



# Analysis of Internet Topologies

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# Roadmap

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- Internet topology and the datasets
- Spectrum of a graph and power-laws
- Power-laws and the Internet topology
- Spectral analysis of the Internet graph
- Conclusions and references



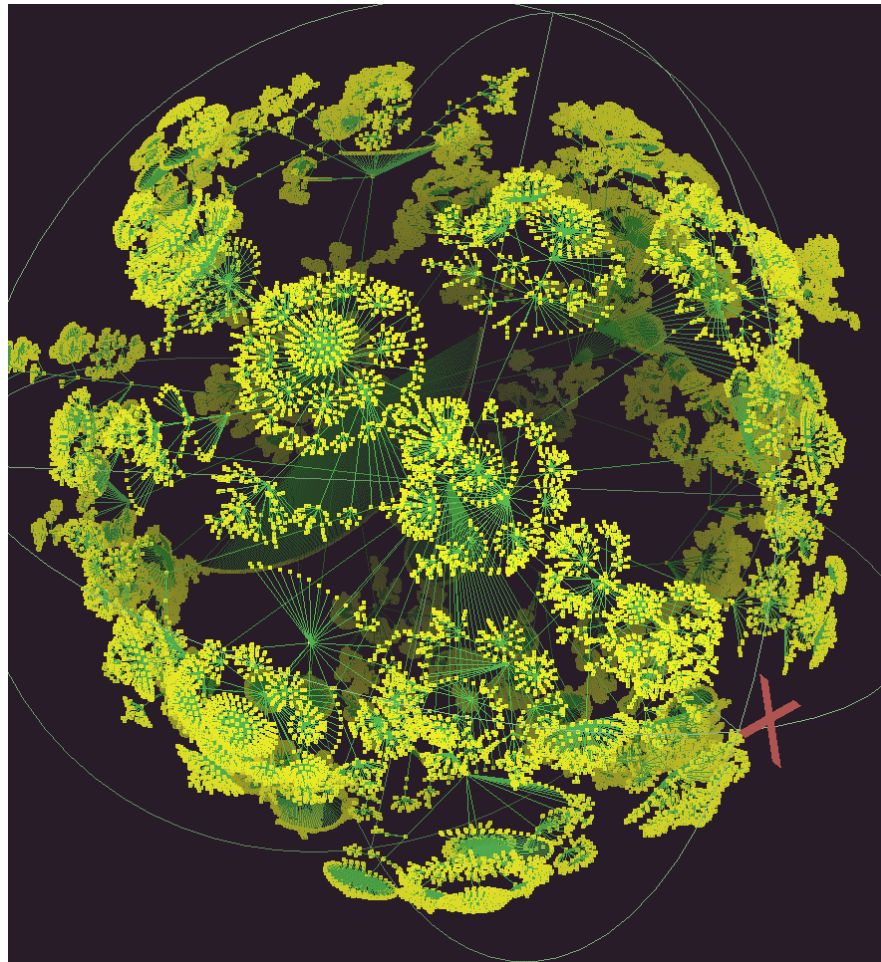
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Ihr (535,102 nodes and 601,678 links)



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# Internet graph

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- Internet is a network of Autonomous Systems:
  - groups of networks sharing the same routing policy
  - identified with Autonomous System Numbers (ASN)
- Autonomous System Numbers:  
<http://www.iana.org/assignments/as-numbers>
- Internet topology on *AS-level*:
  - the arrangement of ASes and their interconnections
- Analyzing the Internet topology and finding properties of associated graphs rely on mining data and capturing information about Autonomous Systems (ASes).



# Internet routing protocol

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- Border Gateway Protocol (BGP):
  - inter-AS protocol
  - used to exchange network reachability information among BGP systems
  - reachability information is stored in **routing tables**



# Internet AS-level data

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Source of data are routing tables:

- **Route Views:** <http://www.routeviews.org>
  - most participating ASes reside in North America
- **RIPE (Réseaux IP européens):** <http://www.ripe.net/ris>
  - most participating ASes reside in Europe
- The BGP routing tables are collected from multiple geographically distributed BGP Cisco routers and Zebra servers.
- Analyzed datasets were collected at 00:00 am on July 31, 2003 and 00:00 am on July 31, 2008.



# Internet AS-level data

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- Data used in prior research (partial list):

	Route Views	RIPE
Faloutsos, 1999	Yes	No
Chang, 2001	Yes	Yes
Vukadinovic, 2001	Yes	No
Mihail, 2003	Yes	Yes

- These research results have been used in developing Internet simulation tools:
  - power-laws are employed to model and generate Internet topologies: BA model, BRITE, Inet2



# Data sets

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Concerns about the use of the two datasets:

- different observations about AS degrees:
  - power-law distribution: [Route Views](#) [Faloutsos, 1999]
  - Weibull distribution: [Route Views](#) + [RIPE](#) [Chang, 2001]
- data completeness:
  - RIPE dataset contains ~ 40% more AS connections and 2% more ASs than [Route Views](#) [Chang, 2001]



# Route Views and RIPE: statistics

- Route Views and RIPE samples collected on May 30, 2003

Number of	Route Views	RIPE
AS paths	6,398,912	6,375,028
Probed ASs	15,418	15,433
AS pairs	34,878	35,225

- AS pair**: a pair of connected ASs
- 15,369** probed ASs (**99.7%**) in both datasets are identical
- 29,477** AS pairs in Route Views (**85%**) and in RIPE (**84%**) are identical



# Core ASs

- ASs with largest degrees

	Route Views		RIPE	
	AS	Degree	AS	Degree
1	701	2595	701	2448
2	1239	2569	1239	1784
3	7018	1999	7018	1638
4	3561	1036	209	861
5	1	999	3561	705
6	209	863	3356	673
7	3356	662	3549	612
8	3549	617	702	580
9	702	562	2914	561
10	2914	556	1	489
11	6461	498	4589	482
12	4513	468	6461	476
13	4323	315	8220	450
14	16631	294	3303	429
15	6347	291	13237	412
16	8220	289	6730	313
17	3257	277	4323	305
18	4766	263	3257	305
19	3786	263	16631	296
20	7132	258	6347	281



# Core ASs

- ASs with largest degrees
- 16 of the core ASs in Route Views and **RIPE** are identical
- Core ASs in Route Views have larger degrees than core ASs in **RIPE**

	Route Views		<b>RIPE</b>	
	AS	Degree	AS	Degree
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# Internet topology

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- Datasets are collected from Border Gateway Protocols (BGP) routing tables.
- The Internet topology is characterized by the presence of various power-laws observed when considering:
  - node degree vs. node rank
  - node degree frequency vs. degree
  - number of nodes within a number of hops vs. number of hops
  - eigenvalues of the adjacency matrix and the normalized Laplacian matrix vs. the order of the eigenvalues

Faloutsos et al., 1999 and Siganos et al., 2003



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# Spectrum of a graph

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- Normalized Laplacian matrix  $NL(G)$ :

$$NL(i, j) = \begin{cases} 1 & \text{if } i = j \text{ and } d_i \neq 0 \\ -\frac{1}{\sqrt{d_i d_j}} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

$d_i$  and  $d_j$  are degrees of node  $i$  and  $j$ , respectively

- The spectrum of  $NL(G)$  is the collection of all eigenvalues and contains 0 for every connected graph component.

Chung et al., 1997



# Power laws: node degree vs. rank

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- The graph nodes  $v$  are sorted in decreasing order based on their degrees  $d_v$  and are indexed with a sequence of numbers indicating their ranks  $r_v$ .
- The  $(r_v, d_v)$  pairs are plotted on the log-log scale.
- The power-law implies:

$$d_v \propto r_v^R,$$

where  $v$  is the node number and  $R$  is the node degree power-law exponent.



