# School of Engineering Science
# Simon Fraser University

## ENSC-380, Summer 2007
## Final Exam
## August 17, 2007

**Name:**

**Student No.:** _ _ _ _ _ _ – _ _ _ _

- Aid allowed: Three double sided A4 formula sheets.
- There are **8 questions** in this exam for a total of 50 marks.
- Please write your **name** on this paper and **return** with your exam booklet
- **Time: 3 Hours**

| Question | Score |
|---|---|
| 1 | /3 |
| 2 | /3 |
| 3 | /4 |
| 4 | /4 |
| 5 | /8 |
| 6 | /8 |
| 7 | /10 |
| 8 | /10 |
| **Total** | /50 |

**Potentially useful Formulas:**

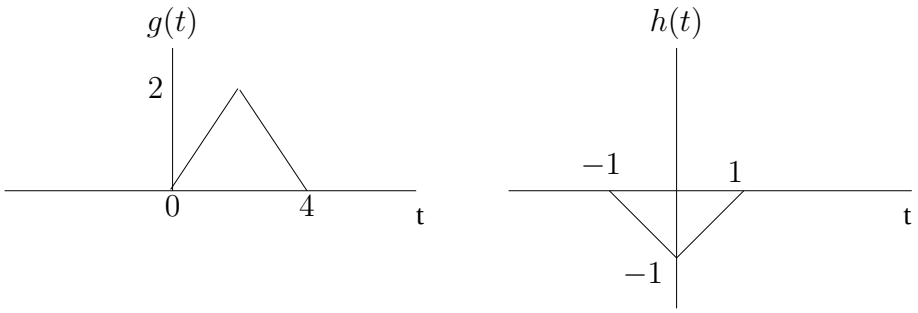
$$\text{sinc}(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{rect}(f)$$

$$\text{comb}(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{comb}(f)$$

$$\alpha^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{z-\alpha}$$

$$n\alpha^n u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z\alpha}{(z-\alpha)^2}$$

1. For the given pair of functions below, determine what transformation has been performed on $g(t)$ to result in $h(t)$:




2. Consider the convolution:

$$g(t) = \text{rect}(t) * \text{comb}(2t)$$

Find and sketch $g(t)$. You can show your work analytically, graphically, or both.

3. The harmonic function for a CT and periodic signal with **representation period of 2(s)** is given:

$$X[k] = 3\delta[k-2] + \delta[k] + 3\delta[k+2]$$

What is the time function associated with this function?

4. Use the bilateral definition of Laplace transform:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

to find the **bilateral Laplace transform** and region of convergence **(ROC)** of

$$x(t) = e^{-2|t|}$$

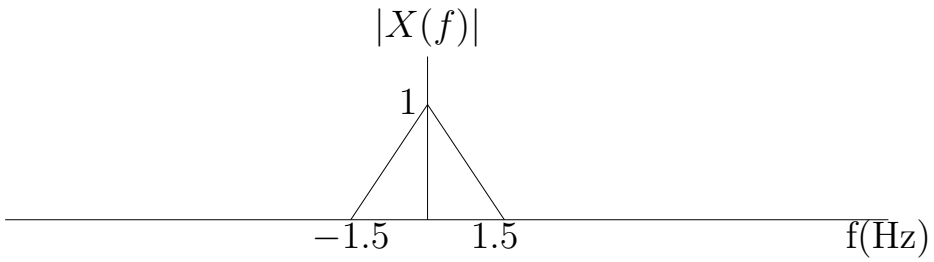
5. Consider an ideal CT-low pass filter (LPF) with a bandwidth of $f_m = 10$ (Hz). The phase response of the filter is given as $\angle H(f) = -6\pi f$ during the pass band of the filter.

   - (a) Sketch the magnitude of the frequency response of this filter, $|H(f)|$.
   - (b) Find the impulse response of this filter, $h(t)$.

6. The CTFT of a signal $x(t)$ with bandwidth $f_m = 1.5$ (Hz) is given below. The signal is sampled every $T_s = 0.5$ (s), resulting in the signal

$$x_\delta(t) = \sum_n x(nT_s)\delta(t - nT_s)$$

   Find and sketch the magnitude of the CTFT of $x_\delta(t)$. Show the important frequency and magnitude values on your graph.

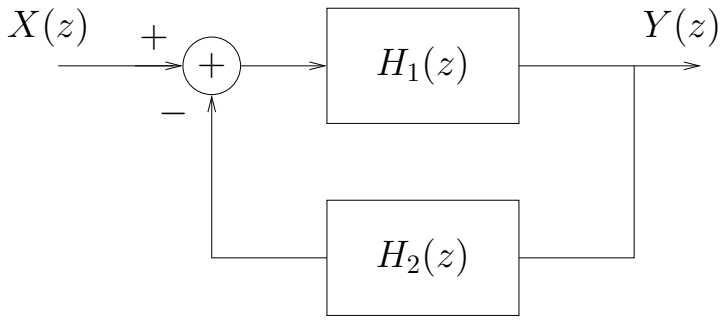


7. Using partial fraction expansion, find the closed form for the inverse z-transform of

$$X(z) = \frac{z^2}{z^2 - z + \frac{1}{4}}$$

8. Consider the DT feedback system below, where

$$H_1(z) = \frac{z}{z-1} \qquad \text{and} \qquad H_2(z) = \frac{z}{z-2}$$




- (a) Find the transfer function $H(z)$ of the overall feedback system.
- (b) Is the system stable? why?