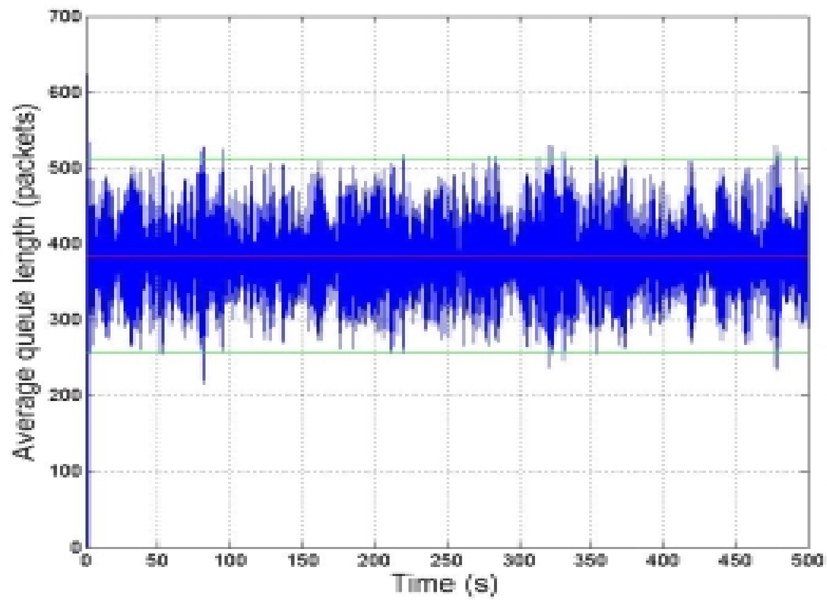
Stable and unstable waveforms of queue length: ns-2 simulations (1)

Stable



K = 758.5 packets (r_0 = 64 ms), Φ = 256 packets, α = 0.01

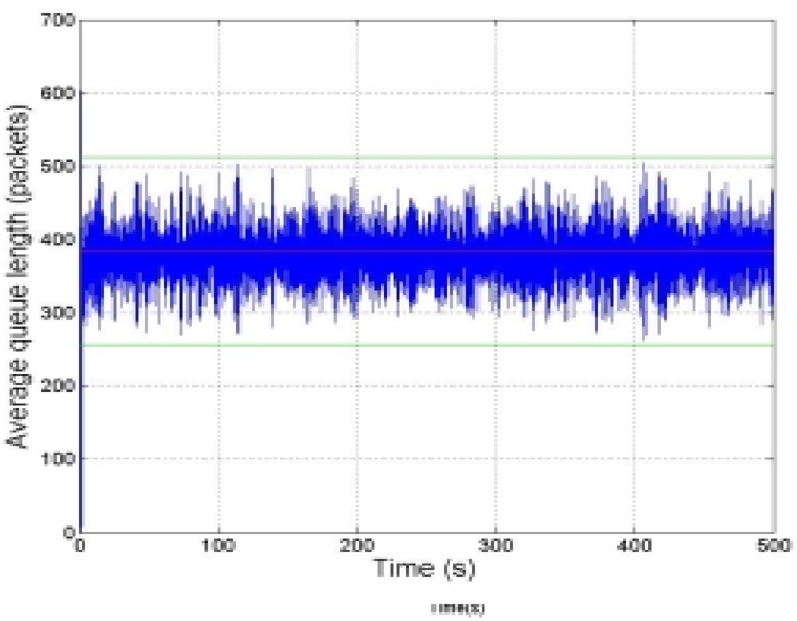
Unstable



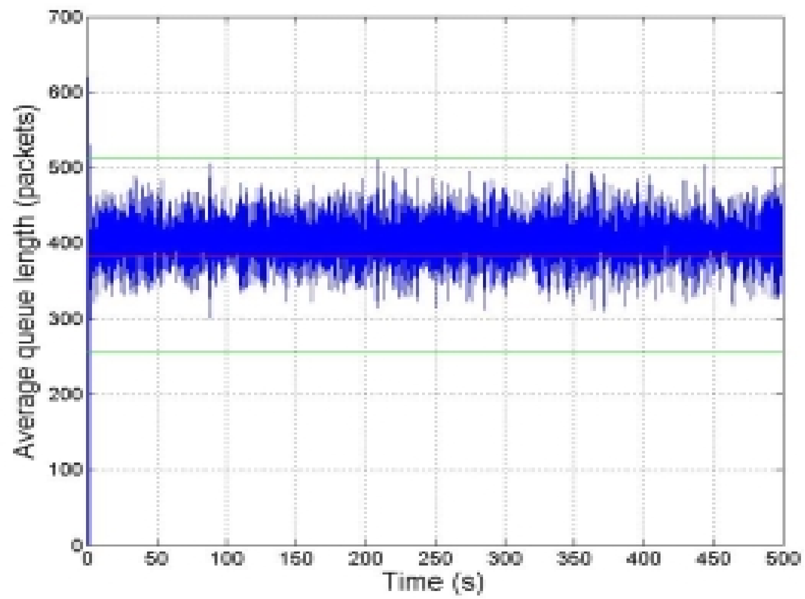
K = 865.2 packets (r_0 = 73 ms), Φ = 256 packets, α = 0.01

Stable and unstable waveforms of queue length: ns-2 simulations (2)

