ENSC 383 Feedback Control Systems

Assignment 2 Solutions

Fall semester, 2007

3.16 (a)

$$\begin{aligned} \text{DC gain} &= \lim_{s\to 0} G(s) \\ &= \lim_{s\to 0} \frac{3}{s^2+2s-3} \\ &= -1 \end{aligned} \tag{1}$$

(b) Response of a step input

$$\begin{aligned} Y(s) &= U(s)G(s) \\ &= \frac{1}{s}\frac{3}{s^2+2s-3} \end{aligned} \tag{2}$$

Final value:

Since the system is unstable, FVT is not applicable. Therefore output y(t) is unbounded.

$$\begin{aligned} y(\infty) &= sY(s)\,|_{s=0} \\ &= -1 \end{aligned} \tag{3}$$

3.19 (a)

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1} + G_2 \tag{4}$$

(b)

$$\frac{Y(s)}{R(s)} = G_7 + \frac{G_1G_3G_4G_6}{(1+G_1G_2)(1+G_4G_5)} \tag{5}$$

(c)

$$\frac{Y(s)}{R(s)} = \frac{G_4G_5(G_6+G_2G_6+G_1G_2G_3)}{(1+G_2)(1+G_4)} + G_7 \tag{6}$$

3.23 The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2 + 2s + K} \tag{7}$$

Then we have

$$\begin{aligned} \omega_n^2 &= K &(8)\\ 2\zeta\omega_n &= 2 &(9) \end{aligned}$$

Then

$$\zeta^2 = \frac{1}{K} \tag{10}$$

Based on the definition of overshoot and requirement

$$\begin{aligned} M_p &= e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 10\% &(11)\\ \frac{\pi\zeta}{\sqrt{1-\zeta^2}} &\geq -\ln 0.1 = 2.3026 &(12)\\ & &(13) \end{aligned}$$

Solving above equation, we have

$$\zeta^2 \geq 0.3495$$

and

$$0 < K \leq 2.8612$$

3.24 The closed loop system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{100K}{s^2 + (25+a)s + 25a + 100K} \tag{14}$$

Therefore,

$$\begin{aligned} \omega_n^2 &= 25a + 100K &(15)\\ 2\zeta\omega_n &= 25 + a &(16) \end{aligned}$$

Based on the requirement of system response, we have

$$\begin{aligned} e^{-\zeta\omega_n t_s} &\leq 0.01 &(17)\\ t_s &= 0.1 &(18)\\ M_p &= e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leq 25\% &(19) \end{aligned}$$

Deriving the above inequalities, we have

$$\begin{aligned} 0.1\zeta\omega_n &\geq -\ln 0.01 = 4.6052 &(20)\\ \frac{\pi\zeta}{\sqrt{1-\zeta^2}} &\geq -\ln 0.25 = 1.3863 &(21) \end{aligned}$$

Solving these two inequalities, we get

$$0.4037 \leq \zeta < 1 \tag{22}$$

From (16) and (20), we have

$$\frac{25+a}{2} \geq 46.052 \tag{23}$$

Therefore,

$$a \geq 67.104$$

From (16) and (22), we obtain

$$\frac{25+a}{2} < \omega_n = \frac{25+a}{2\zeta} \leq \frac{25+a}{2 \times 0.4037} \tag{24}$$

(25)

Hence, based on (15), we have

$$(\frac{a+25}{2})^2 < 25a + 100K \leq 1.534(a+25)^2 \tag{26}$$

Solve the left inequality, we have

$$K > 4.4318$$

Solve the right inequality, we have

$$K \leq 113.35$$

In summary,

$$a \geq 67.104$$

$$4.4318 < K \leq 113.35$$

MATLAB Code:

```
a = 67.10399;
K = 113.3546;
num = 100*K;
den = [1 25+a 25*a+100*K];
sys1 = tf(num, den);
step(sys1)
```