# Power laws: CCDF of a node degree

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- The frequency of a node degree is equal to the number of nodes having the same degree.
- The complementary cumulative distribution function (CCDF)  $D_d$  of a node degree  $d$  is equal to the number of nodes having degree less than or equal to  $d$ , divided by the number of nodes.
- The power-law implies:

$$D_d \propto d^D,$$

where  $D$  is the CCDF power-law exponent.



# Power laws: eigenvalues

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- The eigenvalues  $\lambda_i$  of the adjacency matrix and the normalized Laplacian matrix are sorted in decreasing order and plotted versus the associated increasing sequence of numbers  $i$  representing the order of the eigenvalue.
- The power-law for the adjacency matrix implies:

$$\lambda_{ai} \propto i^\varepsilon,$$

- The power-law for the normalized Laplacian matrix implies:

$$\lambda_{Li} \propto i^L,$$

where  $\varepsilon$  and  $L$  are the eigenvalue power-law exponents.



# Analysis of datasets

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- Calculated and plotted on a log-log scale are:
  - node degree vs. node rank
  - frequency of node degree vs. node degree
  - eigenvalues vs. index
- Linear regression is used to determine the correlation coefficient between the regression line and the plotted data.
- A high correlation coefficient between the regression line and the plotted data indicates the existence of a power-law, which implies that node degree, frequency of node degree, and eigenvalues follow a power-law dependency on the rank, node degree, and index, respectively.



# Analysis of datasets

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- The power-law exponents are calculated from the linear regression lines  $10^a x^b$ , with segment **a** and slope **b** when plotted on a log-log scale.

Source of data are routing tables:

- **Route Views:** <http://www.routeviews.org>
- **RIPE (Réseaux IP européens):** <http://www.ripe.net/ris>

Faloutsos et al., 1999 and Chen et al., 2002

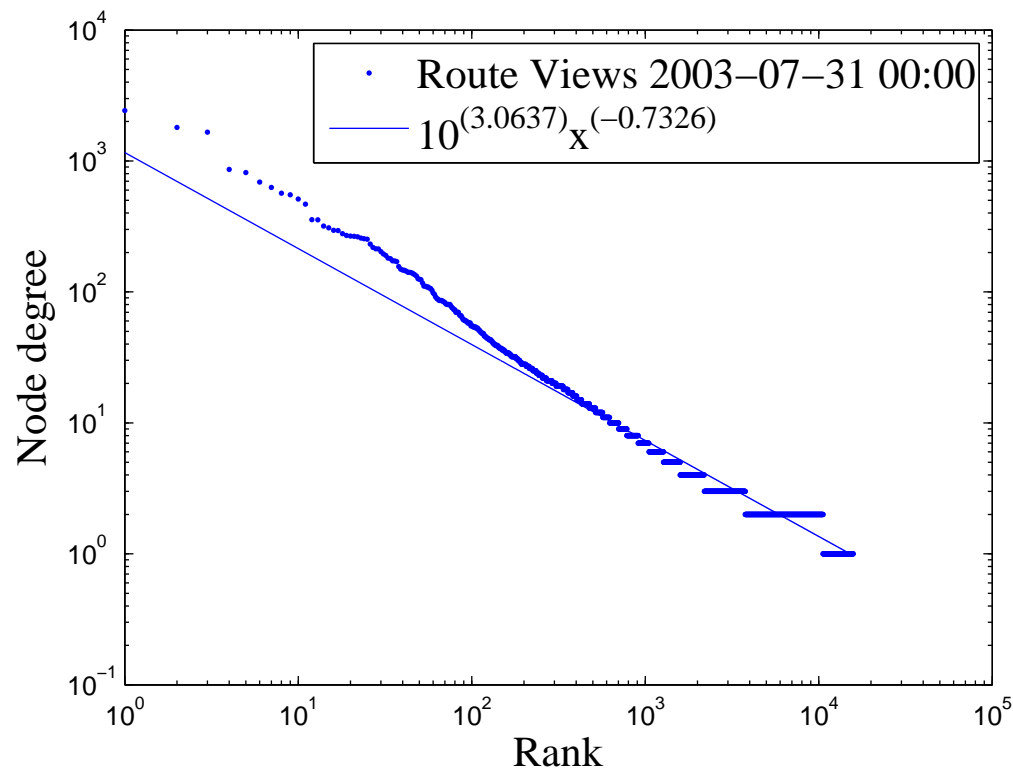


# Roadmap

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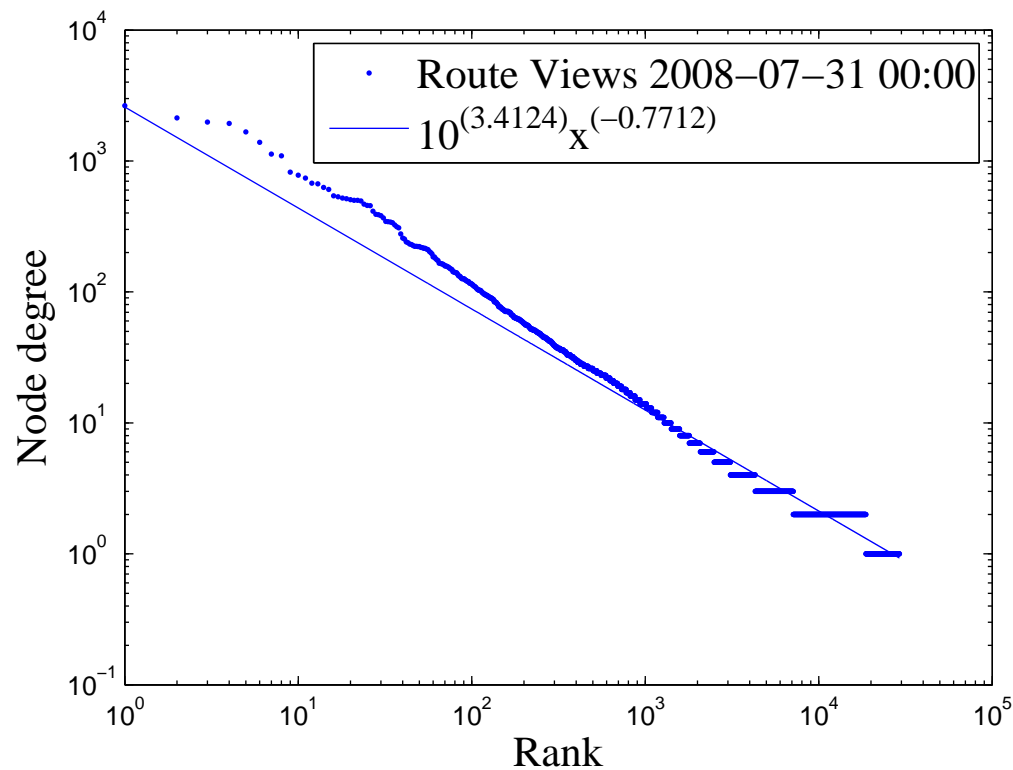
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# Route Views 2003 dataset