Stable



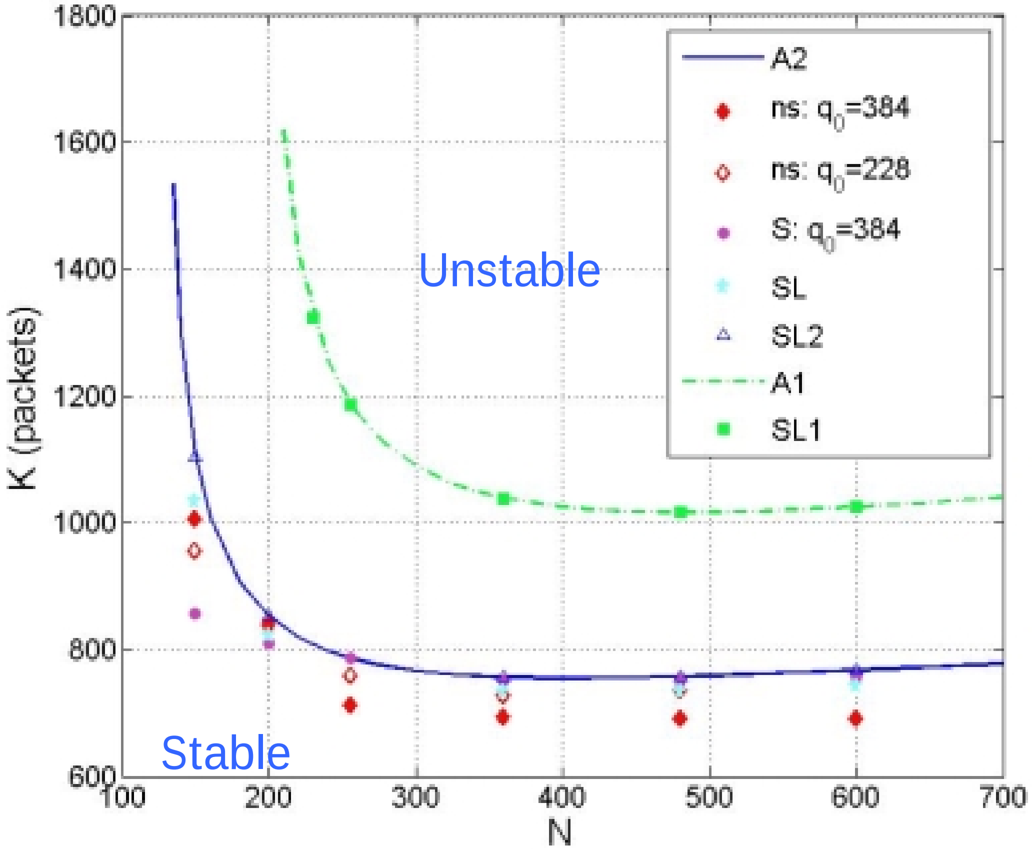
Unstable



K = 758.5 packets (r_0 = 64 ms), Φ = 256 packets, α = 0.01

K = 865.2 packets (r_0 = 75 ms), Φ = 256 packets, α = 0.01

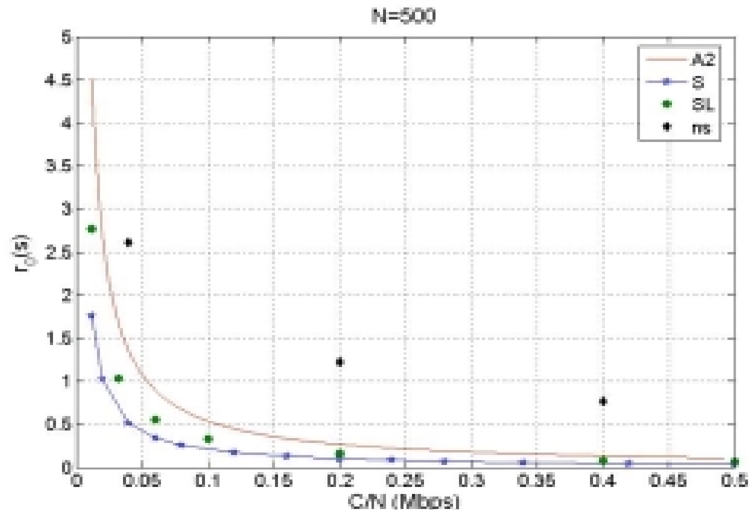
Comparison of simulation methods and approximations

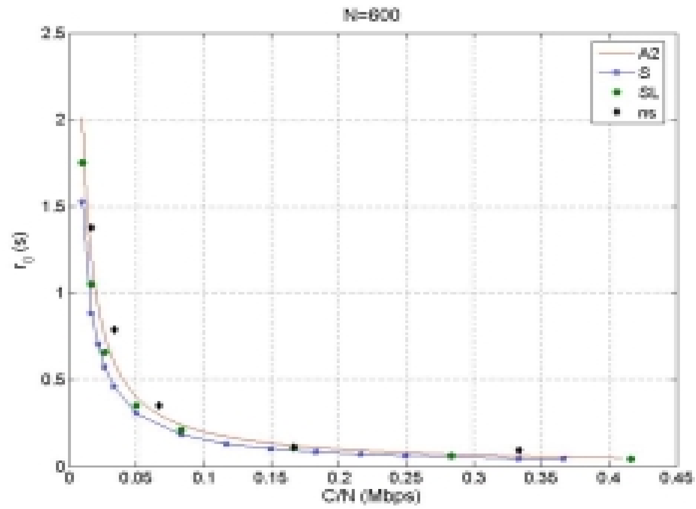


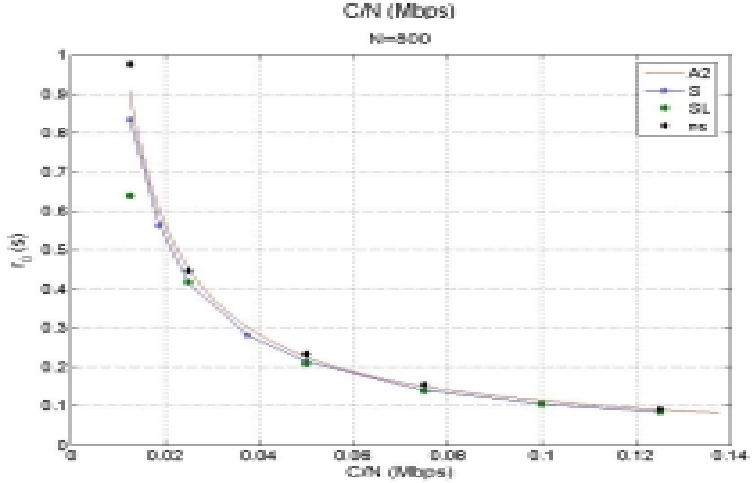
ns	ns-2 simulation
S	Full fluid flow simulation
SL	Linearized fluid flow simulation
SL1	Linearized fluid flow simulation with Padé(0,1) approximation
SL2	Linearized fluid flow simulation with Padé(1,1) approximation
A1	Analytical closed form solution with Padé(0,1) approximation
A2	Analytical closed form solution with Padé(1,1) approximation

