Some Notation

$p \wedge q$ $\quad$ $p \vee q$

$\Rightarrow$ logically implies $\quad$ $\rightarrow$ implies

$\Leftrightarrow$ logically equivalent $\quad$ $\mathcal{U}$ = universe

$A = \{1, 2, 3, 4, 5\}$,
set

$2 \in A$

$6 \notin A$

if $\mathcal{U} = \{\mathbb{Z}^+\}$ $\quad$ $\{x \mid 1 \leq x \leq 5\}$
universe $\quad$ such that

$\subseteq$ subset

$\subset$ proper subset

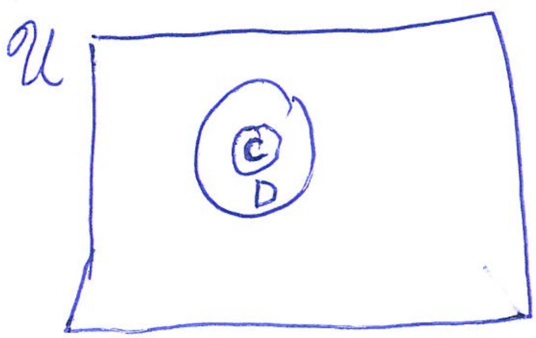
$C \subseteq D \quad D \supseteq C$

C is a subset of D

"For every x"

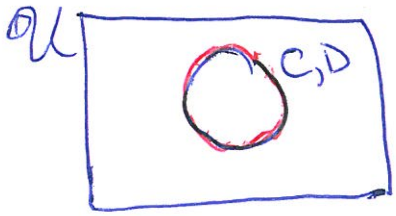
$\forall x \; \{x \in C \Rightarrow x \in D\}$

$\{x \in D \not\Rightarrow x \in C\}$



proper subset

$C \subset D$



$\forall x \; \{x \in C \Rightarrow x \in D\}$ &

$\{x \in D \Rightarrow x \in C\}$

$C \subseteq D \wedge D \subseteq C \Rightarrow C = D$ (set equality)

Finite Sets $\Rightarrow$ Cardinality

$\mathbb{Z}^+$ is an infinite set

$C \subset D \Rightarrow C \subseteq D$

$C \subseteq D \not\Rightarrow C \subset D$

$\exists x$ "There exists an x"
"For some x"

Neither order nor repetition is relevant for a general set wrt set equality

$\{1, 2, 3\} = \{3, 2, 1\} = \{2, 2, 1, 3\} = \{1, 2, 1, 3, 1\}$

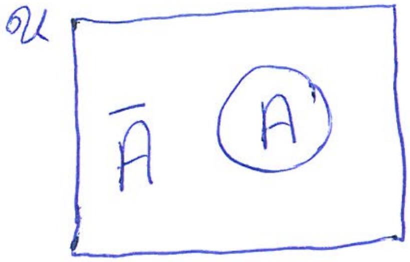
The null set (aka empty set) is the unique set containing no elements. Denoted by $\emptyset$ or $\{\}$. Note that $|\emptyset| = 0$, but $\{0\} \neq \emptyset$ & $\{\emptyset\} \neq \emptyset$

$\{\emptyset\}$ is a set with 1 element (the null set)

The power set of A, $\mathcal{P}(A)$, is the collection (or set) of all subsets of A.

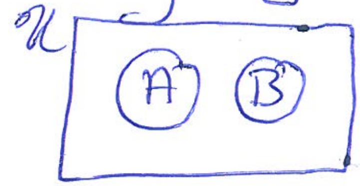
Counting & Venn Diagrams

$\mathcal{U} = A \cup \bar{A}$ & $A \cap A = \emptyset$



then $|A| + |\bar{A}| = |\mathcal{U}|$

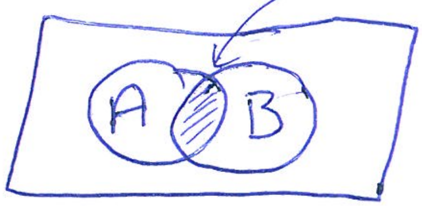
$|A \cup B| = |A| + |B| \longrightarrow$ A & B are mutually disjoint