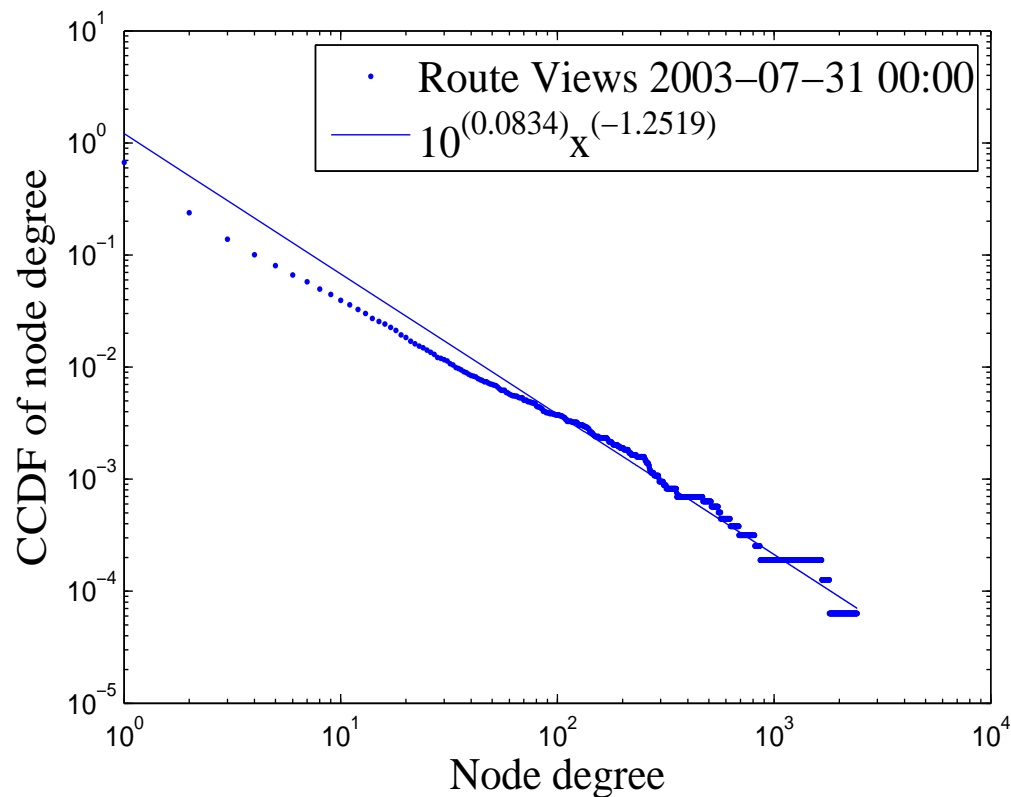
The node degree power-law exponent  $R = -0.7326$   
The correlation coefficient = 0.9661

# Route Views 2008 dataset



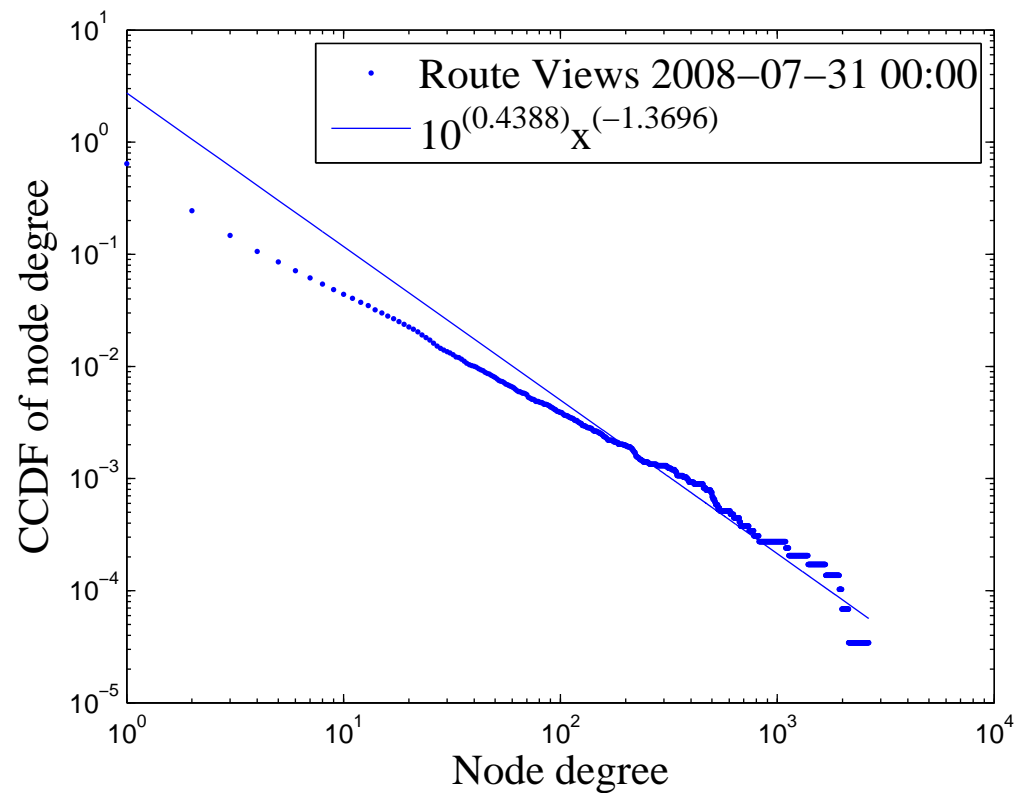
The node degree power-law exponent  $R = -0.7712$   
The correlation coefficient = 0.9686

# Route Views 2003 dataset



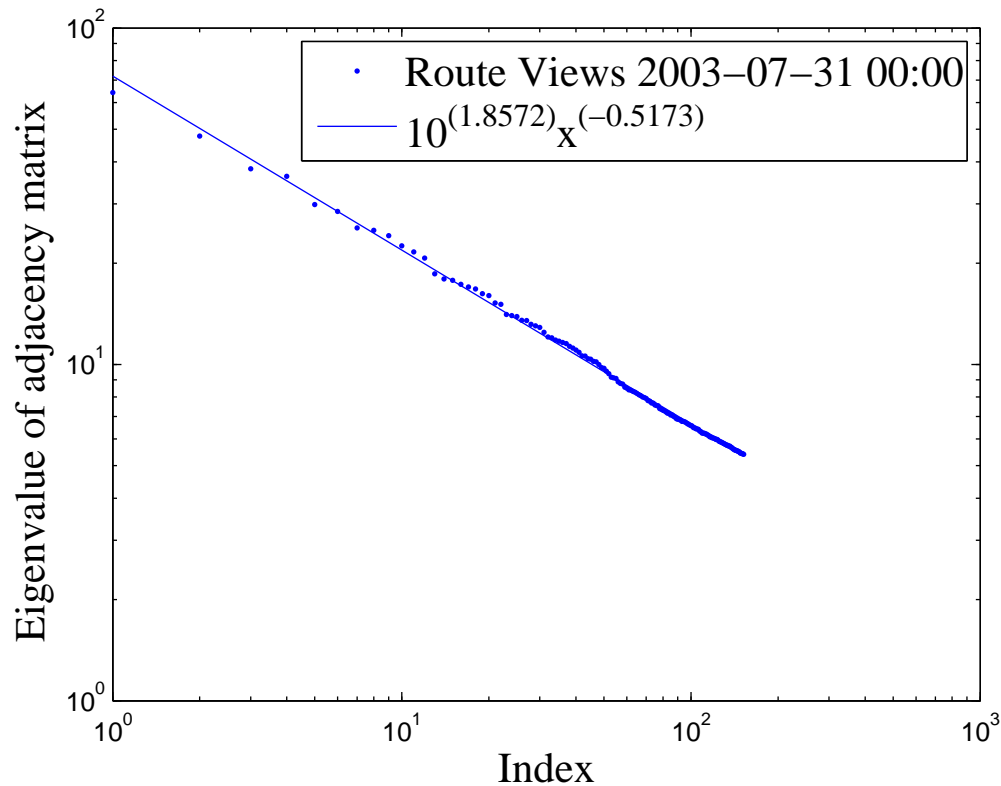
CCDF power-law exponent  $D = -1.2519$   
The correlation coefficient = 0.9810