Φ = 256 packets and α = 0.001

Comparisons: K = Cr_o







Capacity vs. Round Trip Time:

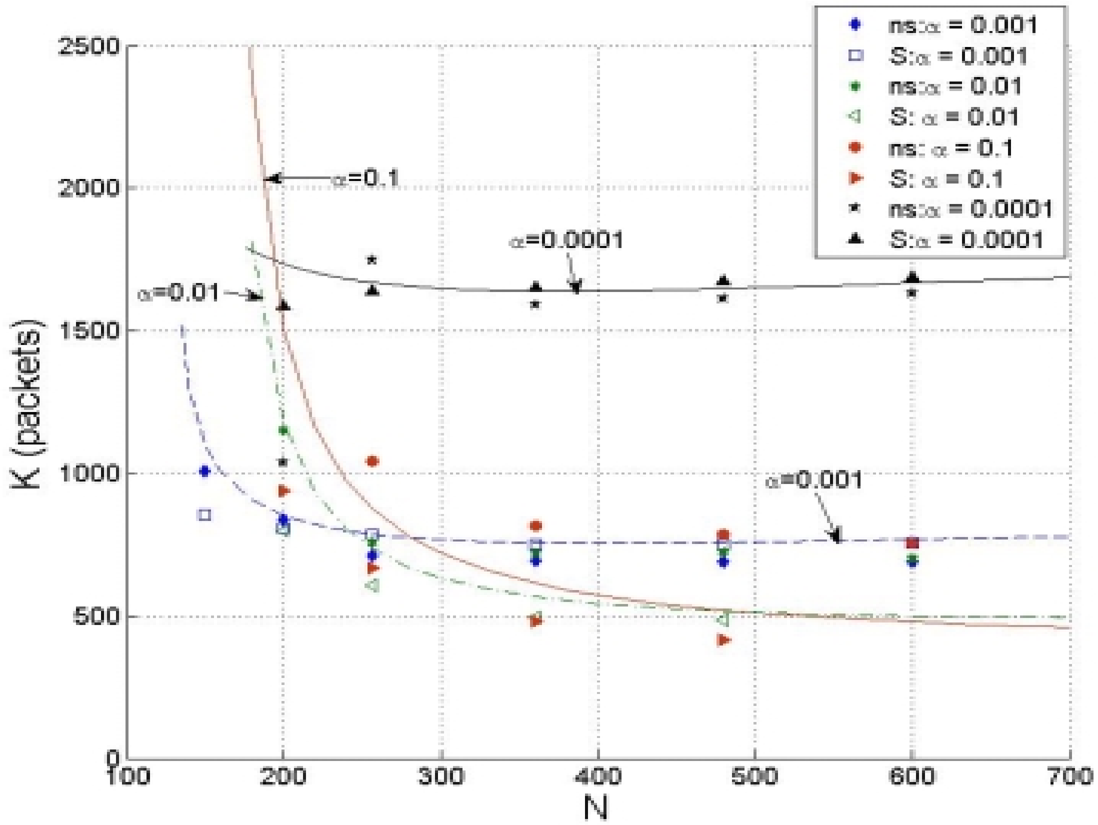
N = 500, 600, and 800

Φ = 800 packets

q_0 = 500 packets

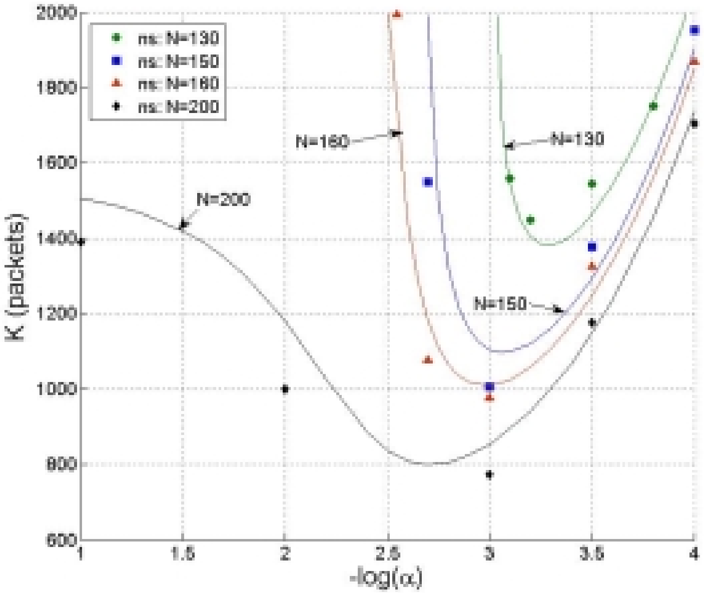
α = 0.001

Comparisons: varying α

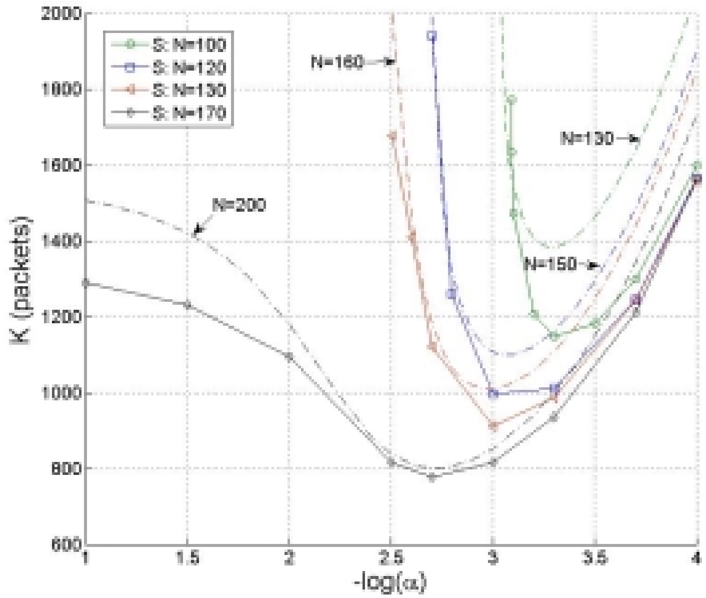


Φ = 256 packets and q_0 = 384 packets

Comparisons: varying N

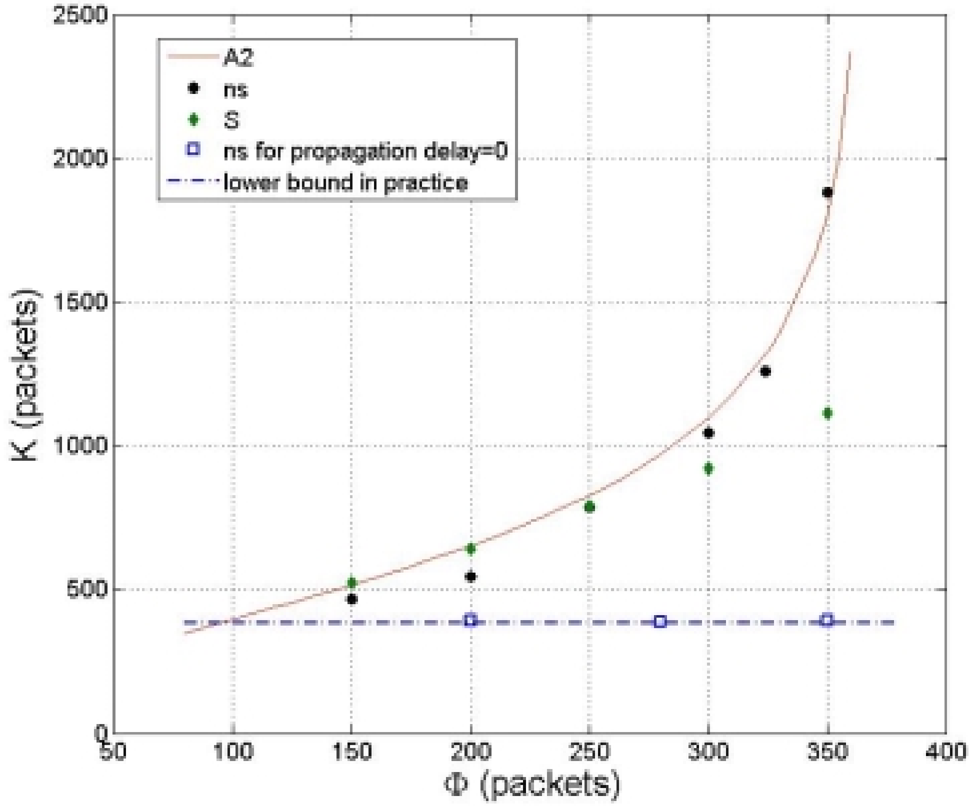


ns-2 results vs. analytical solution for Φ = 256 packets and q_0 = 384 packets



SIMULINK results vs. analytical solution for Φ = 256 packets and q_0 = 384 packets