General Case

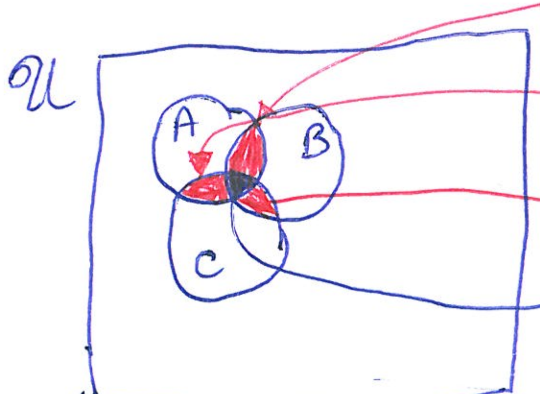
$|A \cup B| = |A| + |B| - \underline{|A \cap B|}$

$\hookrightarrow \emptyset$ if mutually disjoint; otherwise double counted



The Inclusion-Exclusion principle is a counting technique that generalizes the method of obtaining the cardinality of the union of finite sets (Note: this works independent of whether the sets are disjoint or not & can be extended for any number of finite sets that are joined as a union)

$|A \cup B \cup C| = |A| + |B| + |C|$



$- |A \cap B|$

$- |A \cap C|$

$- |B \cap C|$

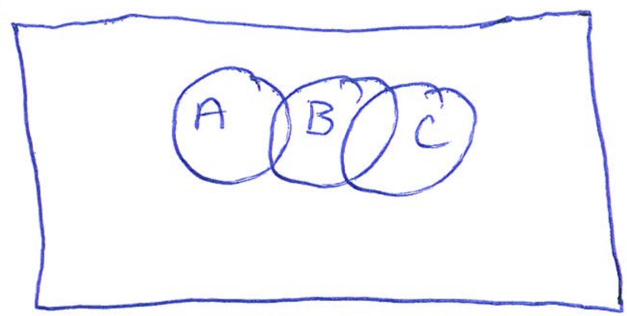
$+ |A \cap B \cap C|$ (subtracted out multiple times; included thrice $|A|$, $|B|$, $|C|$; subtracted thrice this term adds balance)

Generalization for N sets $|X_1 \cup X_2 \cup X_3 \cup \ldots \cup X_N|$

- → Include the cardinality of each individual set
- → Exclude the cardinality of each n-tuple-wise set intersections, where n is even (pairwise, quadruple-wise, etc.)
- → Include the cardinality of m-tuple-wise set intersections, where m is odd (triplewise, quintuple-wise, etc.)

Note: This not only applies to cardinality, but it also applies to Probability i.e. $Pr(A \cup B \cup C)$

$\mathcal{U}$



If the $\mathcal{U}$ is finite, then

$|\bar{A} \cap \bar{B} \cap \bar{C}| = |\overline{A \cup B \cup C}| =$

$|\mathcal{U}| - |A \cup B \cup C|$

$= |\mathcal{U}| - (|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$

* Set Theory Tutorial Questions

① Let $A, B, C \subseteq \mathcal{U}$

Prove: a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$
b) If $A \subset B$ and $B \subseteq C$, then $A \subset C$
c) If $A \subseteq B$ and $B \subset C$, then $A \subset C$
d) If $A \subset B$ and $B \subset C$, then $A \subset C$

② Which of the following are non-empty sets
a) $\{x \mid x \in \mathbb{N}, 2x + 7 = 3\}$
b) $\{x \in \mathbb{Z} \mid 3x + 5 = 9\}$
c) $\{x \mid x \in \mathbb{Q}, x^2 + 4 = 6\}$
d) $\{x \in \mathbb{R} \mid x^2 + 4 = 6\}$
e) $\{x \in \mathbb{R} \mid x^2 + 3x + 3 = 0\}$
f) $\{x \mid x \in \mathbb{C}, x^2 + 3x + 3 = 0\}$

③ Let $A = \{1, 2, 3, 4, 5, 6\}$, give $\mathcal{P}(A)$.

④ Let $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $S, T \subseteq \mathcal{U}$
a) Define a mutually disjoint set, in terms of S and T.
b) Specify 3 different valid sets of elements for S and T, such that S and T are disjoint (mutually disjoint).

⑤ Addition & Multiplication are closed binary operators on $\mathbb{Z}^+$ (i.e. $\{a \in \mathbb{Z}^+, b \in \mathbb{Z}^+ \Rightarrow a+b \in \mathbb{Z}^+\}$). Prove that Subtraction and Division are not closed operators on $\mathbb{Z}^+$

⑥ If $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{7, 8, 9\}$
Find: a) $A \cap B$ b) $A \cup B$ c) $B - A$ d) $A - B$
e) $A - C$ f) $C - A$ g) $A - A$ h) $\mathcal{U} - A$

⑦ Prove $\overline{A \cup B} = \bar{A} \cap \bar{B}$

⑧ Draw Venn Diagrams for all the set relations given here.