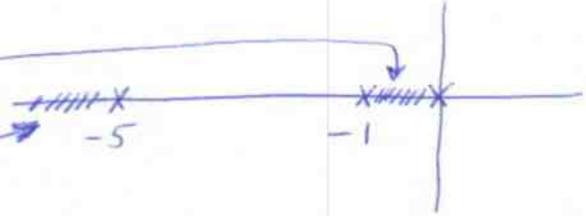
Assignment 4: 5.3, 5.12, 5.16, 5.23, 5.26

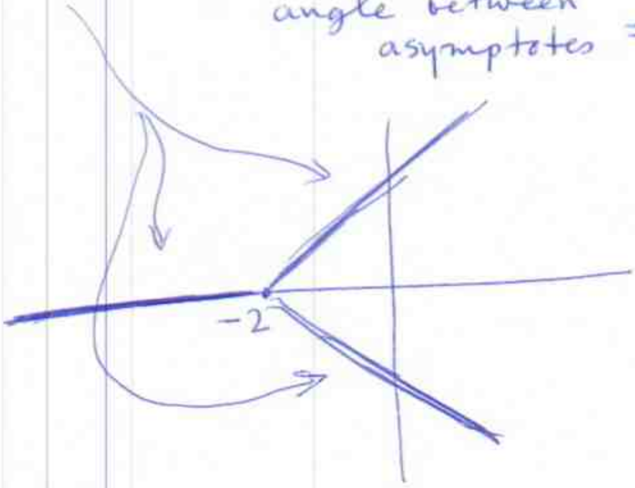
5.3 (a) Real axis segments

$$1 + k\frac{1}{s(s+1)(s+5)} = 0$$



(b) Asymptotes: intercept on real axis $= \dfrac{\Sigma\text{poles} - \Sigma\text{zeros}}{3-0} = \dfrac{-5-1}{3} = -2$

angle between asymptotes $= \dfrac{180 + 360\ell}{3} = 60, -60, 180$ $\quad \ell = 0, \pm 1, \pm 2, \ldots$



(c) To find K for imaginary axis crossing ~~points~~ use Routh's test

$$s(s+1)(s+5) + K = 0 \rightarrow s^3 + 6s^2 + 5s + k = 0$$

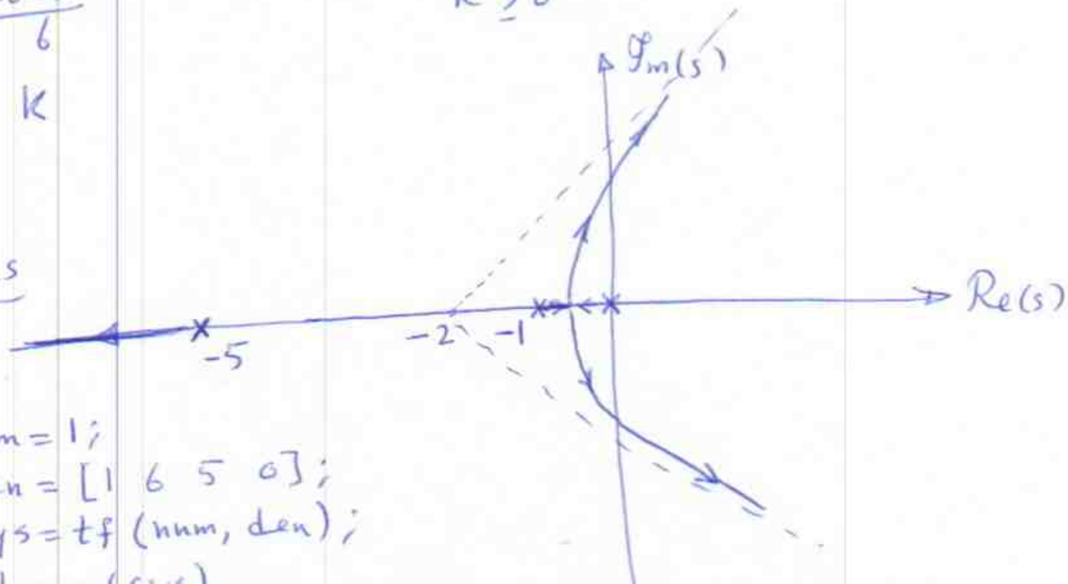
Routh array

$$\left\{\begin{array}{ll} 1 & 5 \\ 6 & K \\ \frac{30-k}{6} & \\ K & \end{array}\right.$$