Comparisons: varying Φ



N = 256, α = 0.001 and q_0 = 384 packets

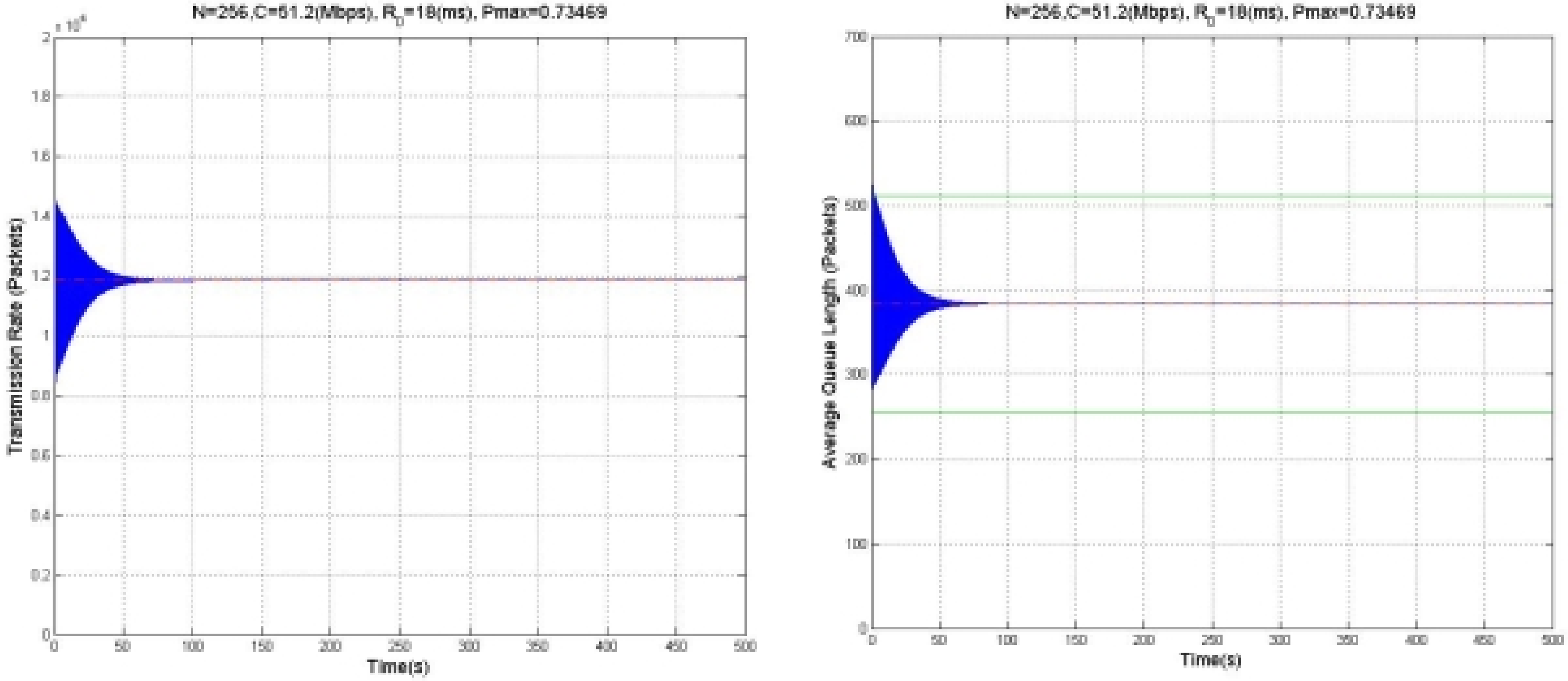
Conclusion and future work

- We have derived stability boundaries of the RED scheme based on an analytical closed-form solution using the Padé(1,1) linearized fluid flow models.
- Very good match between the stability boundaries found from the Padé(1,1) linearized fluid flow model and ns-2 simulations has been achieved.
- The model (verified by ns-2) can be used to predict dynamical behavior of the system.