# Route Views 2008 dataset



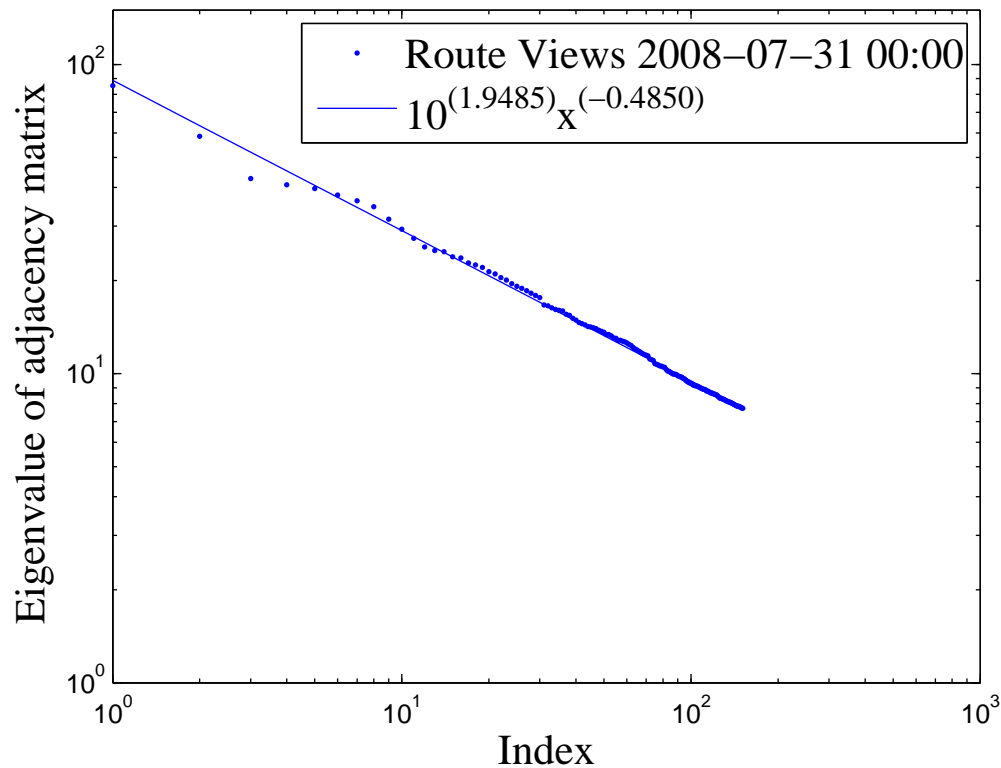
CCDF power-law exponents  $D = -1.3696$   
The correlation coefficients = 0.9626

# Route Views 2003 dataset



The eigenvalue power-law exponents  $\varepsilon = -0.5173$   
The correlation coefficient = 0.9990

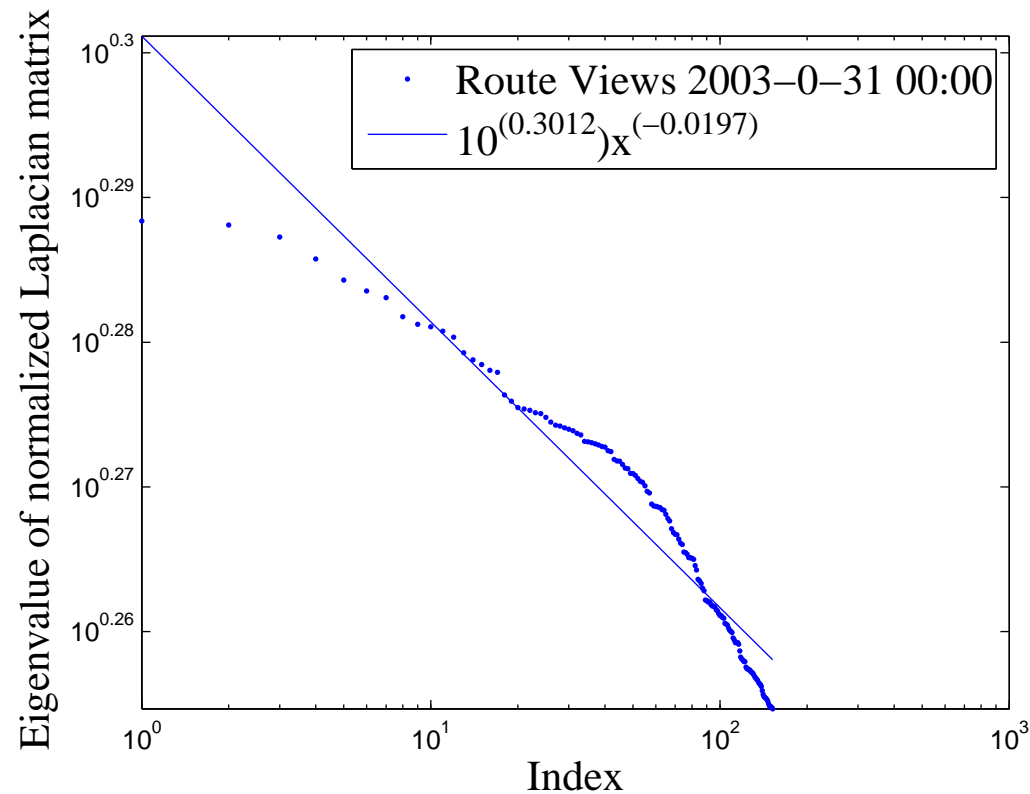
# Route Views 2008 dataset



The eigenvalue power-law exponent  $\varepsilon = -0.4850$

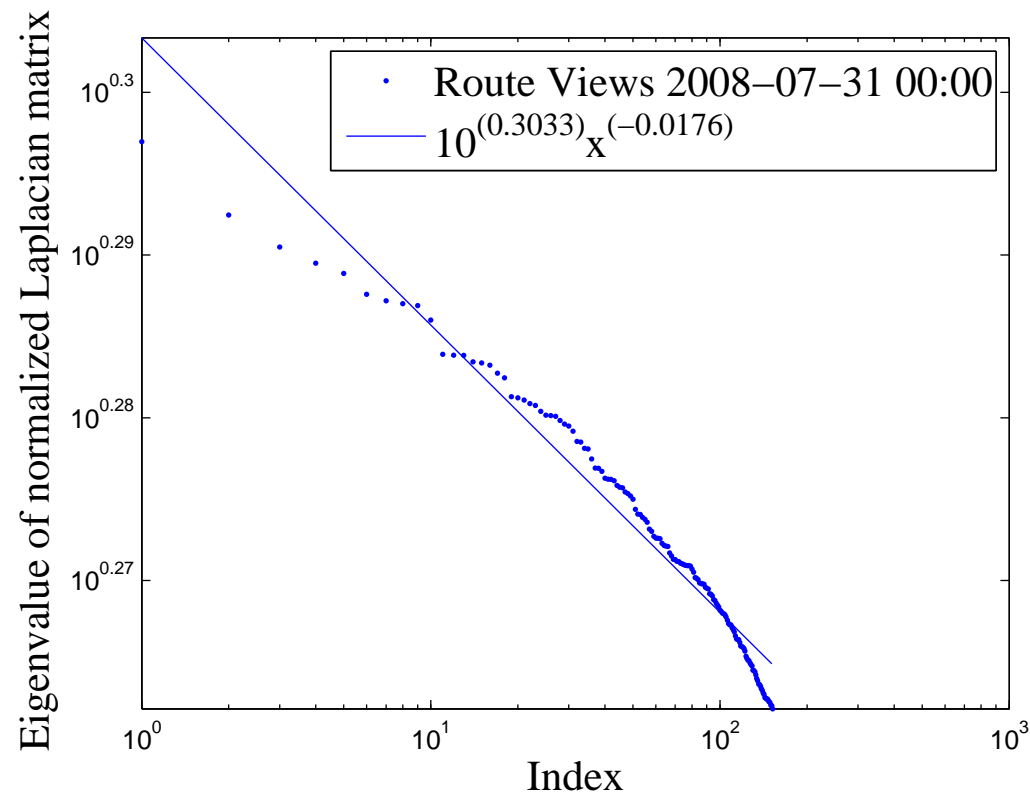
The correlation coefficient = 0.9882

# Route Views 2003 dataset



The eigenvalue power-law exponent  $L = -0.0197$   
The correlation coefficient = 0.9564

# Route Views 2008 dataset



The eigenvalue power-law exponent  $L = -0.0176$   
The correlation coefficient = 0.9783



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# Spectral analysis of the Internet Graph

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- We calculate the **second smallest** and **the largest** eigenvalues and associated eigenvectors of normalized Laplacian matrix.
- Each element of an eigenvector is associated with the AS having the same index.
- ASes are sorted in the ascending order based on the eigenvector values and the sorted AS vector is then indexed.
- The connectivity status is equal to **one** if the AS is **connected** to another AS or **zero** if the AS is **isolated** or is absent from the routing table.