$\Rightarrow 30 - k > 0 \rightarrow k = 30$

$\Rightarrow k \geq 0 \quad\quad k = 0$

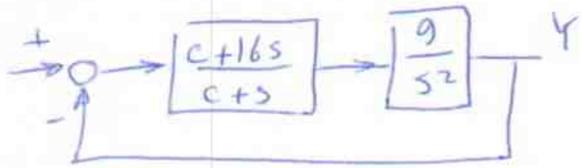
Root locus



MATLAB Code

```
num = 1;
den = [1 6 5 0];
sys = tf(num, den);
```

(5.12)



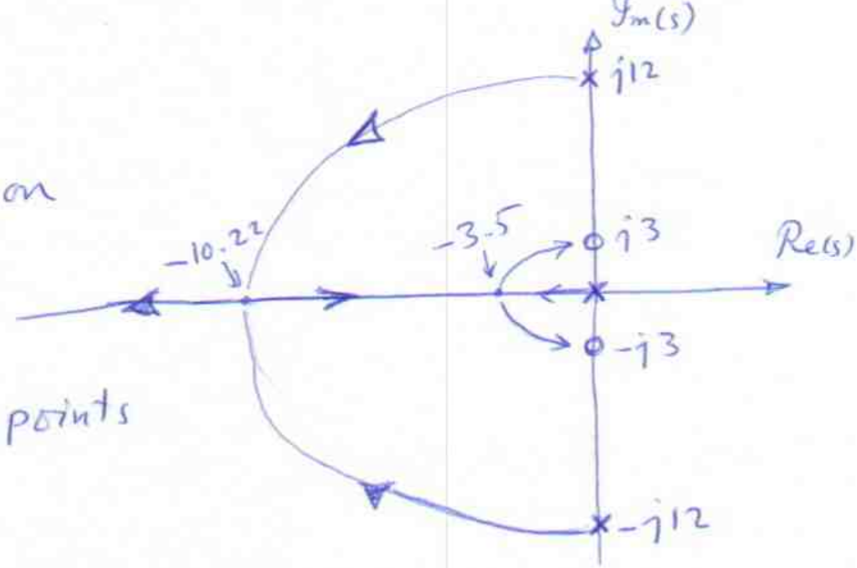
ch. eq. $1 + \frac{(16s+c)9}{s^2(s+c)} = 0$

$\therefore\ s^3 + cs^2 + 144s + 9c = 0$

$s^3 + 144s + c(s^2+9) = 0 \longrightarrow 1 + c\,\frac{s^2+9}{s(s^2+144)} = 0$

Thus $1 + c\,\frac{b(s)}{a(s)} = 0$, where $b(s) = s^2+9$, $a(s) = s^3+144s$

Poles: $s=0$, $s=\pm j12$ Zeros: $s=\pm j3$



* Locus will be to the left of point (0,0) on the real axis

* Break-away, break-in points

$a\frac{db}{ds} - b\frac{da}{ds} = 0$

$(s^3+144s)\,2s - (s^2+9)(3s^2+144) = 0$

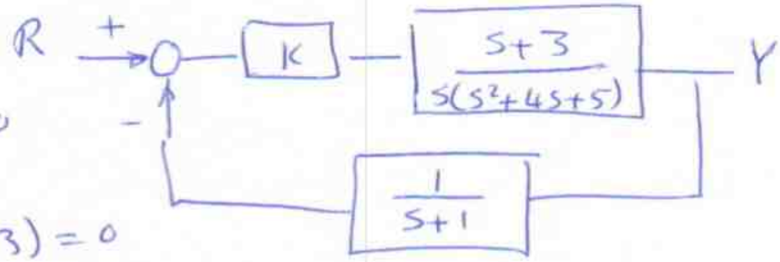
$2s^4 + 288s^2 - 3s^4 - 144s^2 - 27s^2 - 1296 = 0$

$-s^4 + 117s^2 - 1296 = 0 \longrightarrow s^4 - 117s^2 + 1296 = 0$

$\therefore\ s^2 = \frac{117}{2} \pm \sqrt{\left(\frac{117}{2}\right)^2 - 1296}\ \begin{cases} 12.39 \longrightarrow s = \pm 3.5 \\ 104.61 \longrightarrow s = \pm 10.22 \end{cases}$

Hence break in/away points are $\longrightarrow -3.5, -10.22$
(NOTE: +3.5, +10.22 are not acceptable as locus does not lie on +ve real axis)

(5.16)