Thank you!

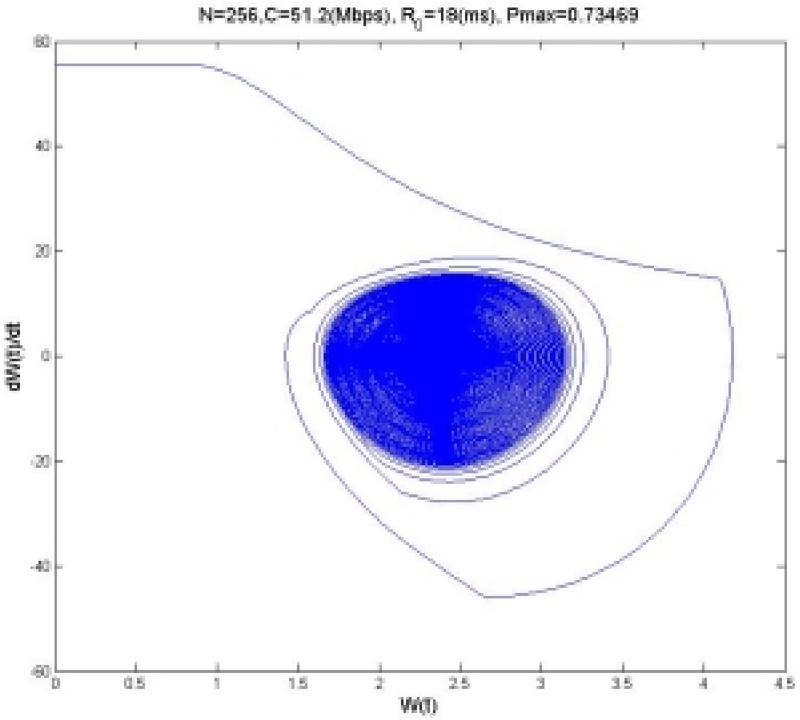
Appendix 1

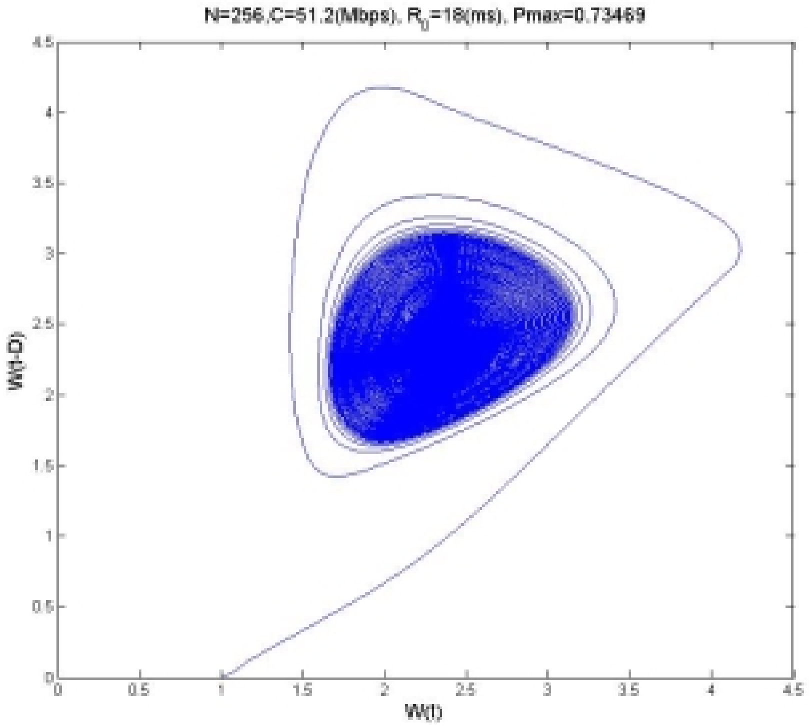
Stable queue length waveforms: Simulink



K = 597.4 packets (r_0 = 51 ms), Φ = 256 packets, α = 0.01

Stable queue length: Simulink





