Stability Analysis of RED Gateway with Multiple TCP Reno Connections

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Presented by Xi Chen
Outline

- Overview of TCP-RED (Random Early Detection) systems
- Fluid-flow model of TCP-RED
- Stability boundaries for TCP-RED
- Numerical verification
- Conclusions and future work
Overview

TCP senders

Bottleneck link

TCP receivers

Bottleneck gateway
TCP Window Congestion Control algorithm

Sender sends $W$ packets at a time

Window size $= W$

- **Additive increase (AI):**
  window size increases by one per round trip time if no loss

- **Multiplicative decrease (MD):**
  window size decreases by half on detection of loss
TCP Window Congestion Control algorithm

Sender sends $W$ packets at a time. Window size = $W$.

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TCP Window Congestion Control algorithm

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- **Additive increase (AI):** window size increases by one per round trip time if no loss
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Sender-receiver connection without RED
Sender-receiver connection with RED

Early notification to sender

Random packet marking

Queue averaging

Sender

Congested router

Round trip time (rtt)

Receiver

Acknowledgements

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Fluid-flow model of TCP-RED (1)

Window size: \[
\frac{dw(t)}{dt} = \frac{1}{r(t)} - \frac{w(t)}{2} \cdot \frac{w(t - r(t))}{r(t - r(t))} p(t - r(t))
\]

- Additive increase
- Multiplicative decrease
- Loss arrival rate

Instantaneous queue length: \[
\frac{dq(t)}{dt} = N \frac{w(t)}{r(t)} - C
\]

-Incoming traffic
- Outgoing traffic

Round trip time: \[
r(t) = \frac{q(t)}{C} + R_0
\]

- Queuing delay
- Propagation delay
Fluid-flow model of TCP-RED (2)

Average queue length:

\[
\frac{dx}{dt} = \frac{\ln(1-\alpha)}{\delta} (x(t) - q(t))
\]

\(\alpha\) : queue averaging weight
\(\delta\) : sampling rate \(\sim 1/C\)

Marking/ dropping probability:

\[
p_b(t) = \begin{cases} 
0 & 0 \leq x(t) < X_{\min} \\
\frac{x(t) - X_{\min}}{X_{\max} - X_{\min}} p_{\max} & X_{\min} \leq x(t) \leq X_{\max} \\
p_{\max} - \frac{x(t) - X_{\max}}{X_{\max} - X_{\min}} (1 - p_{\max}) & X_{\max} < x(t) \leq 2 X_{\max} \\
1 & X_{\max} < x(t) \leq B
\end{cases}
\]
Marking/dropping probability

\[ p_b \]

\[ 1 \]

\[ p_{\text{max}} \]

\[ X_{\text{min}} \]

\[ X_{\text{max}} \]

\[ 2X_{\text{max}} \]

\[ B \]
Steady-state solution and target queue length

Let $N(t) \equiv N$, $C(t) \equiv C$ and $p(t) = \frac{x(t) - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}} p_{\text{max}}$

Equilibrium point $(q_0, r_0, w_0, p_0)$

$$q_0 = \frac{X_{\text{max}} + X_{\text{min}}}{\phi}$$

$$r_0 = \frac{q_0}{C} + R_0$$

$$w_0 = \frac{C r_0}{N}$$

$$p_0 = p_{\text{max}} \frac{1 + (1 - \phi) \frac{X_{\text{min}}}{X_{\text{max}}}}{\phi (1 - \frac{X_{\text{min}}}{X_{\text{max}}})} = 2 \left( \frac{N}{C r_0} \right)^2$$
Linearization and characteristic equation

\[
\begin{align*}
\delta w(t) &= \frac{-N}{r_0^2 C} (\delta w(t) + \delta w(t-r_0)) + \frac{-1}{r_0^2 C} (\delta q(t) - \delta q(t-r_0)) + \frac{-r_0 C^2}{2N^2} \delta p(t-r_0) \\
\delta q(t) &= \frac{N}{r_0} \delta w(t) - \frac{1}{r_0} \delta q(t) \\
\delta \dot{p}(t) &= C \ln(1-\alpha)(\delta p(t) - \beta \delta q(t))
\end{align*}
\]

where:
\[
\begin{align*}
\delta w &= w - w_0 \\
\delta q &= q - q_0 \\
\delta p &= p - p_0 \\
\beta &= \frac{p_{\text{max}}}{X_{\text{max}} - X_{\text{min}}}
\end{align*}
\]

Characteristic equation in Laplace domain:

\[
\lambda^3 + \left( \frac{1}{r_0} + \frac{N}{r_0^2 C} - \alpha_i C \right) \lambda^2 + \left( \frac{2N}{r_0^3 C} - \frac{\alpha_i N}{r_0^2} \right) \lambda - \frac{2\alpha_i N}{r_0^3} + \left( \frac{N}{r_0^2 C} \lambda - \frac{N\alpha_i}{r_0^2} \right) \lambda - \frac{C^3 \alpha_i \beta}{2N} \lambda - \lambda_0 = 0
\]

where: \( \alpha_i = \ln(1-\alpha) \)
Characteristic equation

With Padé(1,1) approximation:

\[ e^{-\lambda r_0} \approx \frac{1 - \frac{r_0 \lambda}{2}}{1 + \frac{r_0 \lambda}{2}} \]

we obtain:

\[ \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \]

where:

\[
\begin{align*}
    a_1 &= \left( \frac{3}{K} - \alpha_1 \right) C \\
    a_2 &= \left( \frac{6N}{K^3} + \frac{2}{K^2} \frac{3\alpha_1}{K} \right) C^2 \\
    a_3 &= \left( \frac{4N}{K^4} - \frac{6N\alpha_1}{K^3} - \frac{2\alpha_1}{K^2} + \frac{2\alpha_1 N}{\Phi K^2} \right) C^3 \\
    a_4 &= -4N\alpha_1 \left( \frac{1}{K^4} - \frac{\alpha_1}{\Phi K} \right) C^4 \\
    K &= Cr_0 \\
    \Phi &= X_{\text{max}} - X_{\text{min}}
\end{align*}
\]
Stability conditions for TCP-RED