# Spectral analysis of the Internet Graph

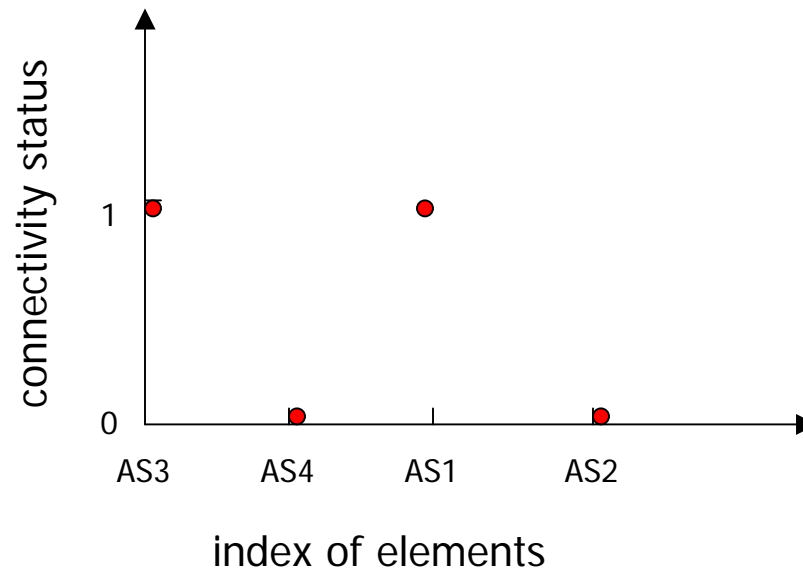
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- The second smallest eigenvalue, called "algebraic connectivity" of a normalized Laplacian matrix, is related to the connectivity characteristic of the graph.
- Elements of the eigenvector corresponding to the **largest eigenvalue** of the normalized Laplacian matrix tend to be positioned close to each other if they correspond to AS nodes with similar connectivity patterns constituting clusters.

Mihail et al., 2003

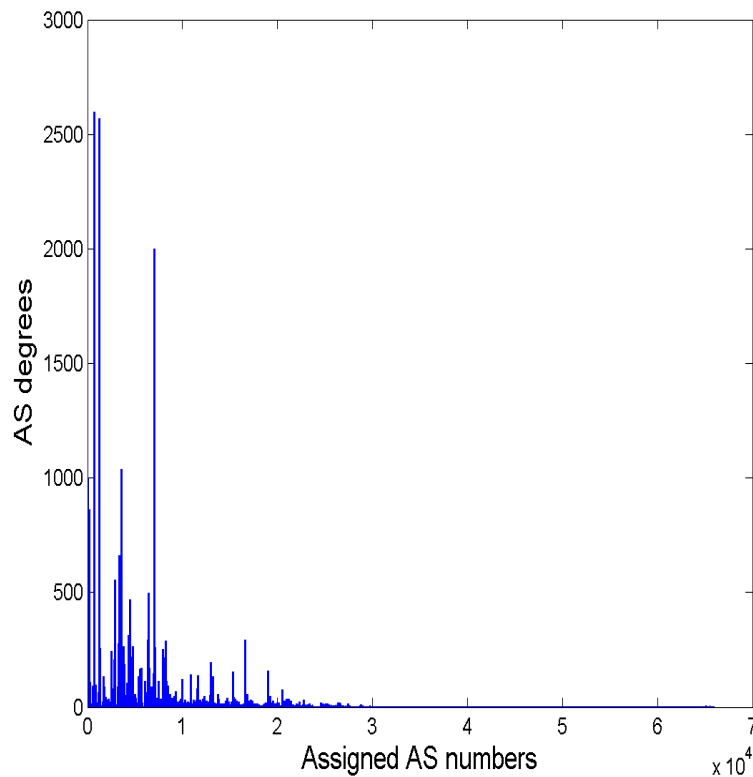
# Characteristic valuation: example

- The second smallest eigenvector: 0.1, 0.3, -0.2, 0
- **AS1**(0.1), **AS2**(0.3), **AS3**(-0.2), **AS4**(0)
- Sort ASs by element value: **AS3**, **AS4**, **AS1**, **AS2**
- **AS3** and **AS1** are connected

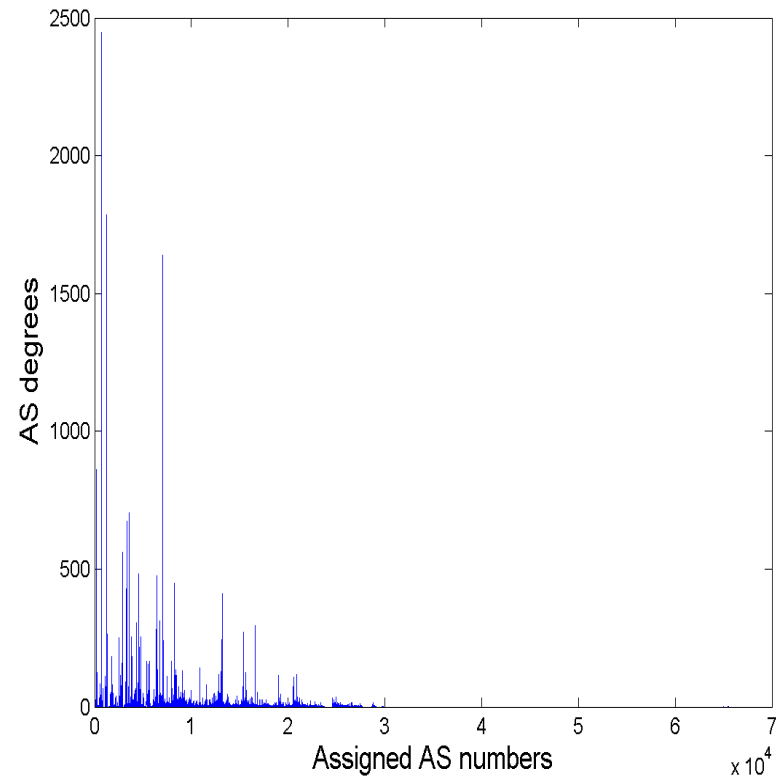


# Spectral analysis of topology data

- Consider all ASs with assigned AS numbers
- AS degree distribution in Route Views and RIPE datasets:



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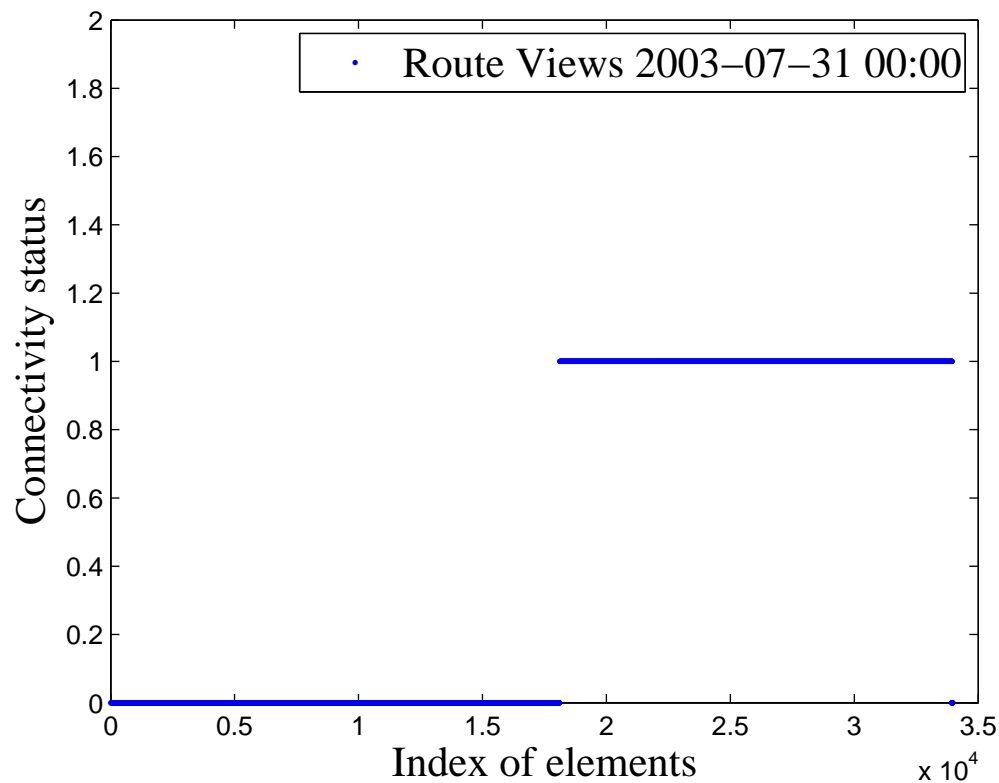


# Spectral analysis: observations

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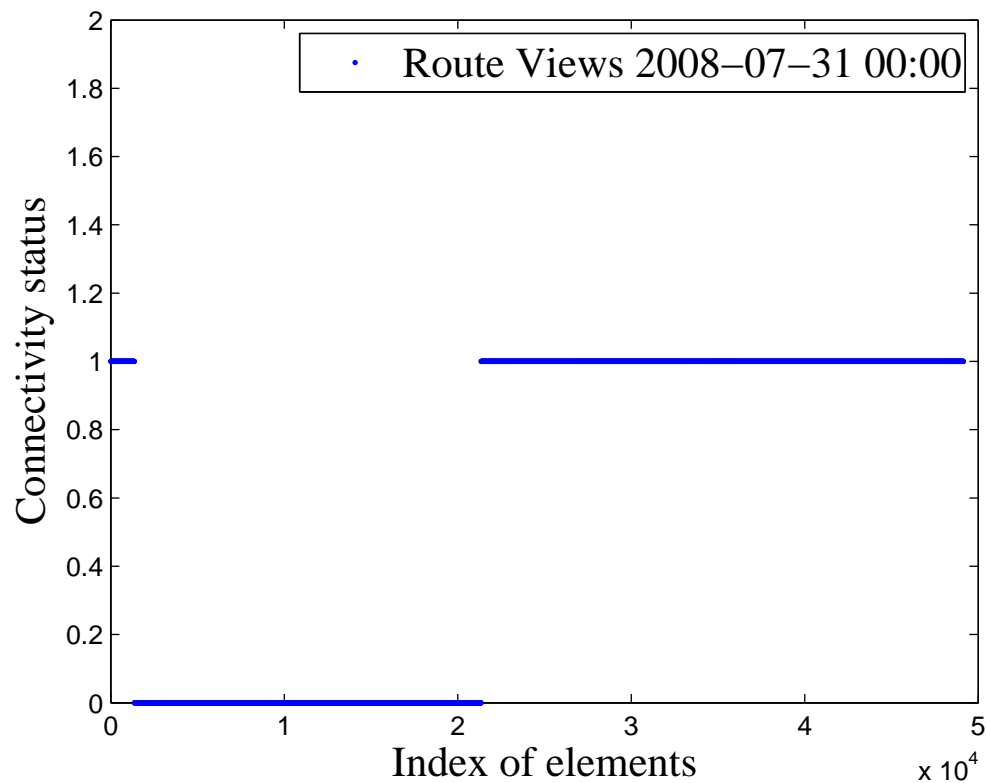
- The **second smallest** eigenvector:
  - separates connected ASs from disconnected ASs
  - Route Views and **RIPE** datasets are similar on a coarser scale
- The **largest** eigenvector:
  - reveals highly connected clusters
  - Route Views and **RIPE** datasets differ on a finer scale

# Route Views 2003 dataset



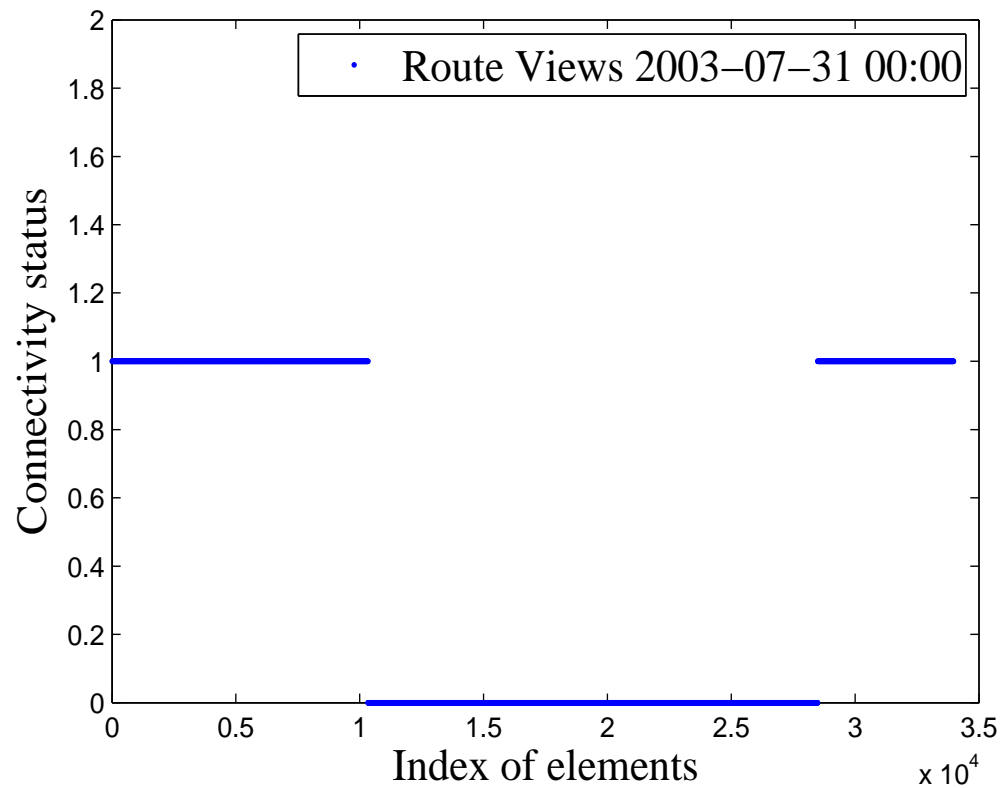
Spectral views of the AS connectivity based on the second smallest eigenvalue.

# Route Views 2008 dataset



Spectral views of the AS connectivity based on the second smallest eigenvalue.

# Route Views 2003 dataset



Spectral views of the AS connectivity based on the largest eigenvalue.