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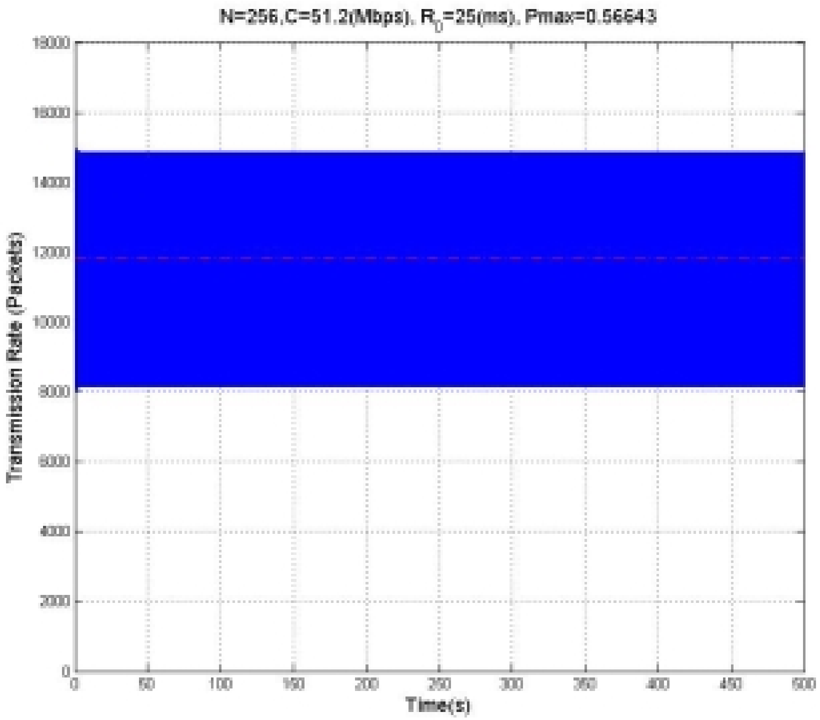
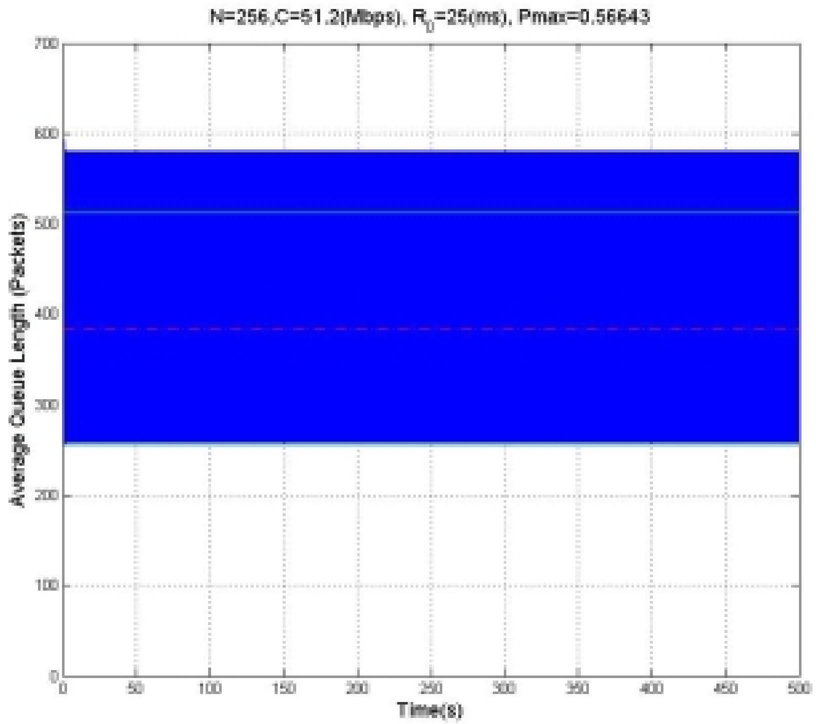
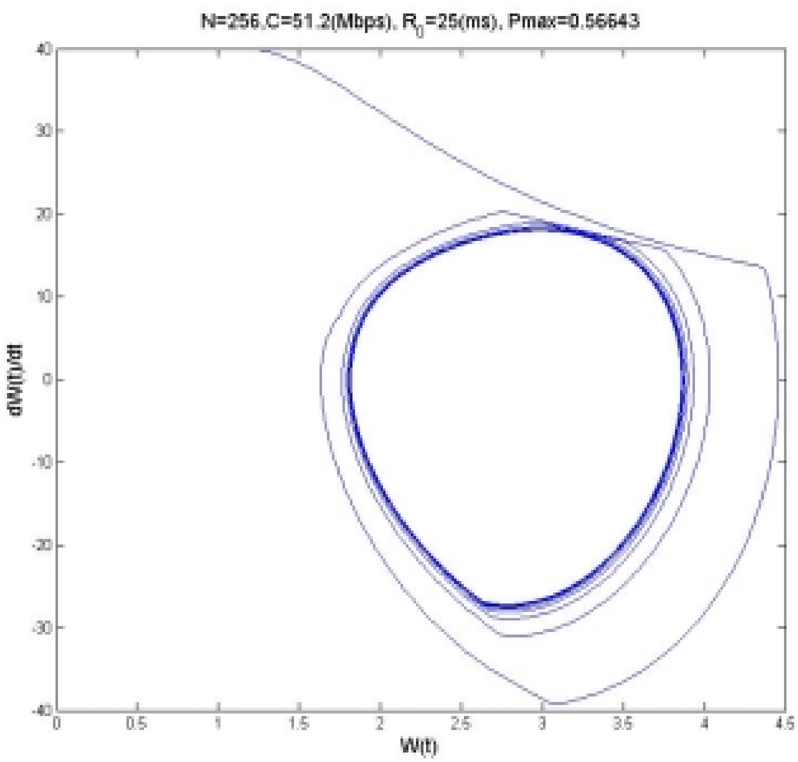
Unstable queue length waveforms: Simulink

