Ch. 4 Solutions : (4.5), (4.10), (4.11), (4.19)

#(4.5) (a) P → $\frac{Y(s)}{R(s)} = \frac{k_p\, G(s)}{1 + k_p\, G(s)} = \frac{19}{5s^2 + 6s + 20}$

Using FVT ⟹ $e_{ss} = 19/20$

Type 0, $k_p = 19$

PD → $\frac{Y}{R} = \frac{19 + 4s}{5s^2 + 10s + 20}$ → FVT → $y_{ss} = \frac{19}{20}$

$e_{ss} = 1/20$ → Type 0, $K_p = 19$

PID → $Y/R = \frac{8s^2 + 38s + 19}{10s^3 + 20s^2 + 40s + 19}$

FVT → $y_{ss} = 1$, $e_{ss} = 0$ → Type 1 with $K_v = 9.5$

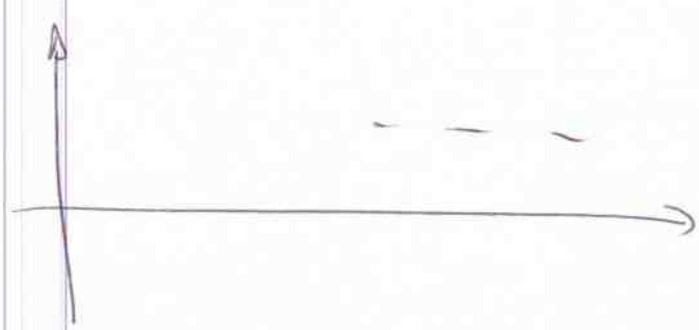
P ⇝ $Y/W = \frac{1}{5s^2 + 6s + 20}$ → $y_{ss} = 1/20$, type 0, $K_p = 19$

PD ⇝ $Y/W = \frac{G}{1 + DG} = \frac{1}{5s^2 + 10s + 20}$ → $y_{ss} = 1/20$, Type = 0

PID → $Y/W = \frac{G}{1 + DG} = \frac{2s}{10s^3 + 20s^2 + 40s + 19}$ → $y_{ss} = 0$ Type 1

(b) The derivative term will reduce oscillatory behavior
The integral term will reduce error but increase oscillatory behavior

(c) plots using MATLAB



(4.19)

(a) $$\frac{Y}{R} = \frac{10(k_I + k_p s)}{s[s(s+1)+20] + 10(k_I + k_p s)}$$

(b) $$\frac{Y}{W} = \frac{10s}{s[s(s+1)+20] + 10(k_I + k_p s)}$$

(c) Ch. eq. $\rightarrow s^3 + s^2 + (10k_p + 20)s + 10k_I = 0$

Routh array

s^3	1	$10k_p + 20$
s^2	1	$10k_I$
s	$10k_p + 20 - 10k_I$	
s^0	$10k_I$	

$\Longrightarrow$ For stability $k_I > 0$, $k_p > k_I - 2$

(d) System is type 1

$K_v = \frac{k_I}{2}$ (for reference tracking)

$K_v = k_I$ (for disturbance rejection)

(4.11)

(a) $$DG = \frac{K(s+\alpha)^2}{(s^2+\omega_o^2)(s+1)s}$$

$$\frac{E}{R} = \frac{1}{1+DG} = \frac{s(s+1)(s^2+\omega_o^2)}{(s+\omega_o^2)(s)(s+1) + K(s+\alpha)^2}$$

Note: At $s = \pm j\omega_o$ the gain E/R is zero

Let $R = \frac{\omega_n}{s^2+\omega_n^2}$ then

$$E(s) = \frac{s(s+1)(s^2+\omega_o^2)}{(s+\omega_o^2)s(s+1) + K(s+\alpha)^2} \frac{\omega_n}{s^2+\omega_n^2}$$

If $\omega_n = \omega_o \longrightarrow e_{ss} = \lim_{s \to 0} sE(s) = 0$

(b) Characteristic eq.:

$$s^4 + (K+\omega_o^2)s^2 + s^3 + (\omega^2 + 2\alpha K)s + K\alpha^2 = 0$$

1	$\omega^2 + K$	$K\alpha^2$
1	$\omega^2 + 2\alpha K$	
$K(1-2\alpha)$	$K\alpha^2$	
$\omega^2 + 2\alpha K + \frac{\alpha^2}{1-2\alpha}$		
$K\alpha^2$		

$\longrightarrow$ If $\alpha = 0.25$, we must have $K > 0$ & $K > 0.25 - 2\omega^2$

4.10

(a) Using block diagram reduction $\rightarrow$ $k_t' = \frac{k k_T}{K_p}$

$$K' = \frac{K_p k K_m}{k}$$

(b) Inner loop may be reduced to

$$\frac{k K_m}{s(1 + \tau_m s + k K_m k_t)}$$

$\rightarrow$ Unity feedback has an integrater

$\Rightarrow$ type is 1

Thus $K_v = \frac{k K_m}{1 + k K_m k_t}$

(c) k_t reduces velocity constant and therefore makes error to a ramp larger