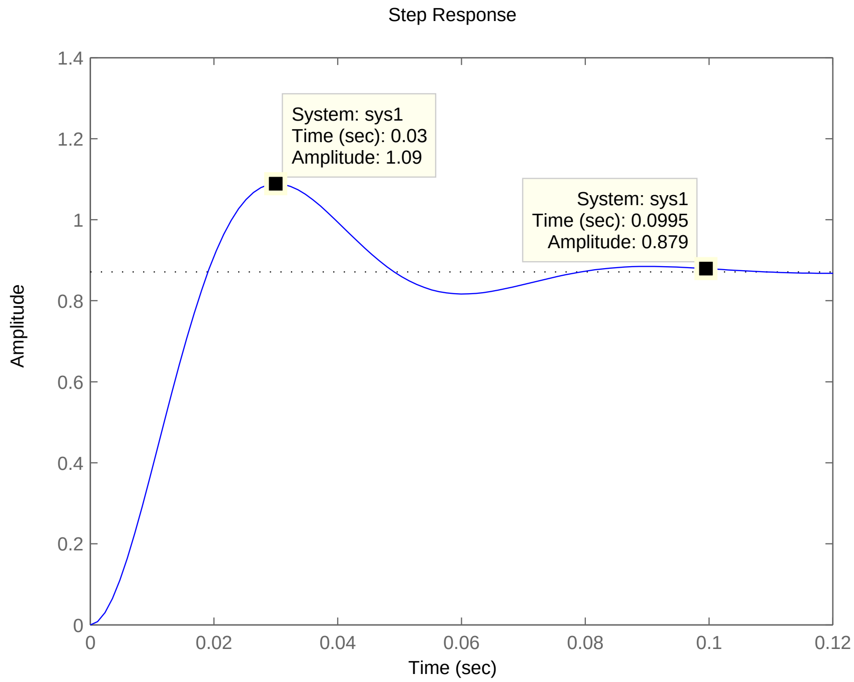

Figure 1: Unit step response in 3.24

3.41 The characteristics equation is

$$1 + G(s) = 0$$

Therefore, we have the following characteristics equation:

$$s^5 + 1.9s^4 + 5.1s^3 + 6.2s^2 + (2 + K)s + 4K = 0$$

The corresponding Routh array is

$$\begin{array}{cccc} s^5 & 1 & 5.1 & 2+K \\ s^4 & 1.9 & 6.2 & 4K \\ s^3 & 1.8368 & \frac{3.8-2.1K}{1.9} & 0 \\ s^2 & \frac{2.1K+7.5882}{1.8368} & 4K & 0 \\ s^1 & \frac{4.41K^2+33.59632K-28.83516}{-1.9(2.1K+7.5882)} & & \\ s^0 & 4K & & \end{array} \tag{27}$$

Therefore, in order to guarantee the stability of the system, we need

$$\frac{2.1K+7.5882}{1.8368} > 0 \tag{28}$$

$$\frac{4.41K^2+33.59632K-28.83516}{-1.9(2.1K+7.5882)} > 0 \tag{29}$$

$$4K > 0 \tag{30}$$

The above conditions can be summarized as

$$K > 0 \quad (31)$$
$$4.41K^2 + 33.59632K - 28.83516 < 0 \quad (32)$$

Solving the above two inequalities, we obtain

$$0 < K < 0.77868$$

The result can be verified by using the Matlab code:

```
K = 0.77869;
p = [1 1.9 5.1 6.2 2+K 4*K];
roots(p)
```

The roots of the polynomial are

$$\begin{array}{r} -0.27747373339235 + 1.91214415607866i \\ -0.27747373339235 - 1.91214415607866i \\ -1.34505326426898 \\ 0.00000036552684 + 0.78757891093995i \\ 0.00000036552684 - 0.78757891093995i \end{array}$$

where two poles are in the right hand side of y-coordinates.

When $K = 0$, there exists a pole equal to 0, which also causes unstable.