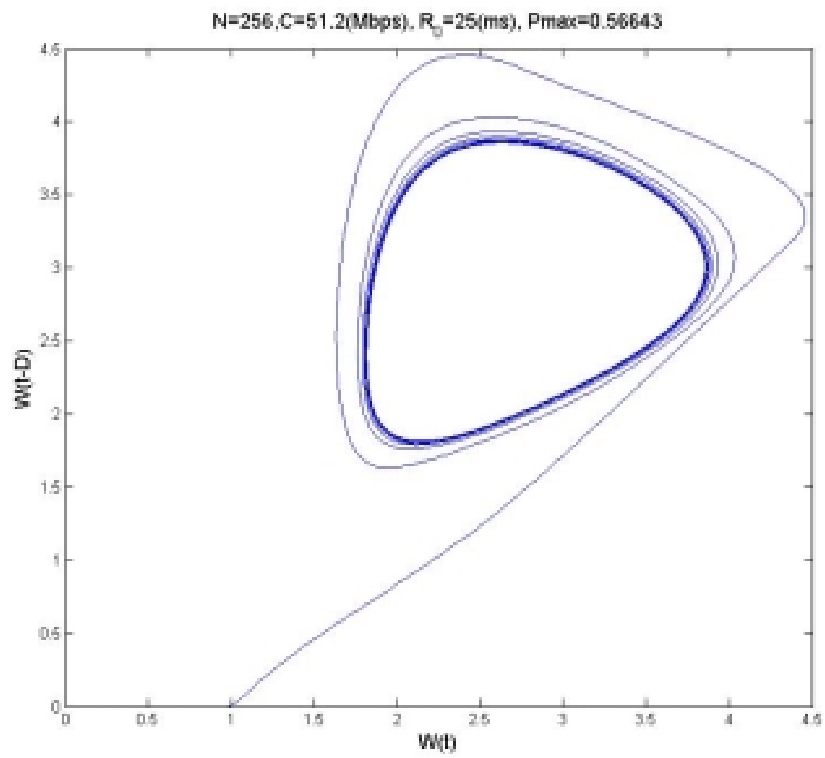




K = 680.3 packets (r_0 = 58 ms), Φ = 256 packets, α = 0.01

Unstable queue length: Simulink





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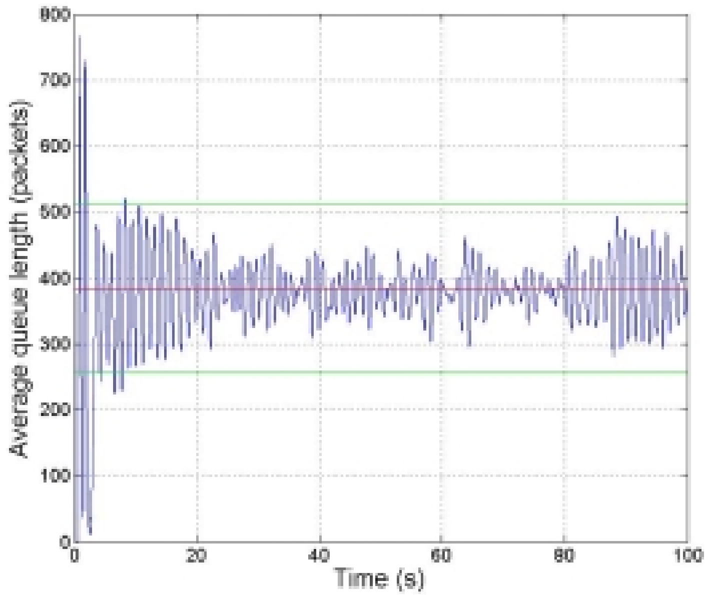
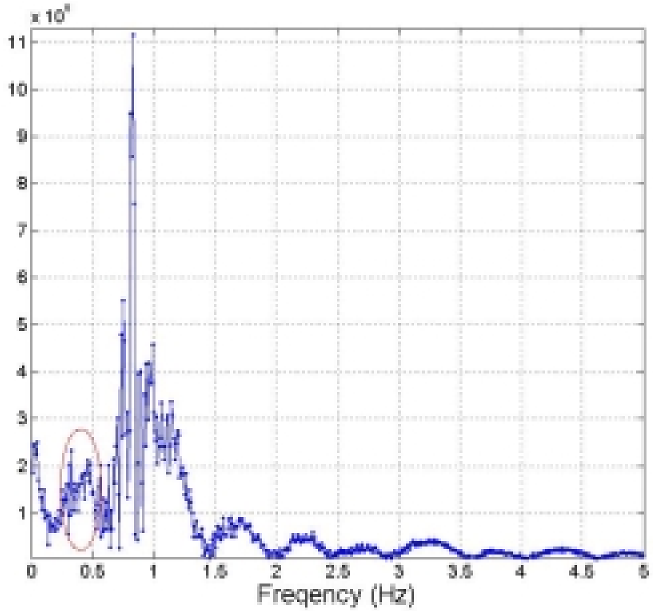
Appendix 2

Stable and unstable queue length waveform: ns-2 (2)

Stable or not ?



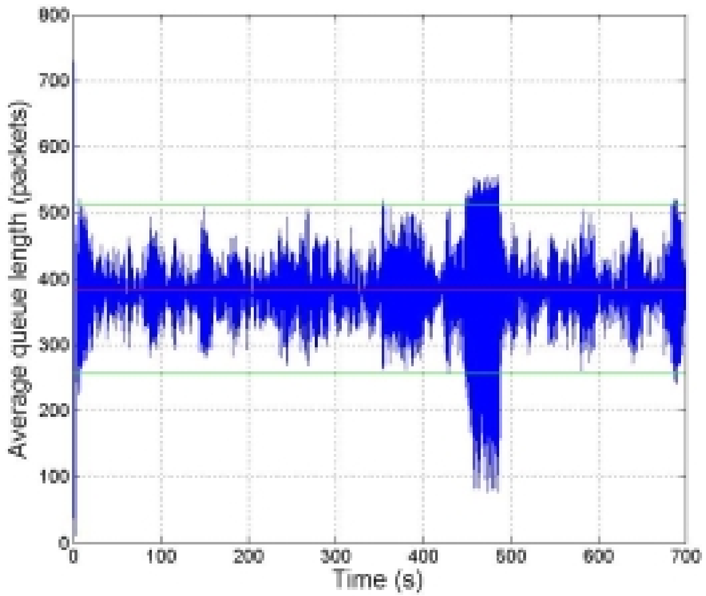


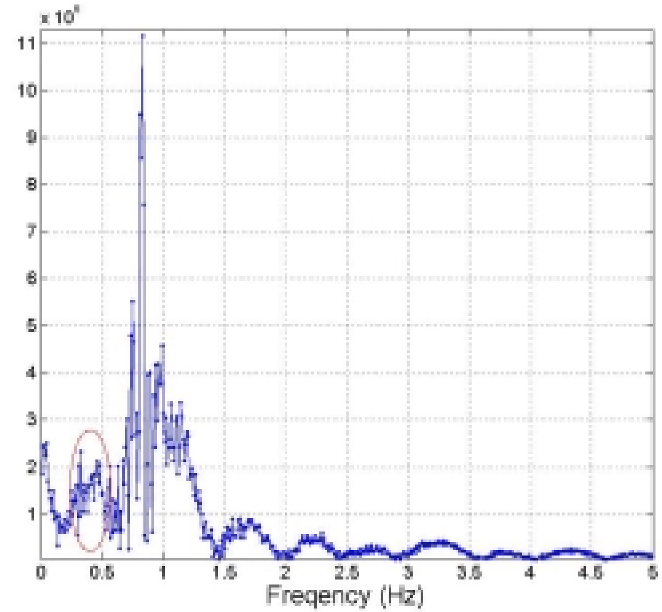
K = 1078.8 packets (r_0 = 155 ms), Φ = 256 packets, α = 0.001