# Route Views 2008 dataset



Spectral views of the AS connectivity based on the largest eigenvalue.



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# Conclusions

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- We have evaluated collected data from the **Route Views** and **RIPE** projects and have confirmed the presence of power-laws in graphs capturing the AS-level Internet topology.
- **spectral analysis** techniques revealed distinct clustering characteristics of **Route Views** and **RIPE** datasets
- The analysis also captured historical trends in the development of the Internet topology over the past five years.
- In spite of the Internet growth, increasing number of users, and the deployment of new network elements, power-law exponents have not changed substantially.



# Conclusions

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- These power-law exponents do not capture every property of a graph and are only one measure used to characterize the Internet.
- However, spectral analysis based on the normalized Laplacian matrix indicated visible changes in the clustering of AS nodes and the AS connectivity.



# References

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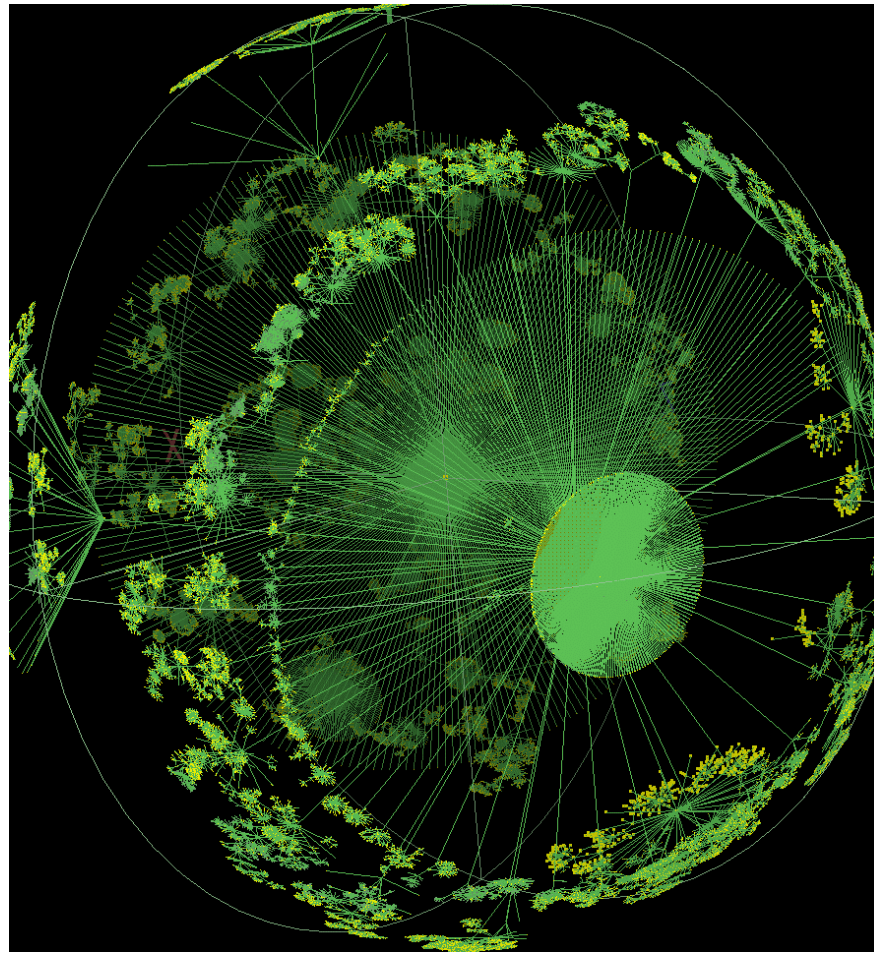
# Resources

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- CAIDA:  
The Cooperative Association for Internet Data Analysis  
<http://www.caida.org/home/>
- Walrus - Gallery: Visualization & Navigation  
<http://www.caida.org/tools/visualization/walrus/gallery1/>
- Walrus - Gallery: Abstract Art  
<http://www.caida.org/tools/visualization/walrus/gallery2/>



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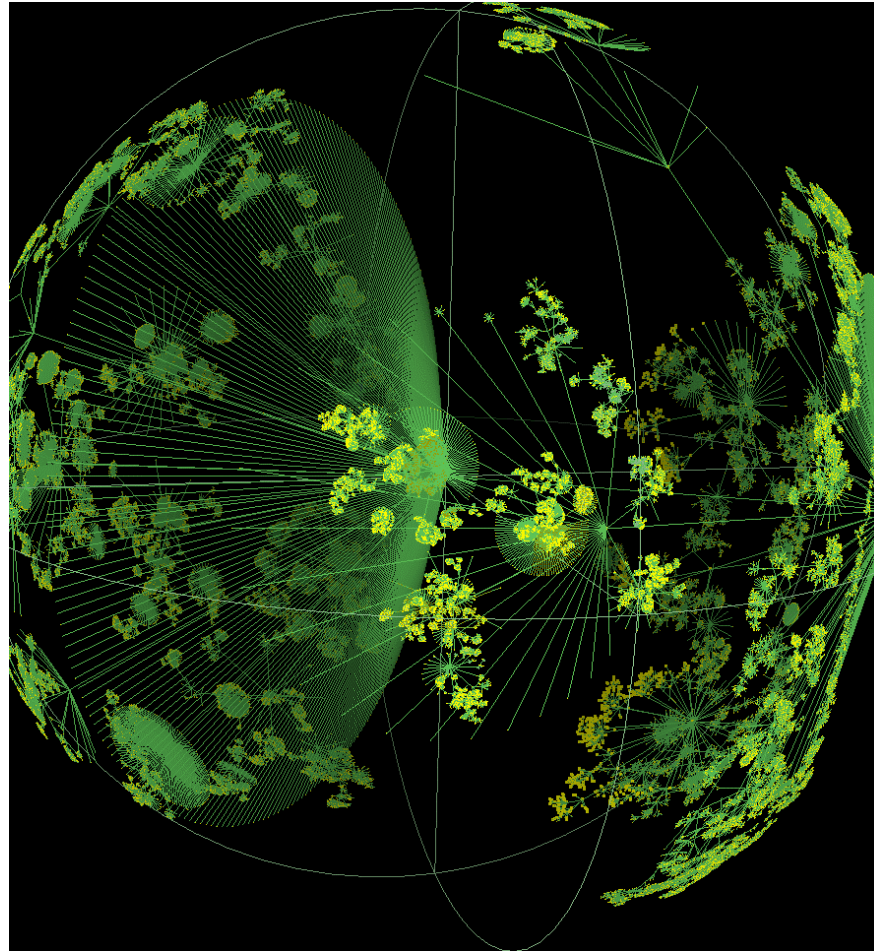
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Ihr (535,102 nodes and 601,678 links)