(a) $1 + K \dfrac{s+3}{s(s+1)(s^2+4s+5)} = 0$

$(s^2+s)(s^2+4s+5) + K(s+3) = 0$

$s^4 + 4s^3 + \underline{5s^2} + s^3 + \underline{4s^2} + 5s + Ks + 3K = 0$

$s^4 + 5s^3 + 9s^2 + (5+K)s + 3K = 0$

Routh's array

1	9	$3K$
5	$5+K$	0
$\frac{40-K}{5}$	$3K$	0
$\frac{200+35K-K^2}{40-K}$		
$3K$		

$\Longrightarrow$ For stability:

$40 - K > 0 \rightarrow K < 40$

$3K > 0 \rightarrow K > 0$

$200 + 35K - K^2 > 0$

OR $K^2 - 35K - 200 < 0$

Roots $= -5, 40$

Thus if $-5 < K < 40$ quadratic term is negative

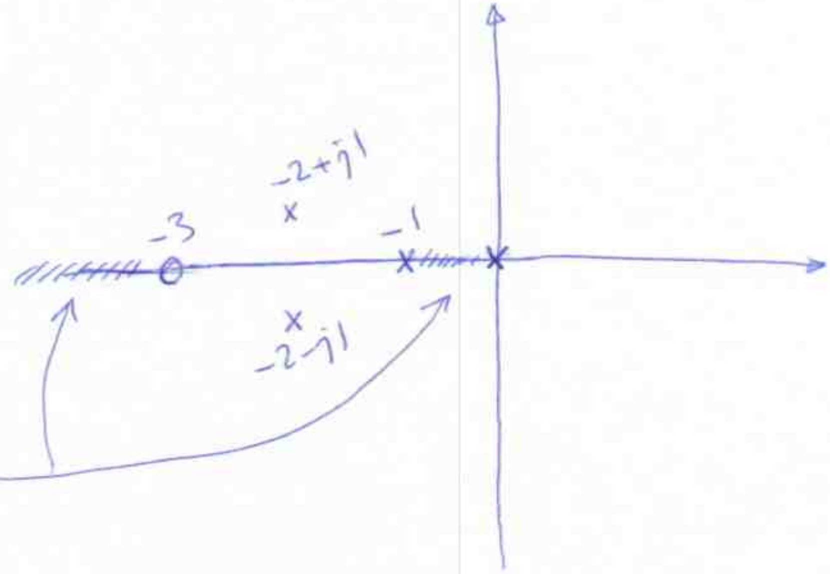
Stability range for K $\Longleftarrow$ $0 < K < 40$

(b) Root locus

* poles $s = 0, -1, -2 \pm j1$

Zeros $s = -3$

* Real axis portion



.... 5.16 continued

* Asymptotes $\longrightarrow$ intercept on real axis $= \dfrac{\sum \text{poles} - \sum \text{zeros}}{4-1}$

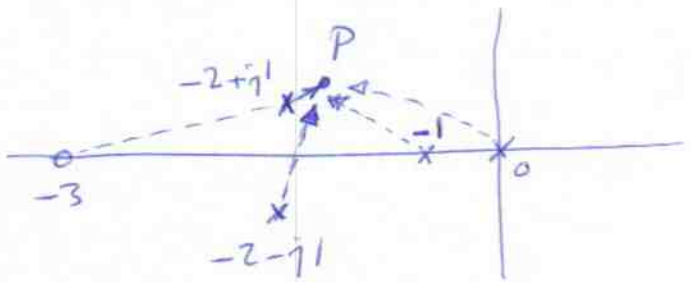
$$= \frac{-1-2-2+3}{3} = -\tfrac{2}{3}$$

$$\text{angles} = \frac{180+360\ell}{4-1} = 60^\circ, -60^\circ, 180$$

$\ell = 0, \pm 1, \pm 2, \dots$

* Angles of departure (~~from~~ complex poles $-2 \pm j1$)

Consider a point P close to $-2+j1$ on the locus. We have:

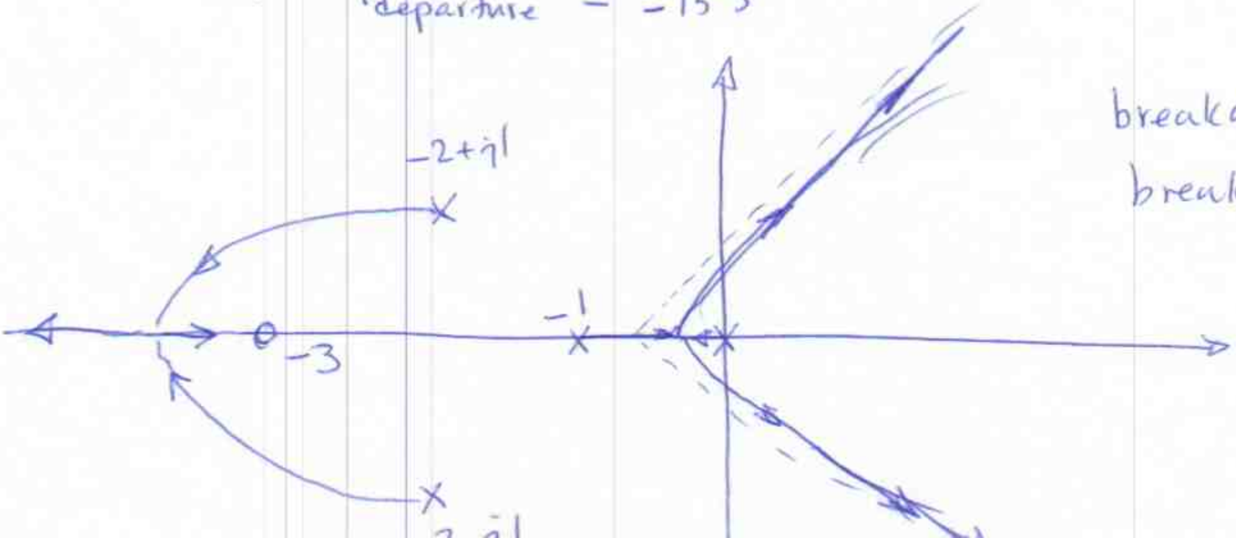


$$\frac{P+3}{P(P+1)(P+2-j1)(P+2+j1)}$$ $\rightsquigarrow$ Use phase condition:

$$360\ell + 180^\circ = \tan^{-1}\frac{1}{1} - \varphi_{departure} - \underbrace{90^\circ}_{\text{angle from } -2-j1} - \tan^{-1}\frac{1}{-1} - \tan^{-1}\frac{1}{-2}$$

OR $$180^\circ + 360\ell = 45^\circ - \varphi_{departure} - 90 - 135^\circ - 153.4^\circ$$

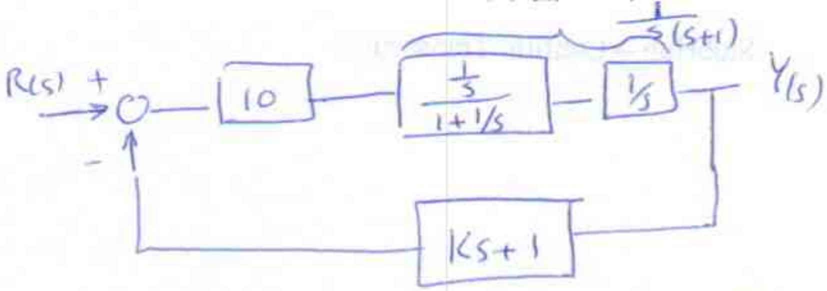
$$\therefore \varphi_{departure} = -153^\circ$$



breakaway $\doteq -0.4363$

breakin $\doteq -3.65$

5.23

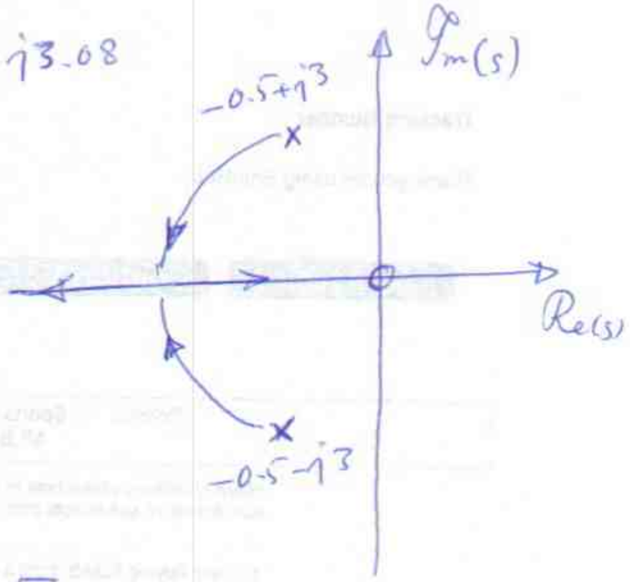


$$L(s) = \frac{10(Ks+1)}{s(s+1)}$$

$$1 + \frac{10(Ks+1)}{s(s+1)} = 0 \rightarrow s^2 + s + 10Ks + 10 = 0 \rightarrow 1 + K\frac{10s}{s^2+s+10} =$$

Zeros $\rightarrow s = 0$

Poles $\rightarrow s = -\frac{1}{2} \pm \sqrt{\frac{1}{2} - 10} = -0.5 \pm j3.08$



$$s^2 + (10K+1)s + 10 = 0$$

char. eq.

