ENSC 383 Feedback Control Systems

Assignment 1 Solutions

Fall semester, 2007

1. Since the equations of motion for the cruise control system is given as

$$\dot{v} + \frac{b}{m}v = \frac{u}{m} \tag{1}$$

and the controller is designed as

$$u = K(v_r - v) \tag{2}$$

the whole system under proportional control law is

$$\dot{v} + \frac{b}{m}v = \frac{K(v_r - v)}{m} \tag{3}$$

which is equivalent to

$$\dot{v} + \frac{b+k}{m}v = \frac{k}{m}v_r \tag{4}$$

To derive the transfer function $H(s) = \frac{V(s)}{V_r(s)}$, we set the initial conditions for (4) as zero. Taking Laplace transform, we have

$$sV(s) + \frac{b+k}{m}V(s) = \frac{k}{m}V_r(s) \tag{5}$$

and

$$H(s) = \frac{V(s)}{V_r(s)} = \frac{\frac{k}{m}}{s + \frac{b+k}{m}} \tag{6}$$

Typical code for the step response using proportional control is as follows.(The code is not necessary the same as mine, but the gain K should be similar)

```
m = 1000; % mass
b = 50;
K = 649; % gain
num = K;
```

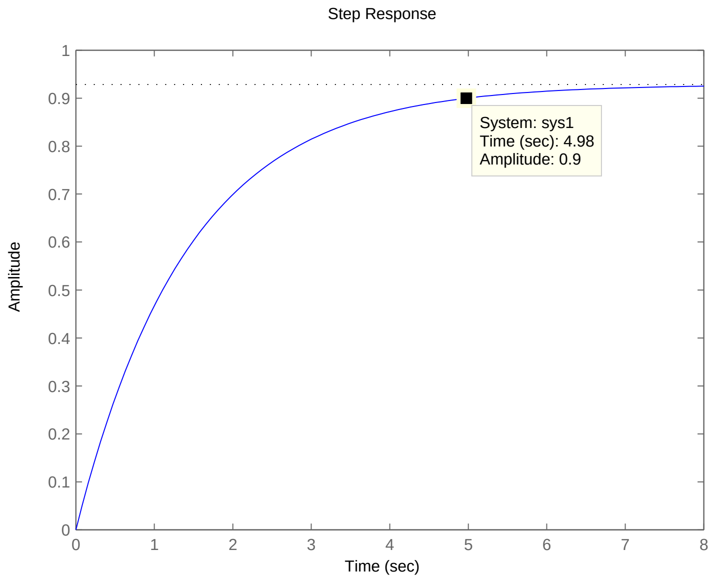

Figure 1: Step response for the question 1

den = [m b+K];
sys1 = tf(num, den); % transfer function
step(sys1)

Remark: The reference speed is unit, which means the output should converge 0.9 within 5 seconds.

2. According to the example 2.11 and example 2.4, we can plot the diagrams for torques on the two disks in Figure 2.

   Applications of Newton's laws on the two disks yields

$$T = J_1\ddot{\theta}_1 + k(\theta_1 - \theta_2) \tag{7}$$
$$J_2\ddot{\theta}_2 = k(\theta_1 - \theta_2) \tag{8}$$

   Based on circuit theory, we have

$$T = K_t i_a \tag{9}$$
$$L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e\dot{\theta}_1 \tag{10}$$

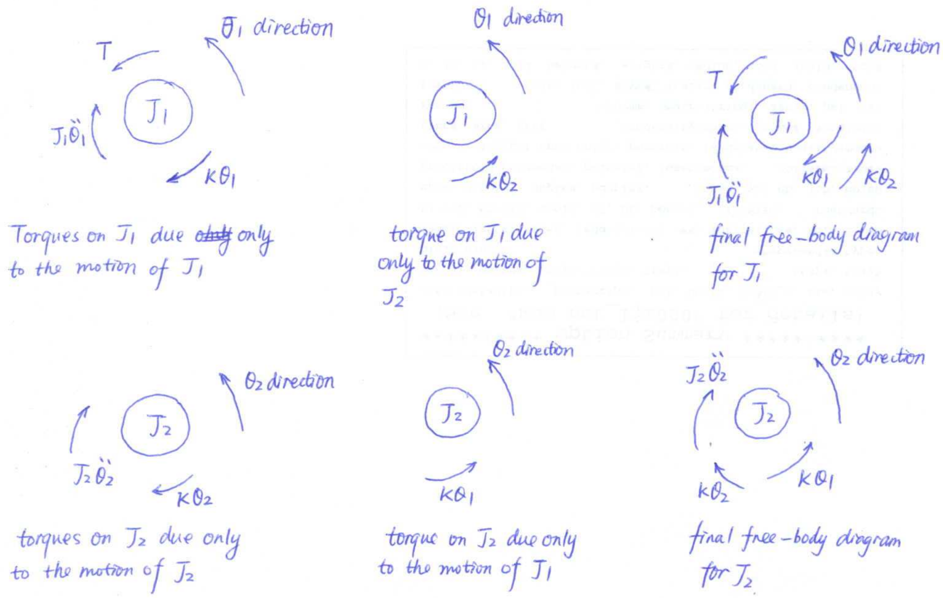

Figure 2: Diagram for the torques on the two disks