FFT of average queue for first 100 seconds K = 1078.8 packets (r_0 = 155 ms), Φ = 256 packets, α = 0.001

Stable and unstable queue length waveform: ns-2 (2)

Unstable





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Stable and unstable queue length waveform: ns-2 (2)

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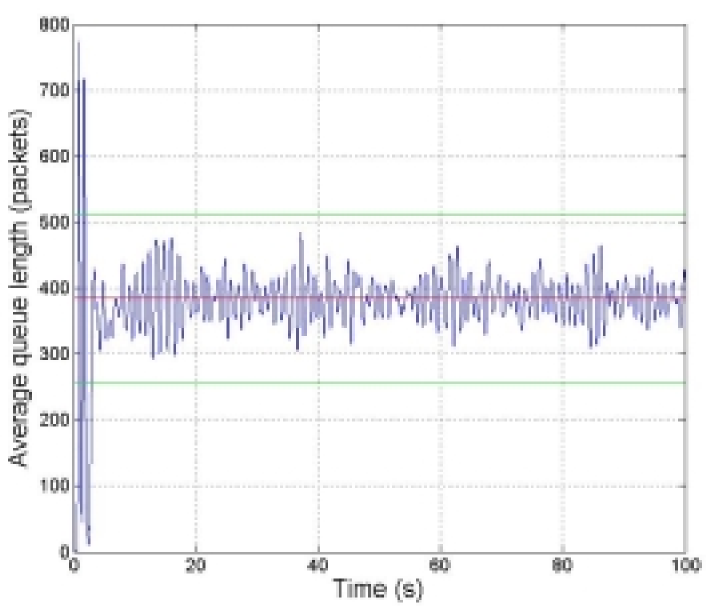
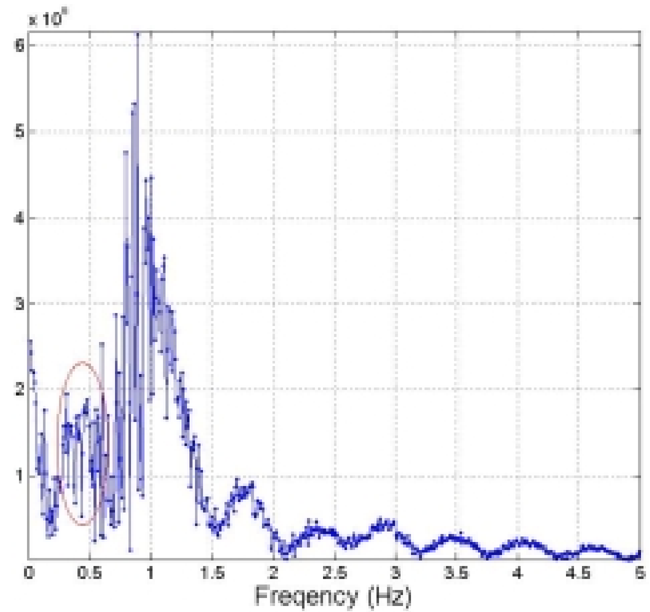


K = 1006.9 packets (r_0 = 145 ms), Φ = 256 packets, α = 0.001



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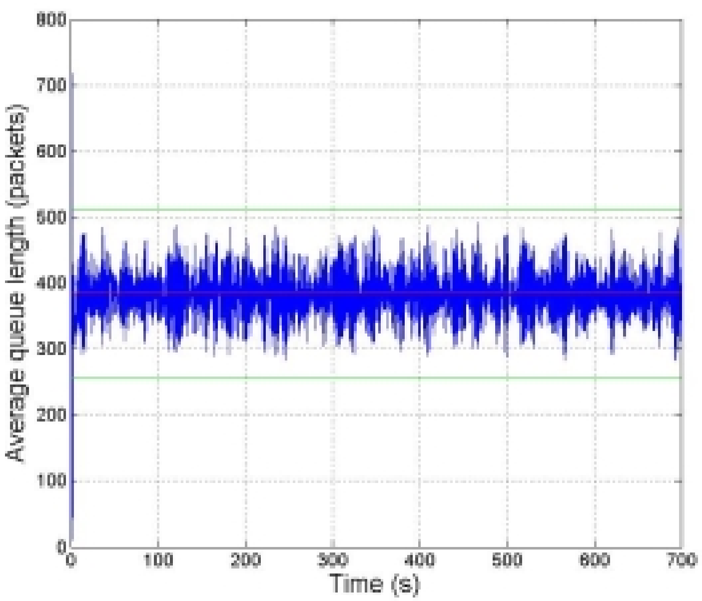
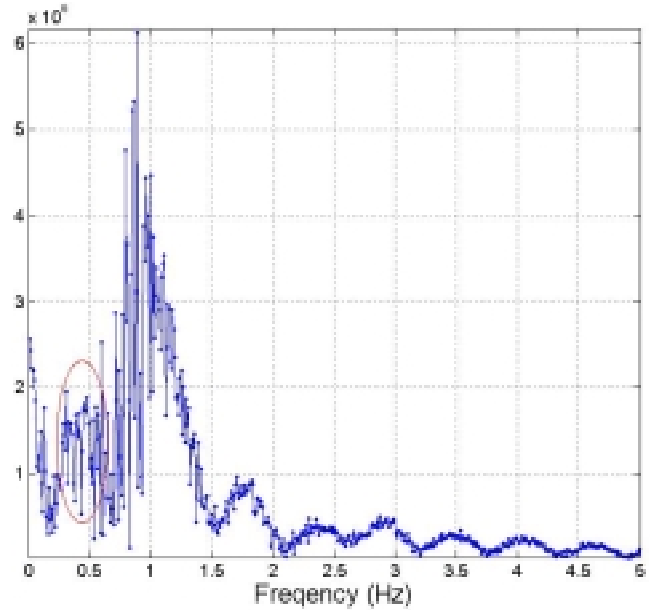
Stable and unstable queue length waveform: ns-2 (2)

Stable





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