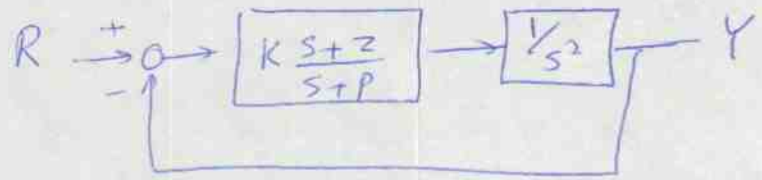
$$\omega_n^2 = 10$$

$$2\zeta\omega_n = 10K+1 \rightarrow 2\zeta\sqrt{10} = 10K+1$$

$$\zeta = 0.5 \rightarrow 2 \times 0.5 \times \sqrt{10} = 10K+1$$

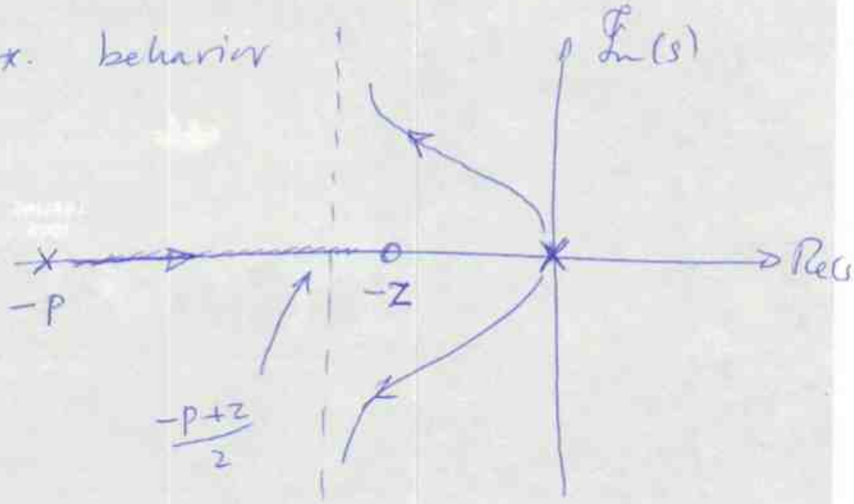
$$\therefore \boxed{K = 0.216}$$

5.26



Dominant poles → $s = -2 \pm j2$

* Root locus shows approx. behavior
* lead compensator → $z < p$



* Asymptotes

$$\frac{\sum \text{poles} - \sum \text{zeros}}{3-1} = \frac{-p+z}{2}$$

angles → $\frac{180 + 360\ell}{3-1} = \pm 90^\circ$

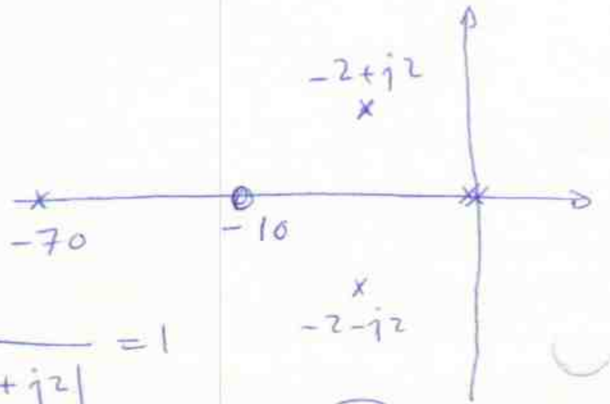
To have dominant poles → $\frac{z-p}{2} \ll -2 \rightarrow z - p \ll -4$
at $-2 \pm j2$

$$\boxed{p \gg 4 + z}$$

For pole to be dominant we also set → $z \gg 2$

Take $\begin{cases} p = 5\,(4+z) \longrightarrow p = 5\,(4+10) = 70 \\ z = 5 \times 2 = 10 \end{cases}$

To find k
use magnitude condition at $-2+j2$



$$k \left| \frac{(s+z)}{s^2(s+p)} \right| = 1 \rightsquigarrow k \frac{|-2+j2+10|}{|2+j2|^2\,|70-2+j2|} = 1$$

$$k \frac{\sqrt{8^2+2^2}}{} = 1 \longrightarrow \boxed{k = 65.9} \approx 66 \Rightarrow \text{Controller } s+10$$