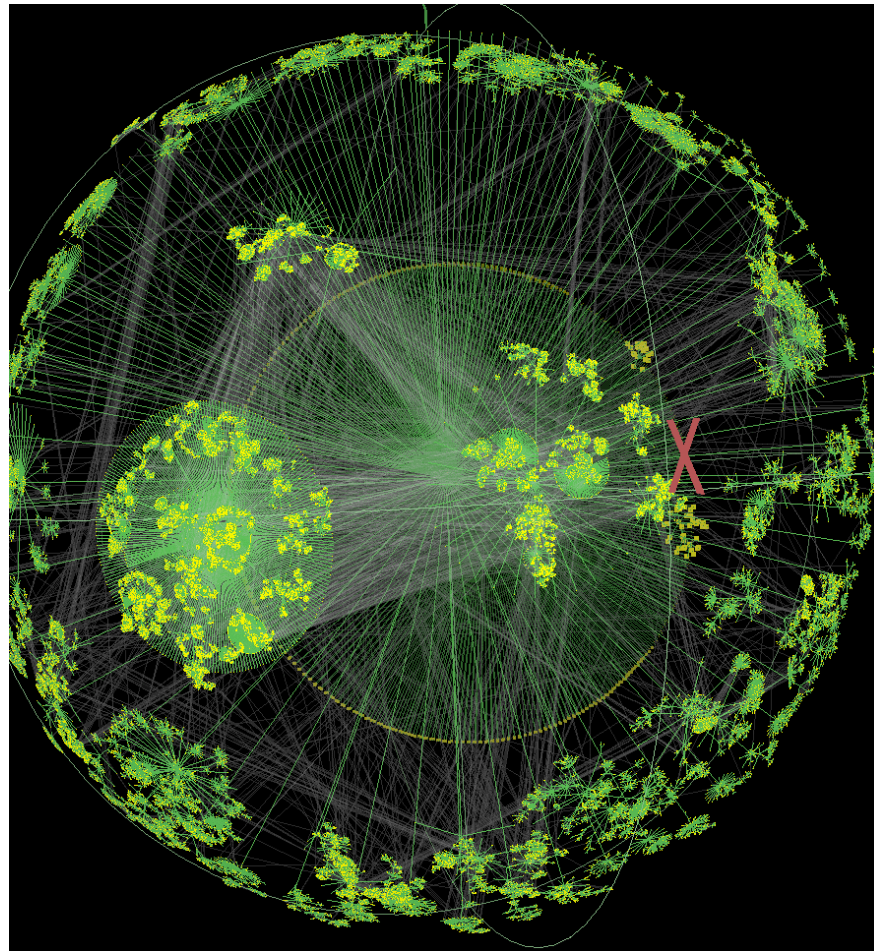


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Ihr (535,102 nodes and 601,678 links)

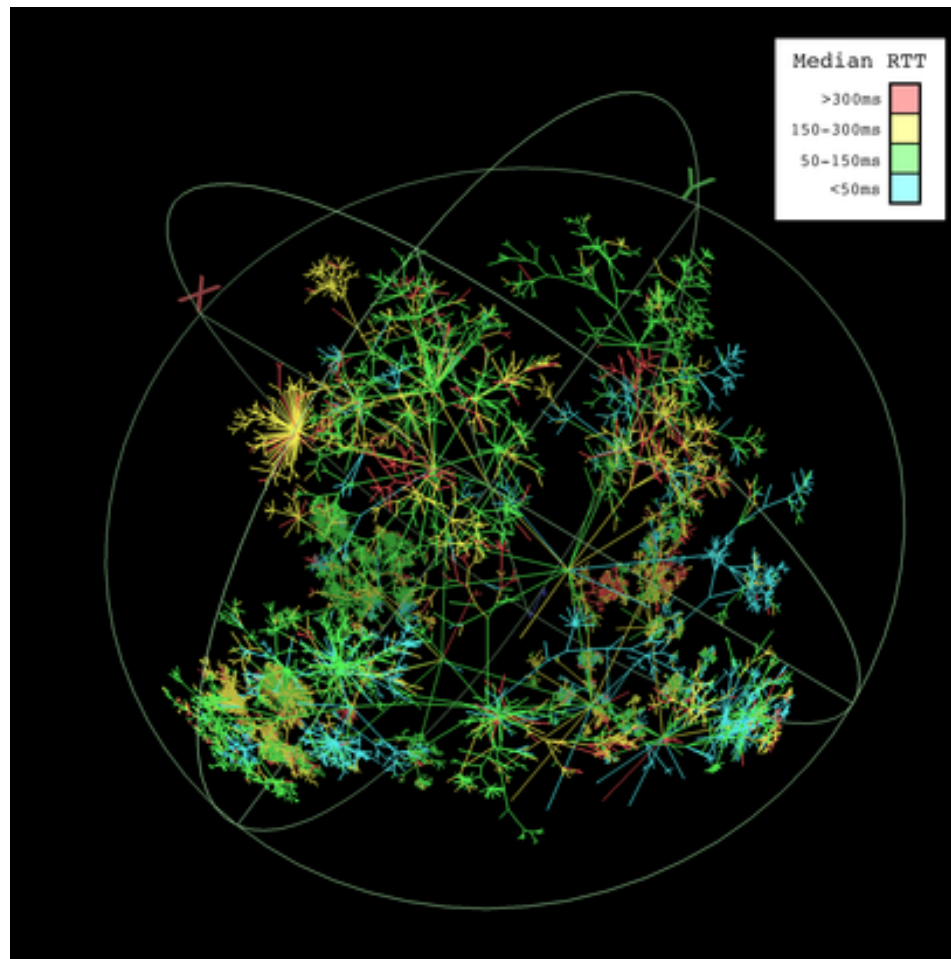


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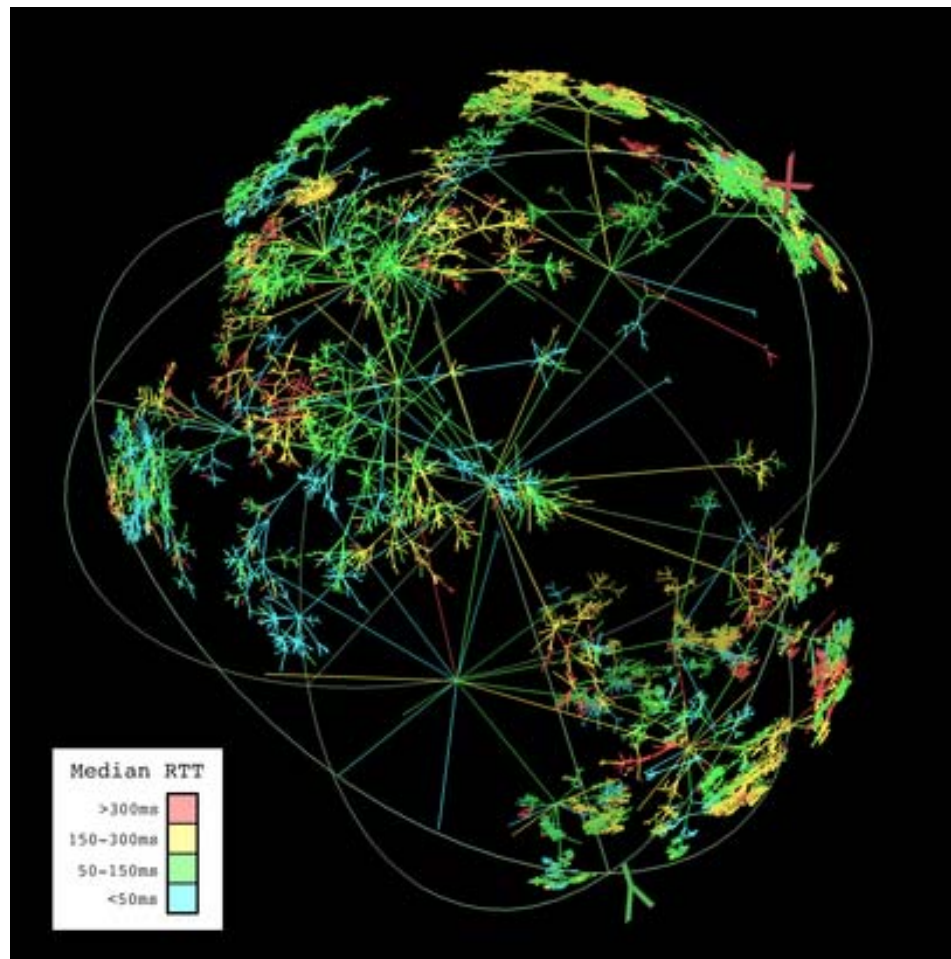
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# Round-trip time measurements (63,631 nodes and 63,630 links)



# Round-trip time measurements (63,631 nodes and 63,630 links)



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