The Routh-Hurwitz stability criterion provides the necessary and sufficient stability conditions for the approximated system in terms of network parameters \((N, \alpha, K, \Phi)\):

\[
\frac{4N}{K^2} \left( - \frac{6N\alpha_1}{K} - 2\alpha_1 + \frac{2\alpha_1 N}{\Phi} \right) \left[ (\frac{3}{K} - \alpha_1)(\frac{6N}{K^2} + \frac{2}{K} - 3\alpha_1) \right. \\
\left. - \frac{4N}{K^3} - \frac{6N\alpha_1}{K^2} - \frac{2\alpha_1}{K} + \frac{2\alpha_1}{K\Phi} \right] + 4N\alpha_1 \left( \frac{3}{K} - \alpha_1 \right)^2 \left( \frac{1}{K} - \frac{1}{\Phi} \right) > 0
\]
Stable region in \((K, N, \alpha)\)

\[ \Phi = 256 \text{ packets} \]
Stable region in \((K, \ N, \ \alpha)\)

\[\Phi = 128 \text{ packets}\]

\[\Phi = 256 \text{ packets}\]

\[\Phi = 384 \text{ packets}\]
Stable and unstable waveforms of queue length: ns-2 simulations (1)

\[ K = 758.5 \text{ packets} \quad (r_0 = 64 \text{ ms}), \quad \Phi = 256 \text{ packets}, \quad \alpha = 0.01 \]

\[ K = 865.2 \text{ packets} \quad (r_0 = 73 \text{ ms}), \quad \Phi = 256 \text{ packets}, \quad \alpha = 0.01 \]
Stable and unstable waveforms of queue length: ns-2 simulations (2)

-Stable

\[ K = 758.5 \text{ packets (} r_0 = 64 \text{ ms)}, \Phi = 256 \text{ packets, } \alpha = 0.01 \]

-Unstable

\[ K = 865.2 \text{ packets (} r_0 = 75 \text{ ms)}, \Phi = 256 \text{ packets, } \alpha = 0.01 \]
Comparison of simulation methods and approximations

<table>
<thead>
<tr>
<th>ns</th>
<th>ns-2 simulation</th>
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<tr>
<td>S</td>
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<td>SL1</td>
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<td>Analytical closed form solution with Padé(1,1) approximation</td>
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$\Phi = 256$ packets and $\alpha = 0.001$

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Comparisons: $K = C r_o$

Capacity vs. Round Trip Time:
$N = 500, 600, \text{ and } 800$
$\Phi = 800 \text{ packets}$
$q_0 = 500 \text{ packets}$
$\alpha = 0.001$
Comparisons: varying $\alpha$

$\Phi = 256$ packets and $q_0 = 384$ packets

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Comparisons: varying N

ns-2 results vs. analytical solution for $\Phi = 256$ packets and $q_0 = 384$ packets

SIMULINK results vs. analytical solution for $\Phi = 256$ packets and $q_0 = 384$ packets
Comparisons: varying $\Phi$

$N = 256$, $\alpha = 0.001$ and $q_0 = 384$ packets
Conclusion and future work

- We have derived stability boundaries of the RED scheme based on an analytical closed-form solution using the Padé(1,1) linearized fluid flow models.
- Very good match between the stability boundaries found from the Padé(1,1) linearized fluid flow model and ns-2 simulations has been achieved.
- The model (verified by ns-2) can be used to predict dynamical behavior of the system.
Thank you!
Appendix 1
Stable queue length waveforms: Simulink

$K = 597.4$ packets ($r_0 = 51$ ms), $\Phi = 256$ packets, $\alpha = 0.01$
$K = 597.4\text{ packets (} r_0 = 51\text{ ms}), \quad \Phi = 256\text{ packets}, \quad \alpha = 0.01$

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Unstable queue length waveforms: Simulink

$K = 680.3$ packets ($r_0 = 58$ ms), $\Phi = 256$ packets, $\alpha = 0.01$

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$\mathcal{K} = 680.3$ packets ($r_0 = 58$ ms), $\Phi = 256$ packets, $\alpha = 0.01$

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Appendix 2
Stable and unstable queue length waveform: ns-2 (2)

Stable or not?

$K = 1078.8$ packets ($r_0 = 155$ ms), $\Phi = 256$ packets, $\alpha = 0.001$

FFT of average queue for first 100 seconds $K = 1078.8$ packets ($r_0 = 155$ ms), $\Phi = 256$ packets, $\alpha = 0.001$
Stable and unstable queue length waveform: ns-2 (2)

Unstable

\[ K = 1078.8 \text{ packets} \quad (r_0 = 155 \text{ ms}), \quad \Phi = 256 \text{ packets}, \quad \alpha = 0.001 \]

FFT of average queue for first 100 seconds
\[ K = 1078.8 \text{ packets} \quad (r_0 = 155 \text{ ms}), \quad \Phi = 256 \text{ packets}, \quad \alpha = 0.001 \]
Stable and unstable queue length waveform: ns-2 (2)

K = 1006.9 packets (r₀ = 145 ms), Φ = 256 packets, α = 0.001

FFT of average queue for first 100 seconds
K = 1006.9 packets (r₀ = 145 ms), Φ = 256 packets, α = 0.001
Stable and unstable queue length waveform: ns-2 (2)

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